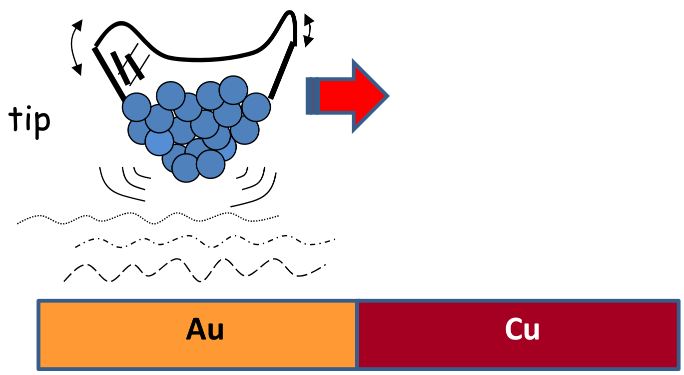
Homework No. 2

Fundamentals of AFM: Part I

Lectures: P1_Wk2_L1 - P1_Wk2_L6

Problem 1: Magnitude of vdW force

A tip of radius 5 nm is sequentially positioned 1 nm above a flat substrate that has two different regions. One region of the substrate is made from Au, the other from Cu.



Q1.1. Assuming the vdW interaction dominates the tip-substrate interaction, and assuming the vdW interaction between a tip atom and a Au atom is the same as the interaction between a tip atom and a Cu atom, which of the following statements is likely to be true?

a) The force acting on the tip when it is over the Au region will be the same as when the tip is over the Cu region

b) The force acting on the tip when it is over the Au region will be about 2 times greater than when the tip is over the Cu region

c) The force acting on the tip when it is over the Au region will be about 70% of the force when the tip is over the Cu region

d) The force acting on the tip when it is over the Au region will be about 25% of the force when the tip is over the Cu region

Correct answer is c).

Solution:

$$F = -\frac{\partial U_{vdW}}{\partial d} = -\frac{HR_{tip}}{6d^2} \qquad H = \pi^2 \rho_{tip} \rho_{metal} C_{vdW}$$

$$\frac{F_{tip-Au}}{F_{tip-Cu}} = \frac{\rho_{Au}}{\rho_{Cu}} \simeq \frac{5.9 \times 10^{28}\,m^{-3}}{8.45 \times 10^{28}\,m^{-3}} = 0.7$$

Values for the number density of Au and Cu atoms are obtained from the chart in the P1_Wk2_L1 lecture.

Problem 2: Surface energies

Highly oriented pyrolitic graphite (HOPG) is a layered form of C that is commonly used as a substrate material used in many AFM studies. It is well documented that most materials deposited on HOPG do not strongly adhere to this substrate. This must imply that

a) HOPG has a very high surface energy
b) HOPG has the remarkable ability to adjust its surface energy depending on the deposited material
c) HOPG must be an amorphous solid
d) HOPG has a very low surface energy

Correct answer is d).

Solution: If nothing sticks to HOPG, then the surface energy of HOPG must be generally lower than the surface energy of the material deposited.

Problem 3: The Derjaguin approximation

The Derjaguin approximation states that

$$F_{plane-sphere}(d) = 2\pi R_{tip}\left(\frac{U_{plane-plane}(d)}{A}\right)$$

If the sphere comes into **contact** with the plane, then $d=a_o$ and we have

$$F_{plane-sphere}(a_o) = 2\pi R_{tip}\left(2\gamma\right)$$

where γ is the surface energy of the tip and substrate (assumed in this case to be of identical materials).

Q3.1 Suppose a Si tip contacts a Si wafer. It is well known that both the tip and wafer are coated with an amorphous, native oxide layer (SiO_2, silica). What would be a good estimate for the surface energy of the tip and wafer?

a) 3 mJ/m^2
b) 30 mJ/m^2
c) 300 mJ/m^2
d) 3000 mJ/m^2

Solution

You can eliminate choice a) and d) immediately, as they are way too low and way too high. A quick search in google using keywords like "typical surface energy silica" gives a value of 340 mJ/m^2, which is displayed on the first google page.

Q3.2 If the tip has a radius of 50 nm, using the value of γ from Q3.1, estimate the adhesive force between the tip and the substrate when they come into contact?

a) 1.9 nN
b) 19 nN
c) 190 nN
d) 1900 nN

Correct answer is c)

Solution:

$$F_{plane-sphere}(a_o) = 2\pi R_{tip}\left(2\gamma\right)$$
$$= 4 \cdot 3.14 \cdot 50 \times 10^{-9} m \cdot 300 \times 10^{-3}\, J/m^2$$
$$= 1.88 \times 10^{-7} N = 190\ nN$$

Problem 4: Hertz contact mechanics

Consider the same Si tip (50 nm radius) pressed against the same flat Si wafer discussed in **Problem 3** above.

Q4.1 What is the approximate Young's modulus for silica to describe the contact mechanics in this situation?

a) 0.066 GPa
b) 0.66 GPa
c) 6.6 GPa
d) 66 GPa

Correct answer is d)

Solution: Refer to the table in lecture P1_Wk2_L4.

Q4.2 From the Hertz model, what is the contact radius when a force of 10 nN is applied to the tip? Assume the Poisson ratio of silica is 0.17.

a) 0.22 nm
b) 2.2 nm
c) 22 nm
d) 220 nm

Correct answer is b)

Solution:
$$a_{Hertz} = \left(\frac{R_{tip}F}{E_{tot}}\right)^{1/3}$$

$$\frac{1}{E_{tot}} = \frac{3}{4}\left(\frac{1-\nu_{tip}{}^2}{E_{tip}} + \frac{1-\nu_{wafer}{}^2}{E_{wafer}}\right)$$

$$= \frac{3}{4}\left(\frac{1-(0.17)^2}{66} + \frac{1-(0.17)^2}{66}\right) = 1.5\left(\frac{0.97}{66}\right) = 0.022\ GPa^{-1}$$

$$E_{tot} = 45.3\ GPa$$

$$a_{Hertz} = \left(\frac{50\times10^{-9}m\cdot10\times10^{-9}N}{45.3\ GPa}\right)^{1/3} = \left(1.10\times10^{-26}m^3\right)^{1/3}$$

$$= 2.2\ nm$$

Q4.3 From the Hertz model, estimate the deformation (indentation) that will result when 10 nN external force is applied.

a) 9.7 nm
b) 0.97 nm
c) 0.097 nm
d) 0.0097 nm

Correct answer is c).

Solution: $$D_{Hertz} = \frac{a_{Hertz}^2}{R_{tip}} = \frac{\left(2.2\times10^{-9}m\right)^2}{50\times10^{-9}m} = 9.68\times10^{-11}m = 97\ pm = 0.097\ nm$$

Q4.4 Typically, the thickness of native SiO_2 layer on Si is about 1 nm. At what applied force will the tip/substrate indentation equal this amount? Use the Hertz model.

a) 320 nN
b) 32 nN
c) 3.2 nN
d) 0.32 nN

Correct answer is a).

Solution: $$D_{Hertz} = \frac{a_{Hertz}^2}{R_{tip}} = \frac{1}{R_{tip}}\left(\frac{R_{tip}F}{E_{tot}}\right)^{2/3}$$

$$\left(D_{Hertz}R_{tip}\right)^{3/2} = \left(\frac{R_{tip}F}{E_{tot}}\right)$$

$$F = \frac{E_{tot}}{R_{tip}}\left(D_{Hertz}R_{tip}\right)^{3/2} = \frac{45.3\times10^{9}\ N/m^2}{50\times10^{-9}m}\cdot\left(1\times10^{-9}m\cdot50\times10^{-9}m\right)^{3/2}$$

$$= 9.06\times10^{17}\ N/m^3\cdot\left(3.53\times10^{-25}m^3\right)$$

$$= 3.2\times10^{-7}N = 320\ nN$$