Homework No. 4

Fundamentals of AFM: Part I

Lectures: P1_Wk4_L1 - P1_Wk4_L6

Problem 1: Specifying the dimensions of a microcantilever

Suppose you want to design a microcantilever from Si that deflects 1 nm when 1nN of force is applied at the end.

Q1.1. Most (if not all) Si cantilevers are made from <100> oriented Si wafers. What is a reasonable estimate for the appropriate Young's modulus for cantilevers made from a <100> oriented Si wafer?

a) about 0.13 GPa
b) about 1.3 GPa
c) about 13 GPa
d) about 130 GPa

Correct answer is d).

Solution: Values for E in the <100> orientation can be found in many textbooks, on the internet, etc. The exact value depends on crystallographic orientation. For example a Young's modulus for a Si <110> wafer is 169 GPa.

Q1.2. Suppose you already have a reliable fabrication process to produce cantilevers that are 30 μm wide. What should be the ratio of the thickness to the length of the microcantilever to achieve the desired performance?

a) 0.01
b) 3.14
c) 0.12
d) 0.0092

Correct answer is a).

Solution:

$$k = \frac{Ewt^3}{4L^3} = \frac{1nN}{1nm} = 1\,N/m$$

$$\frac{t}{L} = \left(\frac{4k}{Ew}\right)^{1/3} = \left(\frac{4 \cdot 1\,N/m}{130\times10^{9}\,N/m^{2}\cdot 30\times10^{-6}\,m}\right)^{1/3} = \left(\frac{4}{3.9\times10^{6}}\right)^{1/3} = 1.01\times10^{-2}$$

Q1.3. If the minimum thickness of the cantilever you can reliably fabricate is 2 μm, what should be the length of cantilever to meet the desired specification?

a) 300 μm
b) 0.32 μm
c) 200 μm
d) 22 μm

Correct answer is c)

Solution:

$$\text{since } \frac{t}{L} = 0.01$$

$$L = \frac{t}{0.01} = \frac{2.0\times10^{-6}\,m}{0.01} = 2.0\times10^{-4}\,m = 200\,\mu m$$

Q1.4. Due to fabrication issues, if the uncertainty in the thickness of the cantilever is ±10%, what is the corresponding uncertainty in the spring constant?

a) ±40%
b) ±30%
c) ±20%
d) ±10%

Solution:

$$k = \frac{Ewt^3}{4L^3}$$
$$\frac{dk}{dt} = 3\frac{Ewt^2}{4L^3}$$
$$dk = 3\frac{Ewt^2}{4L^3}\cdot\frac{t}{t}\cdot dt = 3k\frac{dt}{t}$$
$$= 3k\left(\frac{\pm 0.1t}{t}\right) = \pm 0.3k$$

Q1.5. What is the resonant frequency (in kHz) of the cantilever?

a) 28.6 kHz
b) 10.2 kHz
c) 61.5 kHz
d) 367 kHz

Correct answer is c)

Solution:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m_{eff}}} = \frac{1}{2\pi}\sqrt{\frac{k}{0.24m}}$$
$$m = \rho_{Si}wLt$$
$$\rho_{Si} = 2329\ kg/m^3$$
$$m = 2329\ kg/m^3 \cdot 30\times10^{-6}m\cdot 200\times10^{-6}m\cdot 2\times10^{-6}m$$
$$= 2.79\times10^{-11}kg$$
$$f = \frac{1}{2\pi}\sqrt{\frac{1\ N/m}{0.24\cdot 2.79\times10^{-11}kg}} = \frac{1}{2\pi}\sqrt{\frac{1}{6.7\times10^{-12}s^2}}$$
$$= \frac{1}{2\pi}\cdot 3.86\times10^{5}Hz = 61.5kHz$$

Problem 2: Measuring the Hamaker constant

Assume a tip with radius 10 nm interacts with a flat substrate only through van der Waals forces. The tip is located at the end of a cantilever with a spring constant of 2 N/m. When the tip-substrate gap is determined to be 0.5 nm, the cantilever deflection is measured to be 0.25 nm.

Q2.1. Estimate what must be the Hamaker constant?

a) 0.5×10^{-19} J
b) 1.0×10^{-20} J
c) 2.0×10^{-19} J
d) 7.5×10^{-20} J

Correct answer is d)

Solution: For a sphere-plane, the interaction potential energy is (P1_Wk2_L1):

$$U = -\frac{HR_{tip}}{6d}$$

$$F_{tip-substrate} = -\frac{\partial U}{\partial d} = -\frac{HR_{tip}}{6d^2}$$

$$\left|F_{tip-substrate}\right| = \frac{HR_{tip}}{6d^2} = kq$$

$$H = \frac{6d^2kq}{R_{tip}} = \frac{6\left(0.5\times10^{-9}\,m\right)^2\left(2N/m\right)\left(0.25\times10^{-9}\,m\right)}{10\times10^{-9}\,m}$$

$$= \frac{7.5\times10^{-28}\,Nm^2}{10\times10^{-9}\,m} = 7.5\times10^{-20}\,J$$

Q2.2. Estimate the minimum cantilever spring constant k to avoid jump to contact? Assume a_o = 0.3 nm.

a) 1.0 N/m
b) 2.0 N/m
c) 9.3 N/m
d) 15.3 N/m

Correct answer is c)

Solution:

$$U = -\frac{HR_{tip}}{6d}$$

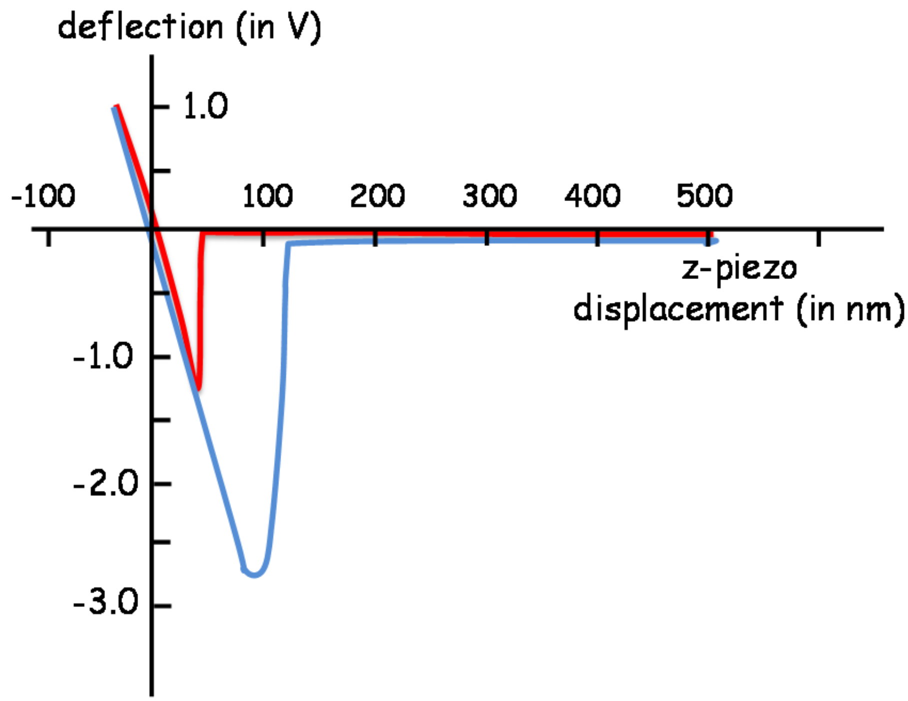
$$F_{tip-substrate} = -\frac{\partial U}{\partial d} = -\frac{HR_{tip}}{6d^2}$$

k should be greater than <u>maximum</u> slope of F(d):

$$k > \left.\frac{\partial F_{tip-substrate}}{\partial d}\right|_{maximum} = \left.\frac{HR_{tip}}{3d^3}\right|_{d=a_o} = \frac{\left(7.5\times10^{-20}\,J\right)\cdot\left(10\times10^{-9}\,m\right)}{3\left(0.3\times10^{-9}\,m\right)^3} = 9.3\ N/m$$

Problem 3. Force vs. distance experiment

Below is a plot of typical data from an AFM deflection vs. displacement experiment. Use this plot to answer the questions that follow below.



Q3.1. From an examination of this data, which curve represents the <u>approach</u> of the tip to the substrate?

a) Blue
b) **Red**

Correct answer is b)

Solution: The red curve indicates the tip jumps to contact when the z-piezo is ~ 50 nm from the substrate. This must be the approach data.

Q3.2. From an examination of this data, is there any compelling evidence for long range van der Waals forces acting between the tip and substrate?

a) Yes
b) No

Correct answer is b)

Solution: The tip jumps to contact when the z-displacement is ~50 nm from the substrate. There is very little if any cantilever deflection before the tip jumps, so this data does not provide evidence for long range van der Waals forces.

Q3.3. The zero for the z-piezo displacement is determined by

a) the location determined by the jump to contact
b) the location determined by the pull-off
c) the location when the cantilever returns to its original, undeflected position after contact with the sample
d) the location at which the set point force is reached

Correct answer is c)

Solution: By inspection, the z=0 position coincides with the condition that the cantilever deflection is zero after the tip contacts the sample.

Q3.4. Suppose this data were taken by indenting a hard tip into a hard substrate. From an examination of this data, estimate roughly the optical sensitivity of the position sensitive detector?

a) 2 nm/V
b) 5 nm/V
c) 20 nm/V
d) 50 nm/V

Correct answer is d)

Solution: By inspection, the slope of the indentation part of the data is about 1V/50 nm, so the optical sensitivity S is about 1/(1V/50nm)=50 nm/V.

Q3.5. If a cantilever with a spring constant of 0.75 N/m was used when this data was acquired, estimate the set point force.

a) 5 nN
b) 18 nN
c) 37 nN
d) 125 nN

Correct answer is c)

Solution: By inspection, the set-point of the tip occurs at a tip deflection of 1.0 V. This means the tip has deflected through approximately 1.0 V x 50 nm/V= 50 nm. With a spring constant of 0.75 N/m, this corresponds to a force F=50 nm x 0.75 N/m = 37.5 nN.

Q3.6. From an examination of this data, can you estimate the pull-off force?

a) 50 nN
b) 100 nN
c) 150 nN
d) Can't make a judgment from the data provided

Correct answer is b)

Solution: By inspection, the pull-off of the tip from the substrate occurs after the tip deflection is 2.7 V x 50 nm/V= 135 nm. With a spring constant of 0.75 N/m, this corresponds to a force F=135 nm x 0.75 N/m = 100 nN.