

Lecture: P1\_Wk2\_L6

Combining contact mechanics with  
intermolecular interactions

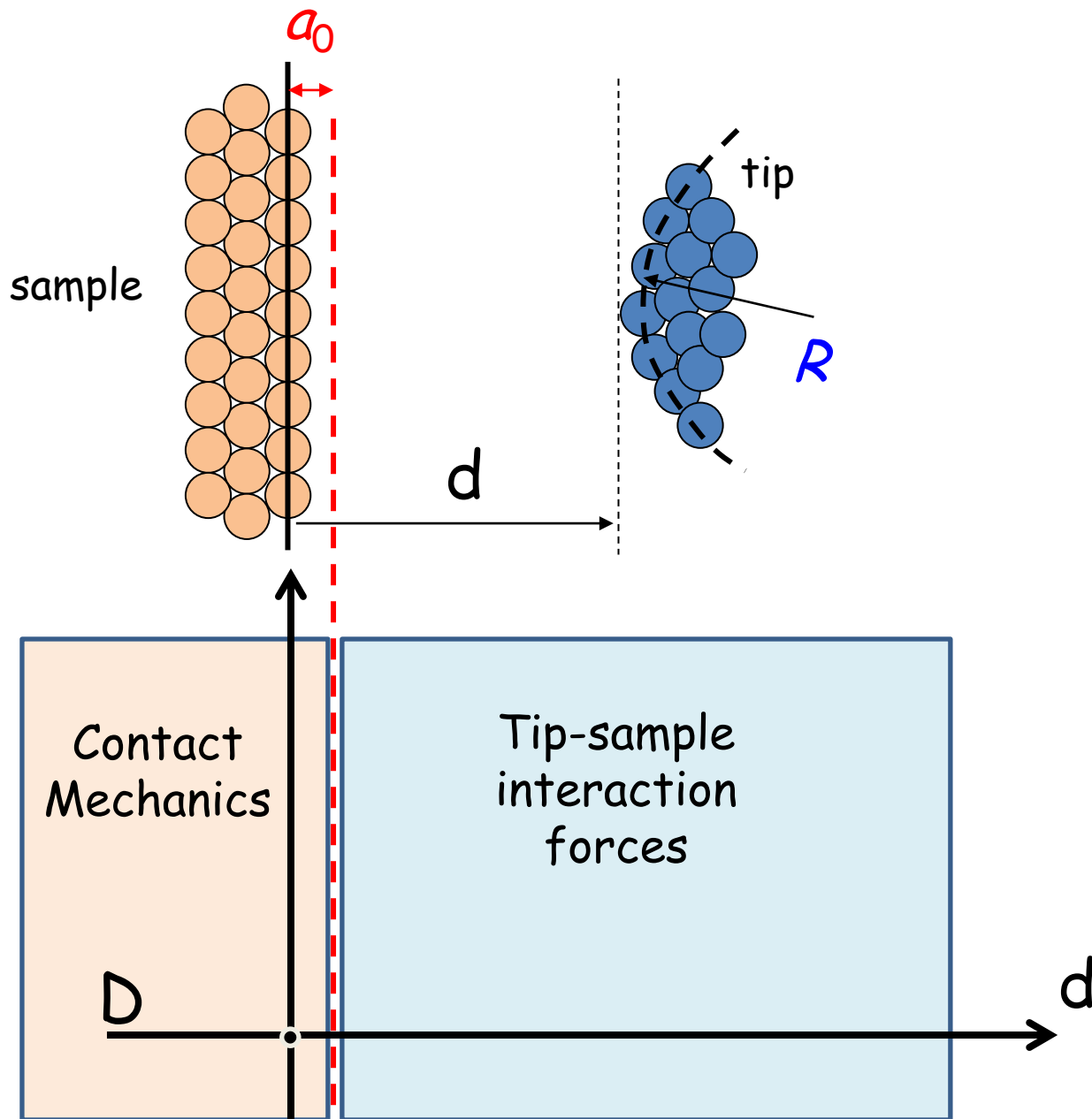
Ron Reifenberger

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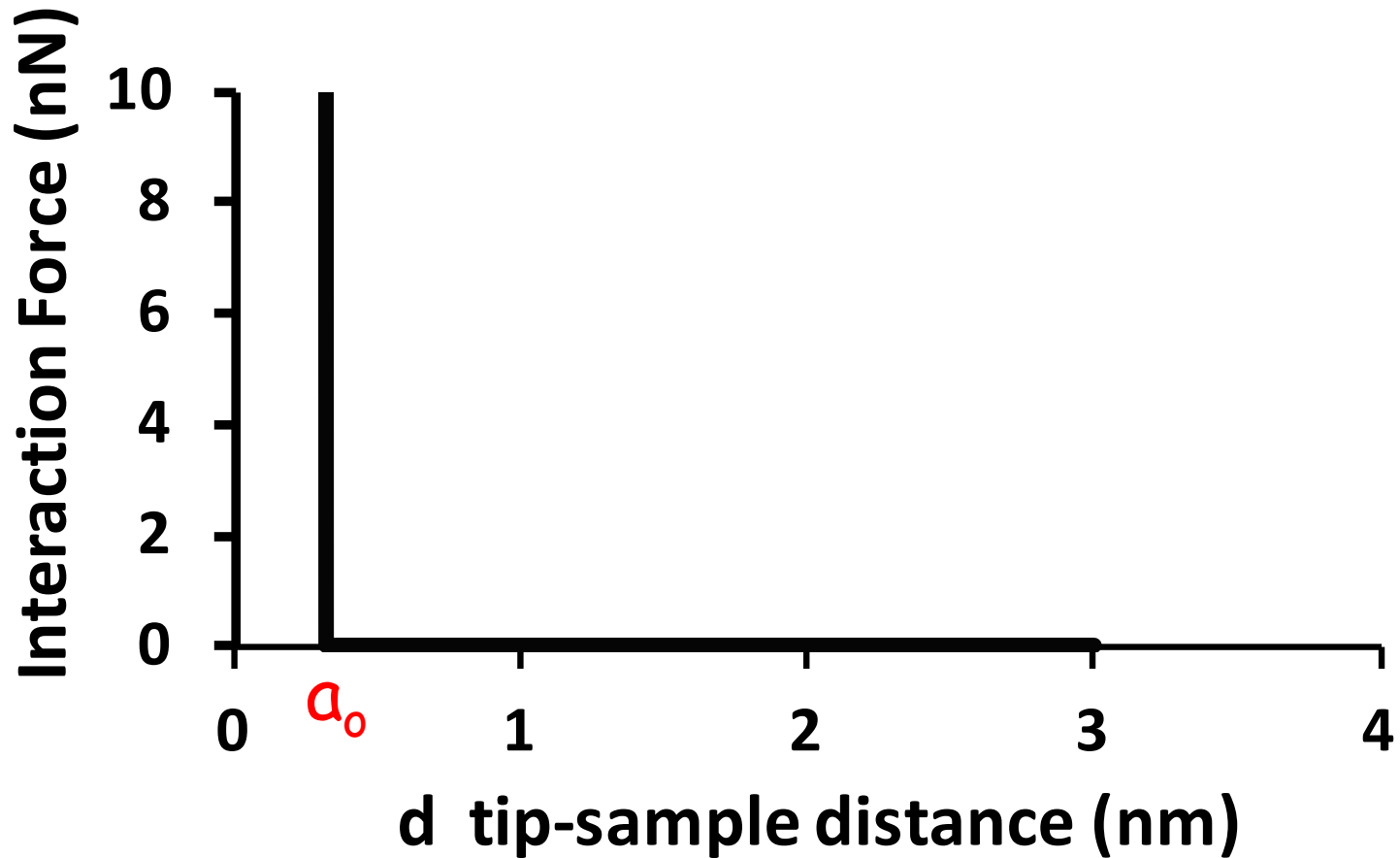
Purdue University

2012

# How to Model?



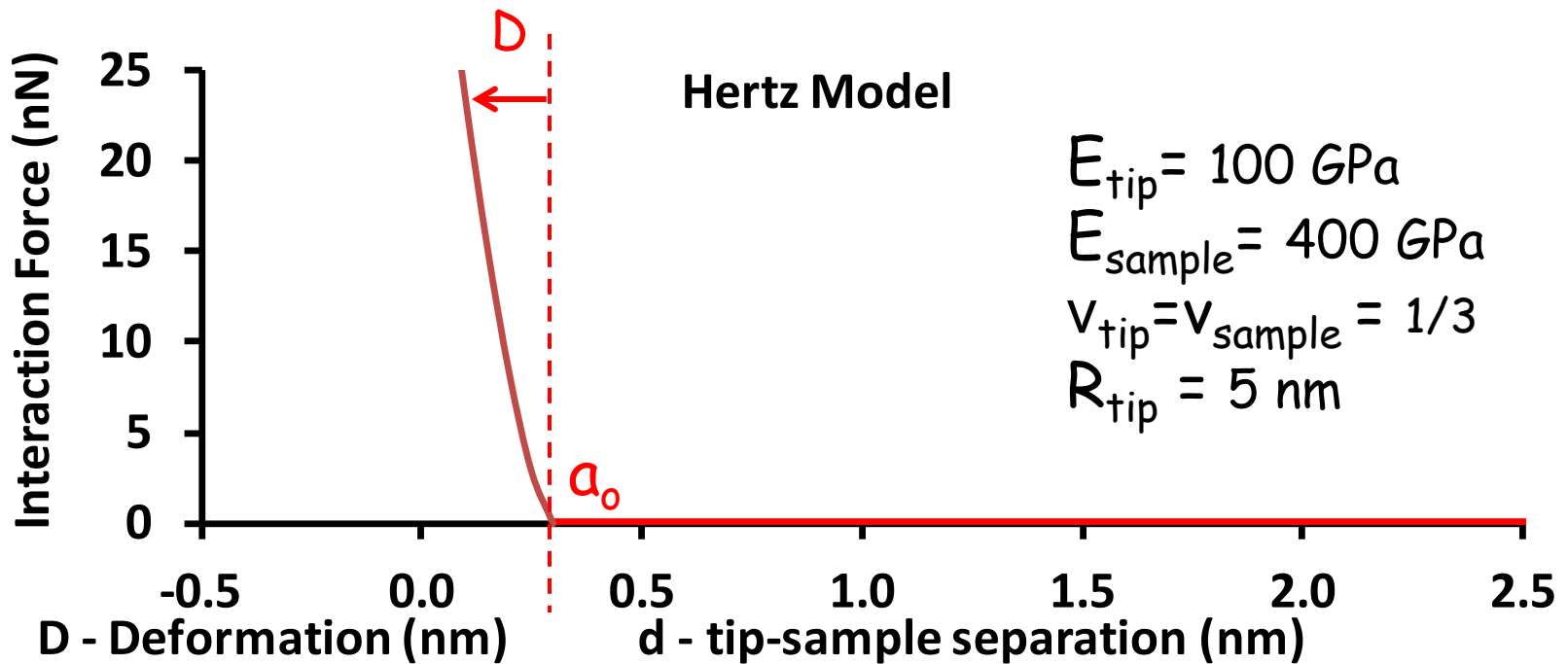
# The infinitely hard tip/sample with no surface forces



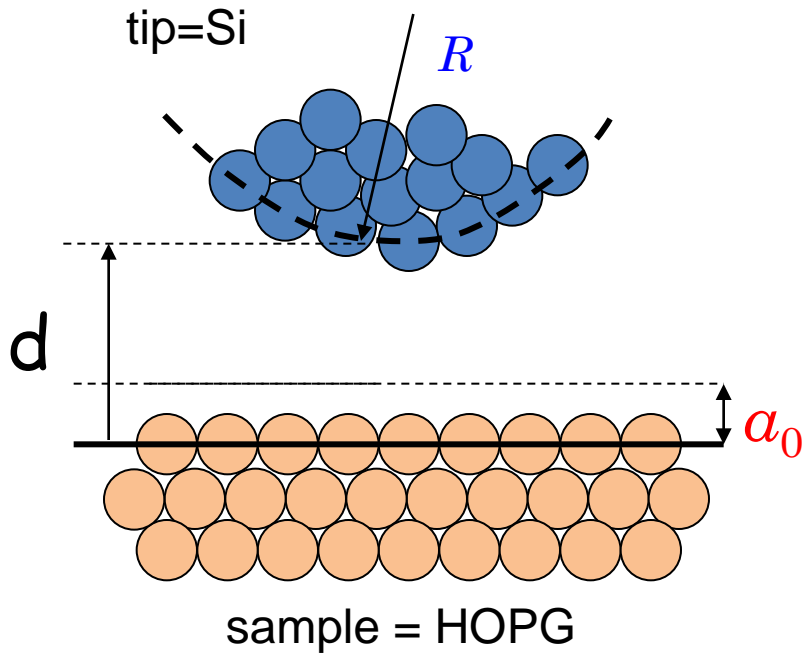
# Hertz Contact - indentation, no surface force

$$F_{\text{Hertz}}(d) = \begin{cases} 0 & d > a_0 \\ \frac{4}{3} E^* \sqrt{R} (a_0 - D)^{3/2} & D \leq a_0 \end{cases}$$

$$E^* = \left[ \frac{1 - \nu_{\text{tip}}^2}{E_{\text{tip}}} + \frac{1 - \nu_{\text{sample}}^2}{E_{\text{sample}}} \right]^{-1}$$



# Combining van der Waals force & DMT contact



$H$  : Hamaker constant (Si-HOPG)

$R$  : Tip radius

$E^*$  : Effective elastic modulus

$a_0$  : Intermolecular distance

$$F_{DMT}(d) = \begin{cases} -\frac{HR}{6d^2} & d > a_0 \\ -\frac{HR}{6a_0^2} + \frac{4}{3}E^* \sqrt{R} (a_0 - D)^{3/2} & D \leq a_0 \end{cases}$$

$$E^* = \left[ \frac{1 - \nu_{tip}^2}{E_{tip}} + \frac{1 - \nu_{sample}^2}{E_{sample}} \right]^{-1}$$

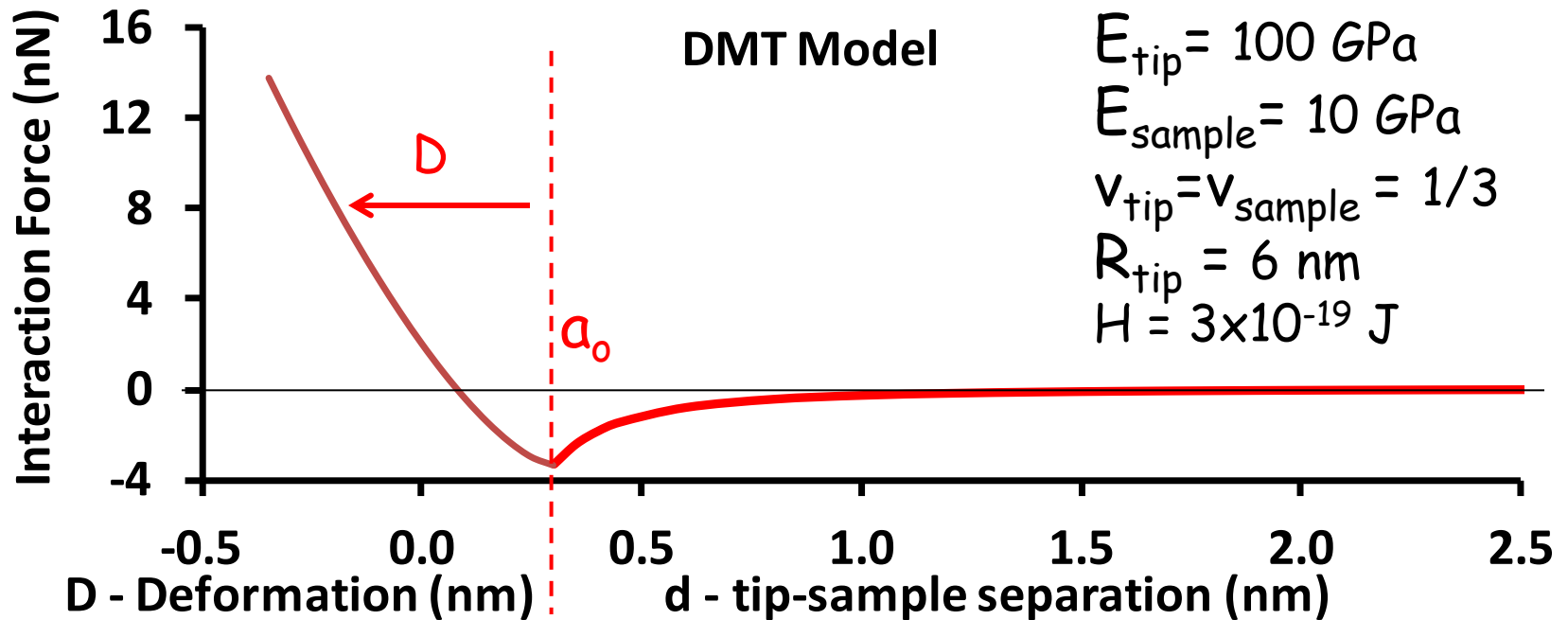
1 click

Raman et al, Phys Rev B (2002), Ultramicroscopy (2003)

# DMT Contact - indentation and surface forces

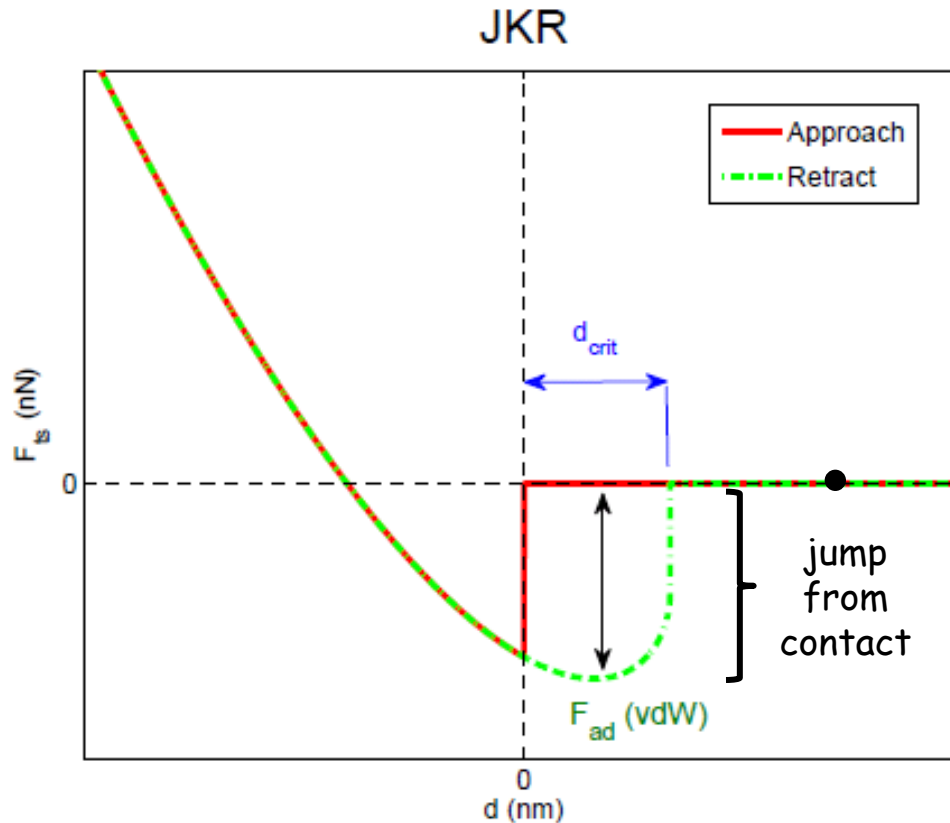
$$F_{DMT}(d) = \begin{cases} -\frac{HR}{6d^2} & d > a_o \\ -\frac{HR}{6a_o^2} + \frac{4}{3}E^* \sqrt{R}(a_o - D)^{3/2} & D \leq a_o \end{cases}$$

$$E^* = \left[ \frac{1 - \nu_{tip}^2}{E_{tip}} + \frac{1 - \nu_{sample}^2}{E_{sample}} \right]^{-1}$$



# JKR Contact

$$F_{jkr}(d) = \begin{cases} 0, & \text{Approaching, } d > 0 \\ \frac{4E^*a^3}{3R} - \sqrt{8\pi W_{jkr}E^*a^3}, & \text{(Approaching and } d < 0) \text{ or (Retracting and } d < d_{crit}) \\ 0, & \text{Retracting, } d > d_{crit} \end{cases}$$

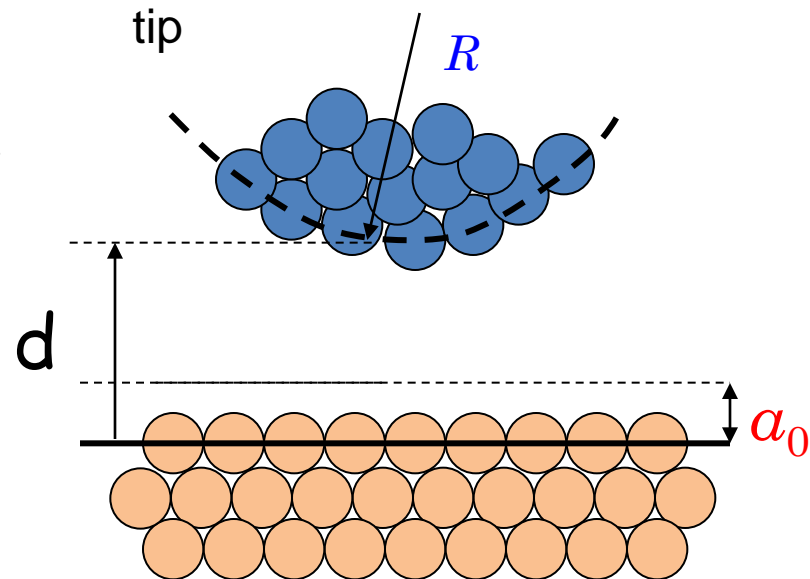


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# The model you choose must fit your experiments

## VEDA Models:

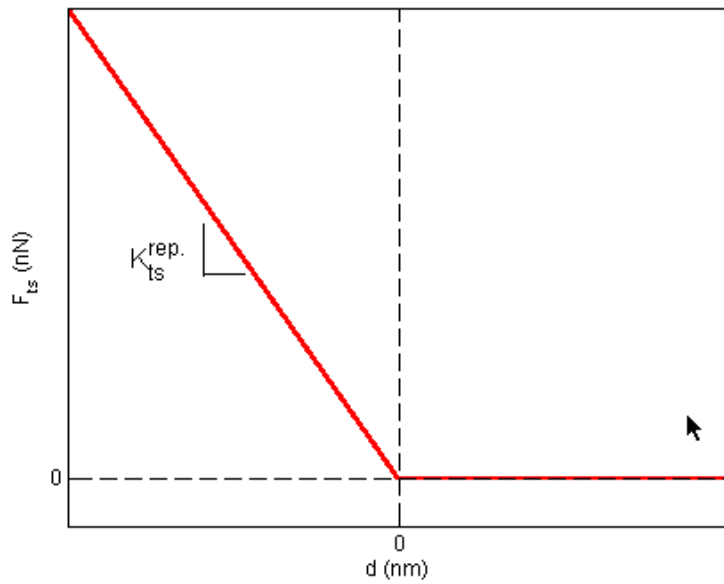
- Linear contact
- Linear attractive/repulsive
- Hertz contact
- DMT contact
- DMT + DLVO interactions (liquids)
- JKR
- Chadwick
- vdW + Morse potential
- vdW + Lennard Jones potential
- Electrostatic force - non contact
- Magnetic dipole



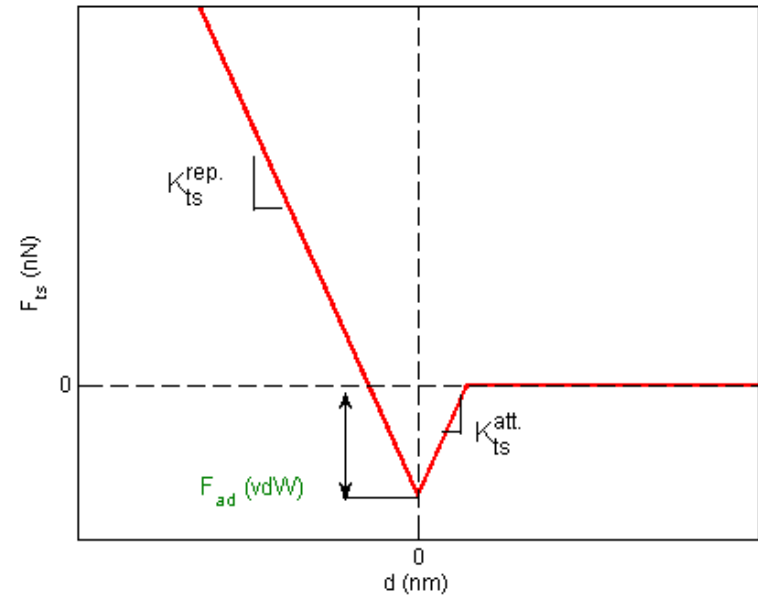


# Plots of a few VEDA models

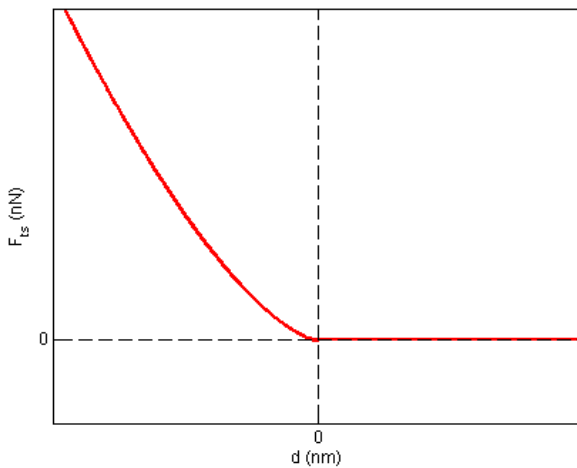
## Linear Contact



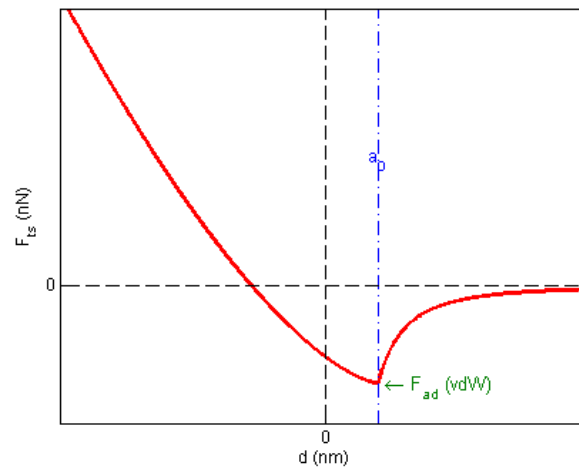
## Linear Attractive/Repulsive



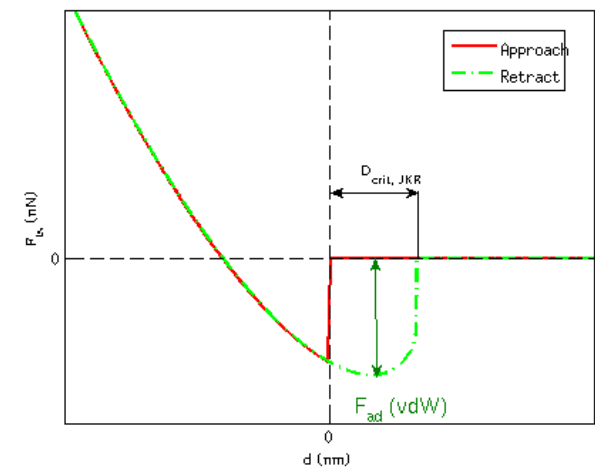
## Hertz



## DMT



## JKR



**Week 3: Brief introduction to VEDA plus  
discussion of AFM Instrumentation**