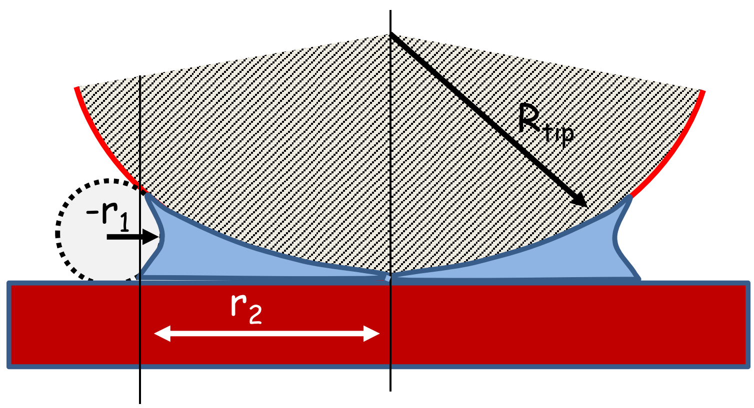
Homework No. 3

Fundamentals of AFM: Part I

Lectures: P1_Wk3_L1 - P1_Wk3_L6

Problem 1: Condensed water between the tip and substrate

A tip of radius R_{tip}= 30 nm comes into contact with a flat plane as shown below. The relative humidity of the ambient air is 40%, so a water meniscus forms as shown schematically in the figure. Assume the temperature is 300 K.



Q1.1. What determines the radii of curvature r_1 and r_2?

a) The ideal gas law
b) The Young-Laplace equation
c) The Kelvin equation
d) The First law of thermodynamics

Q1.2. If a parameter called the effective radius of curvature (r_{eff}) is defined as

$$\frac{1}{r_{eff}} = \frac{1}{r_1} + \frac{1}{r_2},$$

what is r_{eff} for the situation described above?

a) -1.76 nm
b) -0.57 nm
c) 0.3 nm
d) 1.4 nm

Correct answer is b).

Solution:

$$R_{gas} T \, \ell n\left(\frac{P^c_{vap}}{P^f_o}\right) = \gamma V_{mol}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$P^c_{vap} =$ *vapor pressure of liquid having curved surface with radii* r_1, r_2
$P^f_o =$ *vapor pressure of liquid having flat surface*
$\frac{P^c_{vap}}{P^f_o} \equiv$ *Relative Humidity*

$$\gamma \frac{V_{mol}}{R_{gas} T}\bigg|_{water} = \left(0.072 \, J/m^2\right) \cdot \left(\frac{1\,m}{10^9\,nm}\right)^2 \times \frac{\left(18\,cm^3/mole\right) \cdot \left(\frac{1\,nm}{10^{-7}\,cm}\right)^3}{8.314 \, J/K \cdot mole \cdot 300K} = 0.52\,nm$$

$$\frac{1}{r_{eff}} = \left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \frac{R_{gas} T}{\gamma V_{mol}} \ell n\left(\frac{P^c_{vap}}{P^f_o}\right) = \frac{1}{0.52} \ell n\left(\frac{P^c_{vap}}{P^f_o}\right)$$

$$r_{eff}\,(in\ nm) = \frac{0.52}{\ell n\left(\frac{P^c_{vap}}{P^f_o}\right)} = \frac{0.52}{\ell n(0.4)} = -0.57\ nm$$

negative sign specifies concave curvature

Q1.3. What equation determines the pressure difference across the liquid-vapor interface that forms between the tip and substrate?

a) the ideal gas law
b) the Young-Laplace equation
c) the Kelvin equation
d) the First law of thermodynamics

Correct answer is b).

Q1.4. The magnitude of the pressure drop across the liquid-vapor interface is

a) 2.37 GPa
b) 1.06 GPa
c) 0.36 GPa
d) 0.13 GPa

Correct answer is d).

Solution:

$$\left(P_{in}-P_{out}\right)=\gamma\left(\frac{1}{r_1}+\frac{1}{r_2}\right)=\frac{\gamma}{r_{eff}}=\frac{0.072\ J/m^2}{-0.57\times10^{-9}\,m}=-1.26\times10^{8}\ N/m^2=-0.13\ GPa$$

negative sign means $P_{out} > P_{in}$

Problem 2: VEDA simulation of a Hertz contact

Use the default values in VEDA (see table below) to simulate the interaction force of a 10 nm radius tip as a function of tip-sample gap. Select the Hertz contact model.

R_{tip}	10 nm
E_{tip}	130 GPa
v_{tip}	0.3
E_{sample}	1 GPa
v_{sample}	0.3

Q2.1. What force is required to indent the sample by 0.5 nm?

a) 0.09 nN

b) 0.4 nN

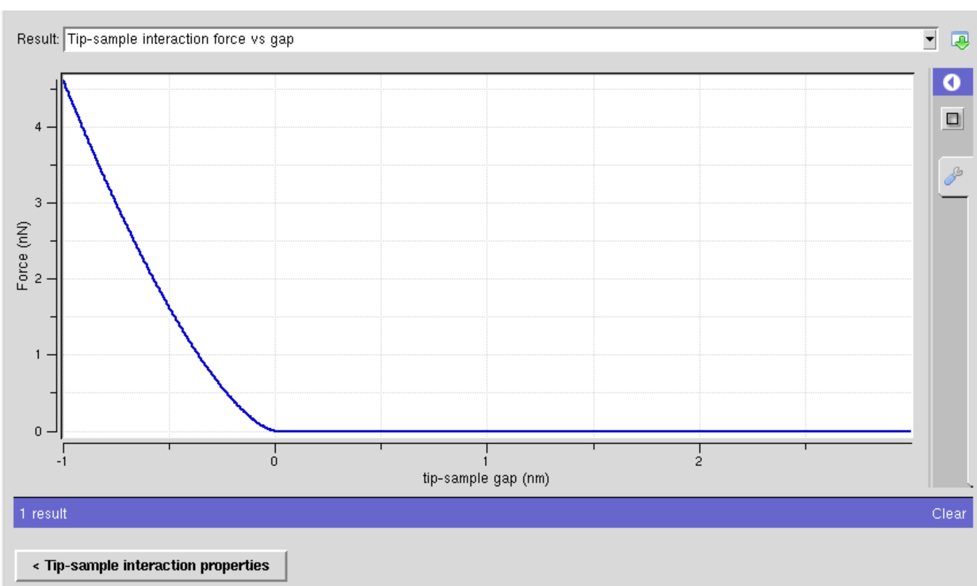
c) 0.9 nN

d) 1.6 nN

Correct answer is d).

Solution:

A screen shot of the relevant VEDA simulation is given below. From this simulation, you can measure the force required to produce a 0.5 nm indentation.



Alternatively, you can cross-check VEDA by directly calculating the force required to produce a 0.5 nm deformation. The relevant calculation is given below. Note the substrate is soft (E=1 GPa).

$$a_{Hertz} = \left(\frac{R_{tip}F}{E_{tot}}\right)^{1/3}$$

$$D_{Hertz} = \frac{a_{Hertz}^2}{R_{tip}} = \frac{1}{R_{tip}}\left(\frac{R_{tip}F}{E_{tot}}\right)^{2/3}$$

$$\frac{1}{E_{tot}} = \frac{3}{4}\left(\frac{1-\nu_1^{\;2}}{E_1} + \frac{1-\nu_2^{\;2}}{E_2}\right)$$

$$= \frac{3}{4}\left(\frac{1-(0.3)^2}{130} + \frac{1-(0.3)^2}{1}\right) = \frac{3}{4}(0.91)\left(\frac{1}{130} + \frac{1}{1}\right)$$

$$= \frac{3}{4}(0.91)(1.007) = 0.688\,GPa^{-1}$$

$$E_{tot} = 1.45\,GPa$$

$$D_{Hertz} = \frac{1}{R_{tip}}\left(\frac{R_{tip}F}{E_{tot}}\right)^{2/3} = \frac{1}{10\times10^{-9}m}\left(\frac{10\times10^{-9}m}{1.45\times10^{9}Pa}\right)^{2/3}(F)^{2/3}$$

$$= 0.5\times10^{-9} = \frac{3.62\times10^{-12}}{10\times10^{-9}}(F)^{2/3}$$

$$F = \left(\frac{0.5\times10^{-9}\cdot10\times10^{-9}}{3.62\times10^{-12}}\right)^{3/2} = \left(1.38\times10^{-6}\right)^{3/2} = 1.62\times10^{-9}N$$

Problem 3. The spring constant of a rectangular microcantilever

Whenever a dynamical system follows the rule that an applied force causes a displacement Δz from equilibrium which varies linearly with the applied force, we can write that the system can be described by the equation

$$F_{applied} = k\,\Delta z$$

where k is called the spring constant of the system.

Calculate the spring constant for a Si microcantilever that is 300x10^{-6} m long, 50x10^{-6} m wide and 1.2x10^{-6} m thick.

a) 2.8 N/m
b) 1.3 N/m
c) 0.83 N/m
d) 0.14 N/m

Correct answer is d).

Solution:

$$q = \left(\frac{4L^3}{Ewt^3}\right)F \;\Rightarrow F = \left(\frac{Ewt^3}{4L^3}\right)q \quad \Rightarrow k = \frac{Ewt^3}{4L^3}$$

$$k = \frac{Ewt^3}{4L^3} = \frac{179\,GPa \cdot 50\times10^{-6}m \cdot \left(1.2\times10^{-6}m\right)^3}{4\cdot\left(300\times10^{-6}m\right)^3} = \frac{179\times10^9 \cdot 50\times10^{-6}m \cdot \left(1.2\times10^{-6}m\right)^3}{4\cdot\left(300\times10^{-6}m\right)^3}\frac{N}{m^2}\cdot m$$

$$= 0.14\,{}^{N}\!/\!{}_{m}$$

Problem 4. A question about floor vibration

The vibration criteria VC-E states that floor vibrations should have an rms velocity less than 3 μm/s for all frequencies less than 100 Hz.

Q4.1. Assume that floor vibrations are primarily caused by different pieces of equipment scattered around the building that have moving parts that undergo cyclic motion when energized. This equipment might reasonably cause the floor to oscillate sinusoidally at some frequency f (in Hz) with a constant amplitude z_o. If this is the case, what expression might be used to approximate the floor's vibration as a function of time?

a) $z(t) = z_o (t-t_o)/f$
b) $z(t) = z_o(t-t_o)^2 f$
c) $z(t)=z_o \sin(2\pi ft)$
d) $z(t) = z_o(t-t_o)^2 \sin(2\pi ft)$

Q4.2. If a floor meets the VC-E specification, what is the corresponding amplitude of the floor vibration at 50 Hz?

a) 0.6 nm
b) 1.4 nm
c) 3.8 nm
d) 13.9 nm

Solution: If the floor oscillates sinusoidally with time, then the velocity of vibration of the floor will be given by taking a time derivative of the displacement. So if the amplitude of the floor vibration is represented by z(t), we have

$$z(t) = z_o \sin(2\pi f t)$$
$$v(t) = \frac{dz(t)}{dt} = 2\pi f z_o \cos(2\pi f t) = v_o \cos(2\pi f t)$$
$$v_{rms} = 3\ \mu m/s = \frac{v_o}{\sqrt{2}} = \frac{2\pi f z_o}{\sqrt{2}}; \quad f = 50\ Hz$$
$$z_o = \frac{\sqrt{2}\cdot 3x10^{-6}\ m/s}{2\pi f} = \frac{1.414\cdot 3x10^{-6}\ m/s}{2\pi\cdot 50/s} = 13.9\ nm$$

Problem 5. VEDA simulation of a DMT contact

Use VEDA to simulate the interaction force of a 10 nm radius tip as a function of tip-sample gap. Select the DMT contact model. Enter the values given in the table below. Under the tip-sample interaction tab, select the DMT calculation option: *"Enter Hamaker constant and intermolecular distance; autocalculate adhesion force".* This will allow you to input the Hamaker constant explicitly. The questions below can be answered by analyzing the simulation that is graphed when the "tip-sample interaction force vs. gap" is selected in the "Result:" tab.

R_{tip}	10 nm
E_{tip}	130 GPa
v_{tip}	0.3
E_{sample}	100 GPa
v_{sample}	0.3
a_o	0.2 nm
H	1×10^{-19} J

Q5.1. What is the interaction force when the tip-sample gap is 1 nm?

a) 0.6 nN, repulsive
b) 0.17 nN, attractive
c) 3.8 nN, attractive
d) 4.2 nN, attractive

Q5.2. What is the largest attractive interaction force?

a) 0.6 nN
b) 0.17 nN
c) 3.8 nN
d) 4.2 nN

Q5.3. What is the indentation when the applied force is 5 nN?

a) 0.07 nm
b) 0.13 nm
c) 0.36 nm
d) 0.42 nm

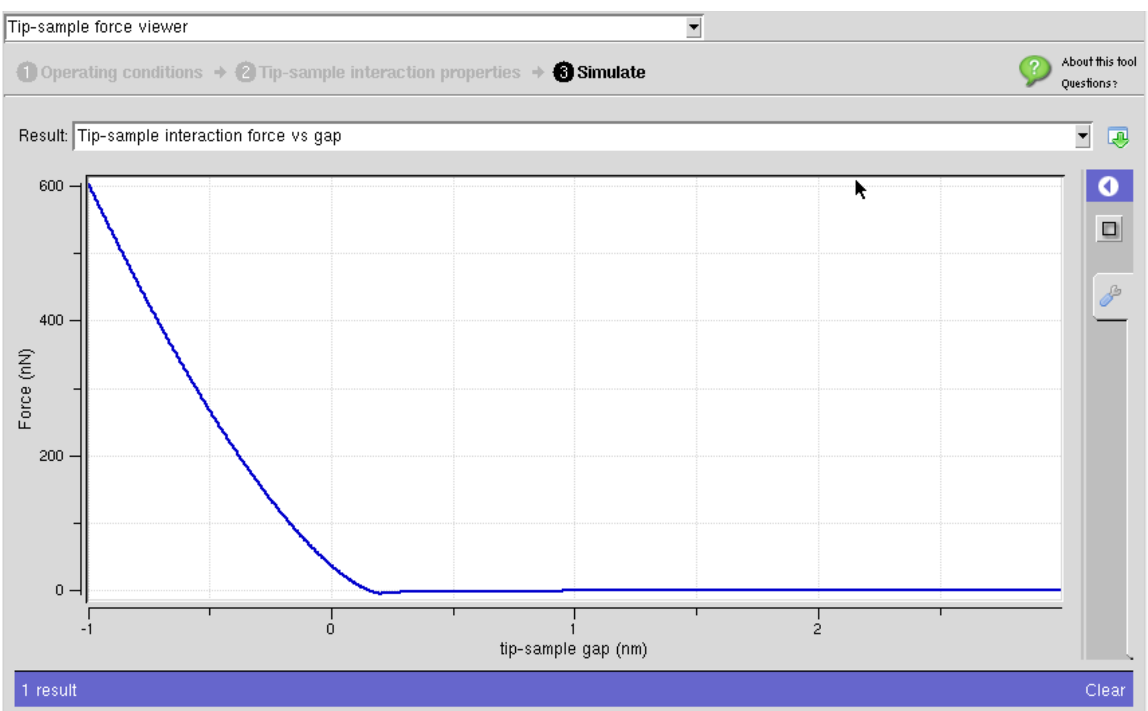
Q5.4. The indentation determined in Q5.3 is

a) mostly due to the tip being crushed when it comes into contact with the substrate
b) mostly due to a deformation of the substrate when the tip comes into contact with it
c) split roughly equally between the tip being crushed and the sample being indented
d) can't tell from the information provided

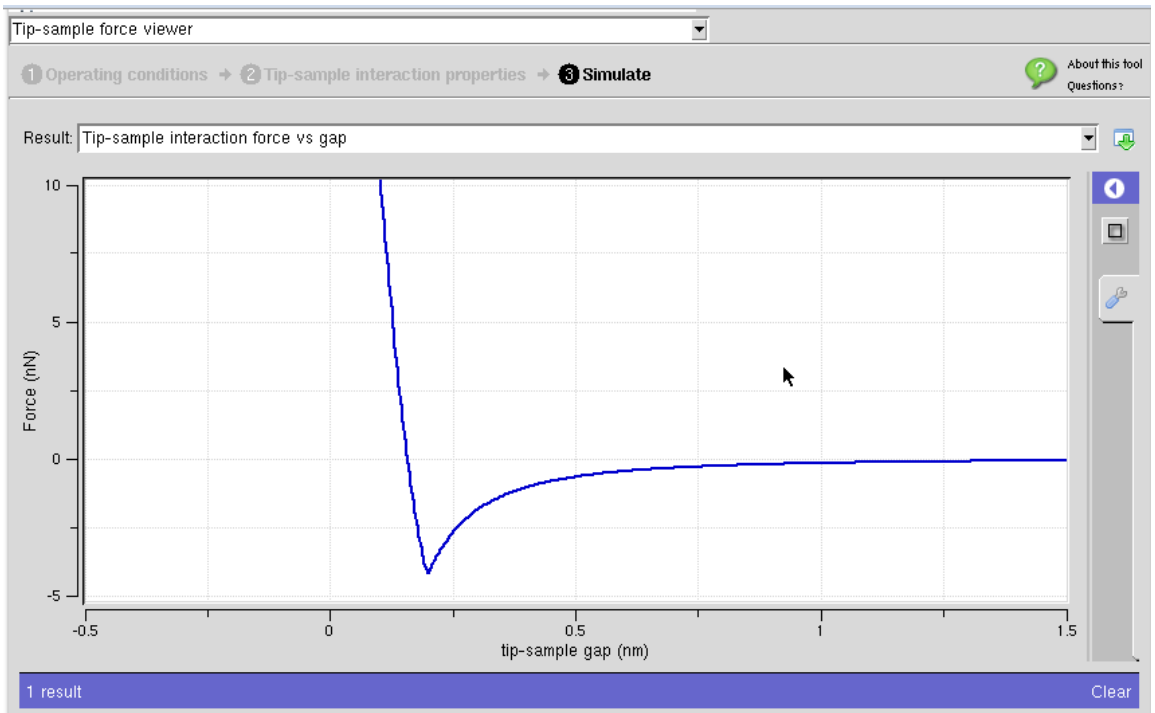
Q5.5. Before contact, how does the interaction force depend on the tip-sample gap d?

a) $F \propto d$
b) $F \propto d^{-1}$
c) $F \propto d^{2}$
d) $\mathbf{F \propto d^{-2}}$

Solution to Q5.1-Q5.5: This problem is designed to make sure you can use VEDA to answer some simple questions. After pressing the simulate button, you should see a plot similar to the screen shot given below.



From the plot provided with this simulation, it is difficult to read off the various quantities that will answer the questions that have been asked. So, you need to zoom into a narrower range of the plot to better display the region of interest. This can be accomplished by double clicking on the x and y axis and entering new limits that will narrow the plotting range. A typical plot obtained following this procedure is shown below.



From this plot, using the mouse-positioned cross-hair cursor, you can read off the answers to the questions asked above.

To answer Q5.5, perhaps it is easiest to measure the force at a few tip-substrate gaps (maybe 5) using the mouse-positioned cross-hair. Then

$$\text{if} \quad F = \text{constant} \times d^n$$

we must have

$$\log_{10}(F) = \log_{10}(\text{constant}) + n\log_{10}(d)$$

So a plot of $\log_{10}(F)$ vs. $\log_{10}(d)$ should give a straight line with a slope equal to n.

Some typical values I measured are given in the table below:

d (nm)	F (nN)	$Log_{10}(d)$	$Log_{10}(F)$
1.25	-0.106	0.097	-0.975
1.00	-0.167	0.000	-0.777
0.75	-0.295	-0.125	-0.530
0.50	-0.667	-0.301	-0.176
0.25	-2.62	-0.602	0.418

From a log-log plot, I get n=-1.99, so the correct answer to Q5.5 would be (d).

Of course this is what you might expect from the discussion of the vdW force between a tip and a flat plane in Week 2 of the lecture notes.