Fundamentals of Atomic Force Microscopy
Part 2: Dynamic AFM Methods

Week 2, Lecture 4
Connections between amplitude reduction, phase, frequency shift, and excitation

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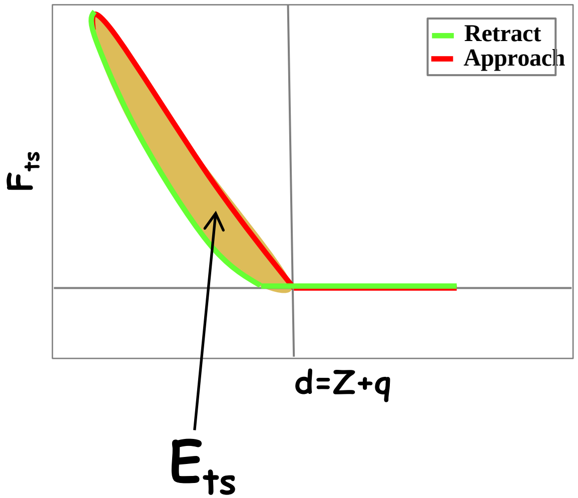
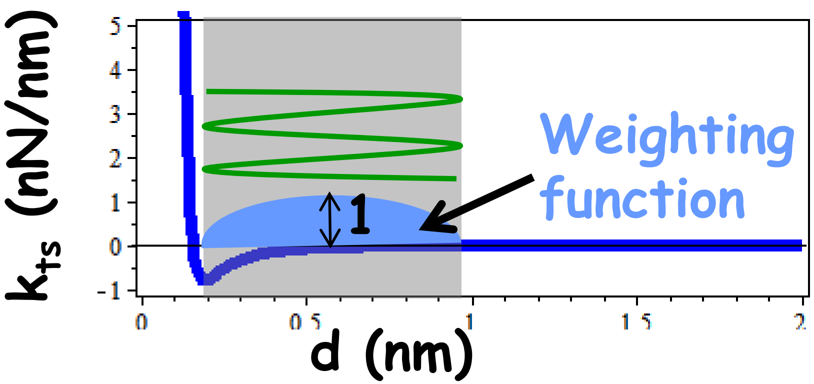
From the last lecture

$$V_{ts}(Z,A) = \left\langle F_{ts}^{CONS} \bullet q \right\rangle = \frac{kA^2}{2}\left(1 - \frac{\omega^2}{\omega_0^2}\right) - \frac{AF_0}{2}\cos\phi \qquad \text{("Virial")} \qquad (1)$$

$$E_{ts}(Z,A) = -\int_0^{2\pi/\omega_0} F_{ts}^{DISS} \bullet \dot{q}\, dt = \pi A F_0 \sin\phi - \pi k A^2 \frac{\omega}{Q\omega_0} \qquad \text{("Dissipation")} \qquad (2)$$





Connections between amplitude reduction, frequency shift and dissipation

Eliminating ϕ from (1) and (2) (see Appendix) we arrive at

$$\frac{kA}{F_0} = \frac{1}{\sqrt{\left(\frac{\omega'^{\,2}_0 - \omega^2}{\omega_0^{\,2}}\right)^2 + \left(\frac{\omega}{\omega_0 Q'}\right)^2}} \qquad (3)$$

$$\tan(\phi) = \frac{\left(\frac{\omega}{\omega_0 Q'}\right)}{\left(\frac{\omega'^{\,2}_0 - \omega^2}{\omega_0^{\,2}}\right)} \qquad (4)$$

where

$$\omega'_0 = \omega_0 - \omega_0 \frac{V_{ts}(Z,A)}{kA^2} \qquad (5)$$

$$\frac{1}{Q'} = \frac{1}{Q} + \frac{\omega_0 E_{ts}}{\omega \pi k A^2} \qquad (6)$$

In the limit of infinitesimal amplitude, recall $V_{ts}(Z,A) \sim \frac{1}{2} k_{ts}(Z) A^2$,

leading to a classical result of the linearized analysis:

$$\omega'_0 = \omega_0 \left(1 - \frac{k_{ts}}{2k}\right)$$

AM-AFM or Tapping mode

Assume $\omega = \omega_0$, and F_0 is constant

Recall, $F_0 = kA^{far} / Q$ when $\omega = \omega_0$

where A^{far}, $\phi^{far} = \pi / 2$ are the amplitude and phase far from the surface

the equations reduce to

$$V_{ts}(Z,A) = -\frac{kAA^{far}}{2Q}\cos\phi \qquad (7)$$

$$E_{ts}(Z,A) = \frac{\pi kAA^{far}}{Q}\sin\phi - \frac{\pi kA^2}{Q} \qquad (8)$$

$$\frac{A}{A^{far}} = \frac{1/Q}{\sqrt{\left(-2\frac{V_{ts}(Z,A)}{kA^2}\right)^2 + \left(\frac{1}{Q} + \frac{E_{ts}(Z,A)}{\pi kA^2}\right)^2}} \qquad (9)$$

$$\tan(\phi) = \frac{\left(\frac{1}{Q} + \frac{E_{ts}(Z,A)}{\pi kA^2}\right)}{\left(-2\frac{V_{ts}(Z,A)}{kA^2}\right)} \qquad (10)$$

$\frac{A}{A^{far}}$ called amplitude setpoint ratio

AM-AFM or tapping mode

- While scanning Z adjusts so that E_{ts} and V_{ts} reach values where the desired amplitude reduction is achieved
- Topography is the set of those Z values which achieve a constant amplitude by adjusting E_{ts} and V_{ts} pointwise on the sample
- In tapping mode (AM-AFM) Ets and Vts are n ot independent, rather they are related through Eqs. (9, 10)
- Phase lag depends both on conservative and dissipative interactions

FM-AFM

ω changes to keep phase fixed

From (4)

$$\omega = \omega'_0 = \omega_0 - \omega_0 \frac{V_{ts}(Z,A)}{kA^2} \text{ !!!} \qquad (11)$$

and F_0 changes to keep A fixed

From (3)

$$F_0 = kA\frac{\omega}{\omega_0 Q'} = kA\frac{\omega}{\omega_0}\left(\frac{1}{Q} + \frac{\omega_0 E_{ts}}{\omega \pi k A^2}\right) \qquad (12)$$

And Z changes to keep a constant frequency shift

$$\triangle\omega_0 = \omega'_0 - \omega_0 = -\omega_0 \frac{V_{ts}(Z,A)}{kA^2} \qquad (13)$$

FM-AFM

- Topography is set of those values of Z that keep V_{ts}/A^2 constant
- Drive frequency ω tracks effective natural frequency ω'_0
- F_0 picks up the dissipative interactions

Appendix

Starting with

$$V_{ts}(Z,A)=\frac{kA^2}{2}\left(1-\frac{\omega^2}{\omega_0^2}\right)-\frac{AF_0}{2}\cos\phi \qquad (\text{"Virial"}) \qquad (14)$$

$$E_{ts}(Z,A)=\pi AF_0\sin\phi-\pi kA^2\frac{\omega}{Q\omega_0} \qquad (\text{"Dissipation"}) \qquad (15)$$

we can rewrite (14) and (15) as

$$\sin\phi=\frac{kA}{F_0}\frac{\omega}{\omega_0}\left(\frac{\omega_0}{\omega}\frac{E_{ts}(Z,A)}{\pi kA^2}+\frac{1}{Q}\right) \qquad (16)$$

$$\cos\phi=\frac{kA}{F_0}\left(\frac{\omega_0^2\left(1-\frac{2}{kA^2}V_{ts}(Z,A)\right)-\omega^2}{\omega_0^2}\right) \qquad (17)$$

Define

$$\frac{1}{Q'}=\frac{1}{Q}+\frac{\omega_0}{\omega}\frac{E_{ts}(Z,A)}{\pi kA^2} \qquad (18) \qquad \text{and} \qquad \left(\omega'_0\right)^2=\omega_0^2\left(1-\frac{2}{kA^2}V_{ts}(Z,A)\right) \qquad (19)$$

If $\frac{V_{ts}(Z,A)}{kA^2}<<1$ then

$$\omega'_0=\omega_0\left(1-\frac{V_{ts}(Z,A)}{kA^2}\right)$$

Eliminating j from (16) and (17) and using (18) and (19) we arrive at

$$\frac{kA}{F_0}=\frac{1}{\sqrt{\left(\frac{\omega'^2_0-\omega^2}{\omega_0{}^2}\right)^2+\left(\frac{\omega}{\omega_0Q'}\right)^2}} \qquad (20)$$

$$\tan(\phi)=\frac{\left(\frac{\omega}{\omega_0Q'}\right)}{\left(\frac{\omega'^2_0-\omega^2}{\omega_0{}^2}\right)} \qquad (21)$$