Principles of Electronic Nanobiosensors

Unit 2: Settling Time

Lecture 2.1: Shape of a Surface

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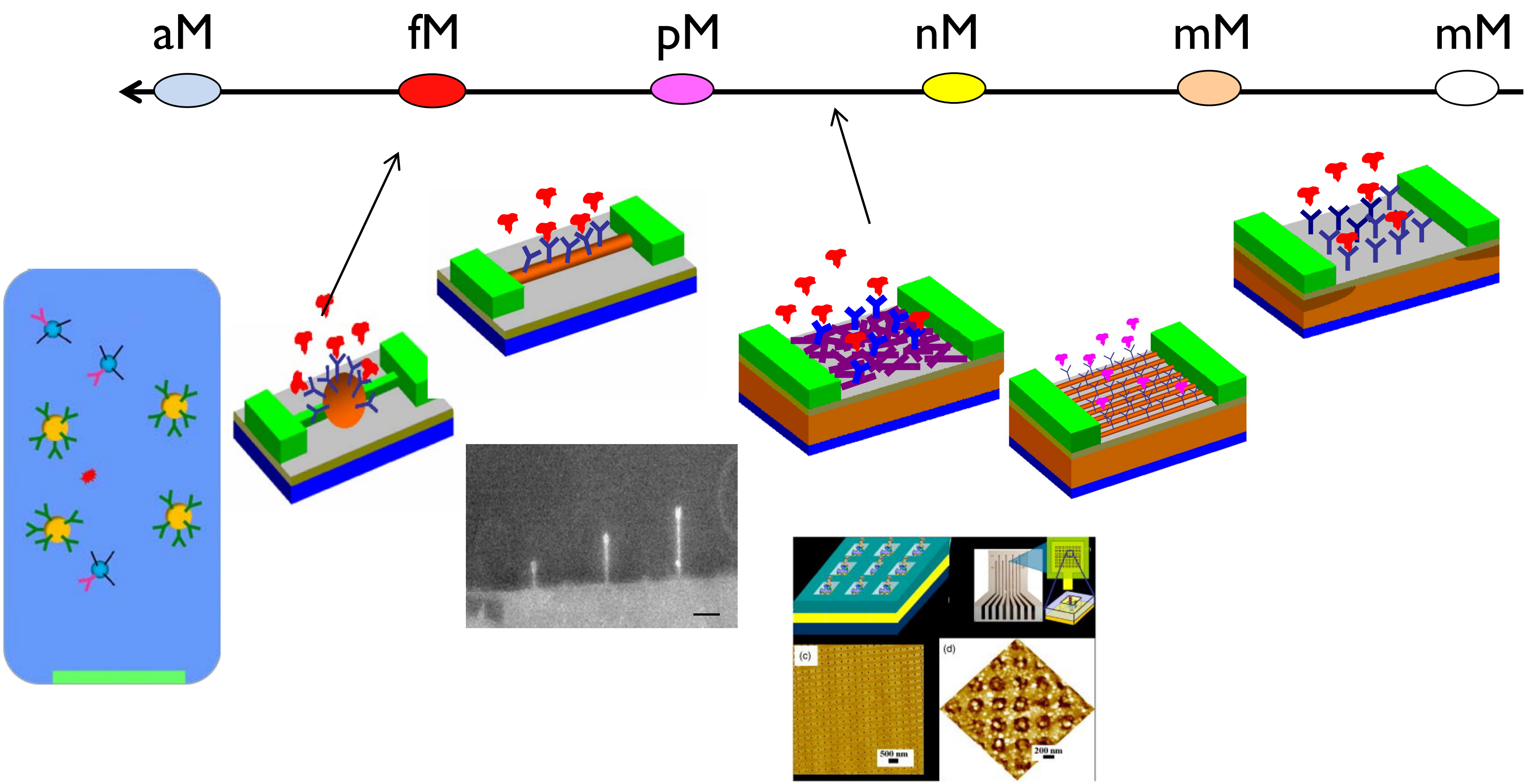
Outline

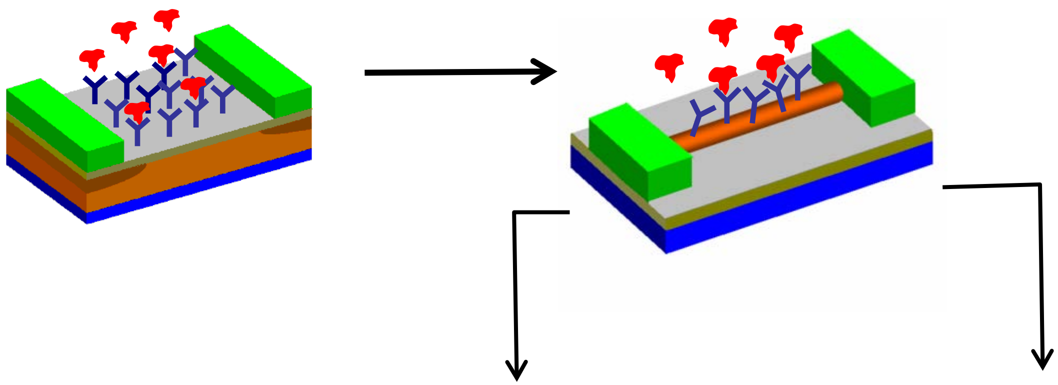
- Recap of lecture 1.3
- Shape of a object described by fractals
  - Regular fractals
  - Irregular fractals
- Outline of the course
- Summary and conclusion

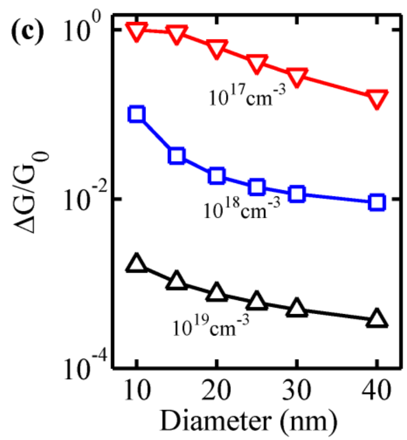
Nanobiosensors have high sensitivity



Is there something fundamental about the geometry?

Geometry is Important, but exactly how ?





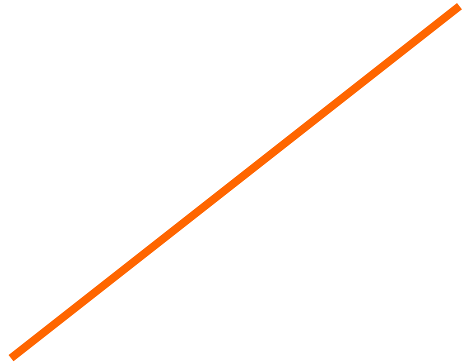
Berg, 1963

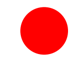
Geometry of electrostatics

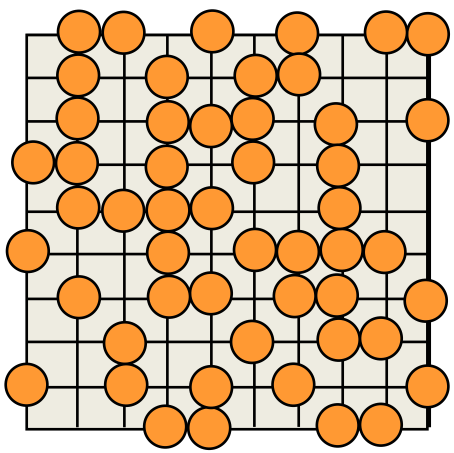
Geometry of diffusion

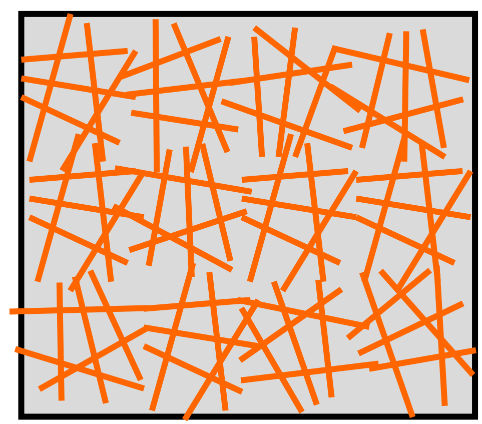
Basic concepts: dimension of a surface

D=2 D=1 D=0









D=?

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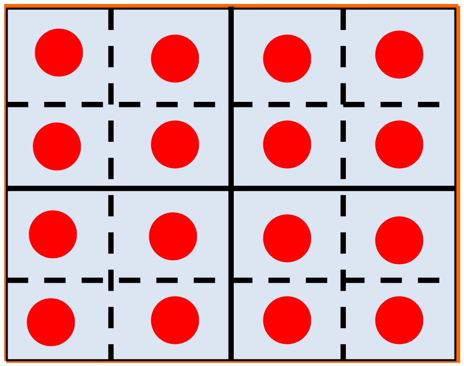
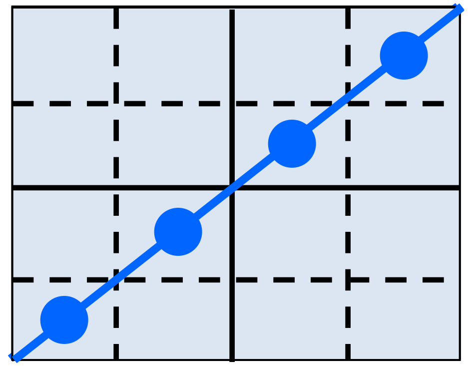
Classification of surfaces

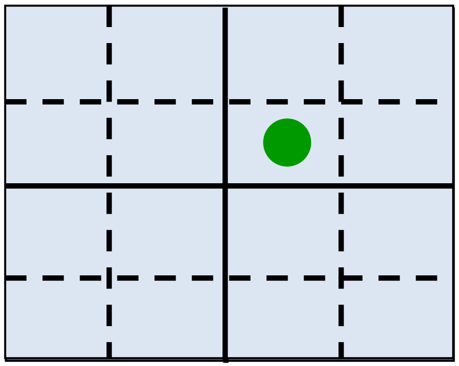
Fractal Dimension (D_F)- Box counting technique



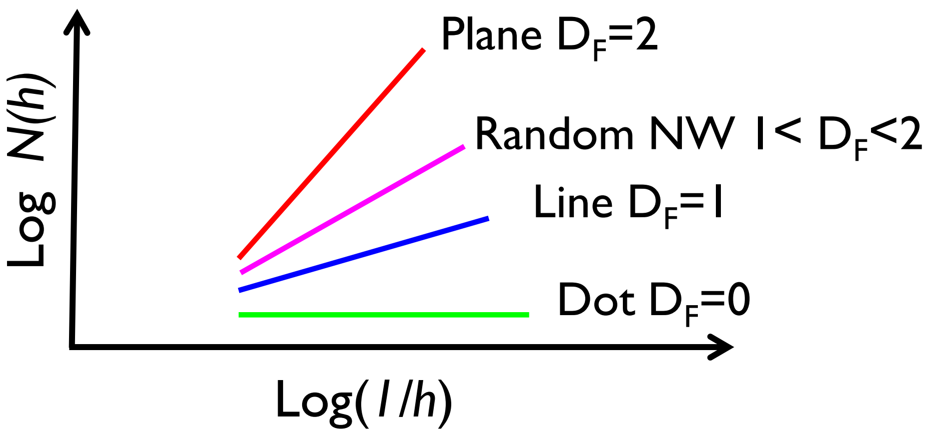
$N(h) \sim h^2$



$N(h) \sim h^1$



$N(h) \sim h^0$



Example: Regular 1D fractals

h	1/3
N	2

h	1/9
N	4

h	1/27
N	8

h	$1/3^n$
N	2^n

$$D_{F,1} = \frac{\log(N)}{\log(1/h)} = \frac{\log(2^n)}{\log(3^n)} = 0.63$$

Bigger than a point, but smaller than a line

In general, $D_{F,1} = \dfrac{\log(m)}{\log(n)}$

…. keep m piece

of n pieces

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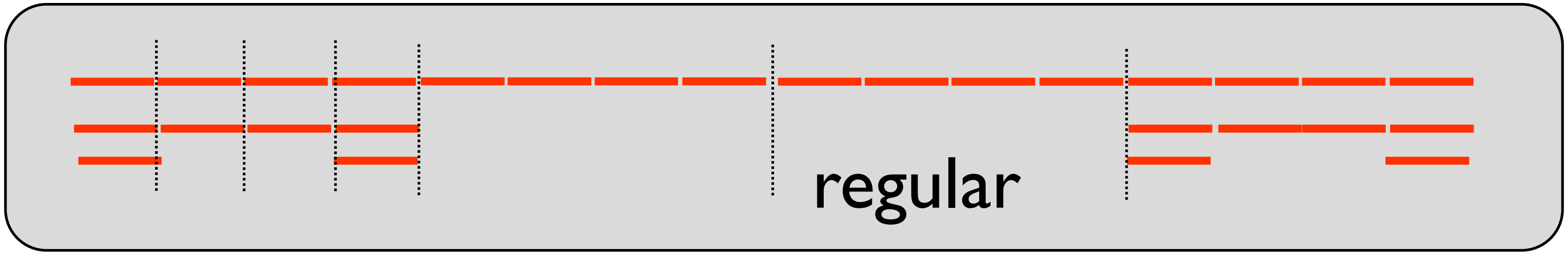
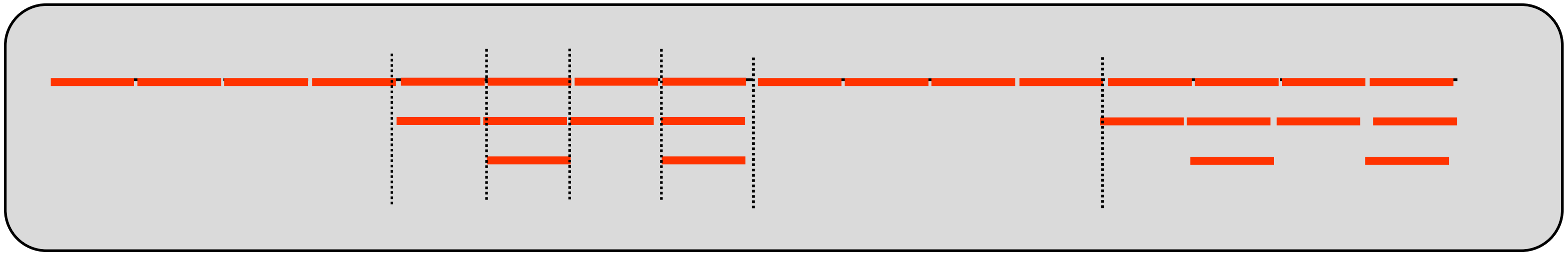
Regular and irregular 1D fractals

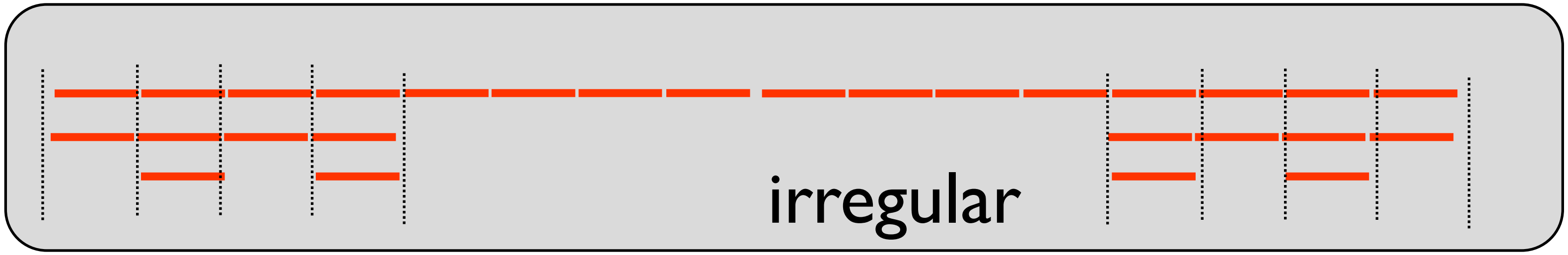
$$D_{F,1} = \frac{\log(m)}{\log(n)}$$

…. keep m piece

of n pieces







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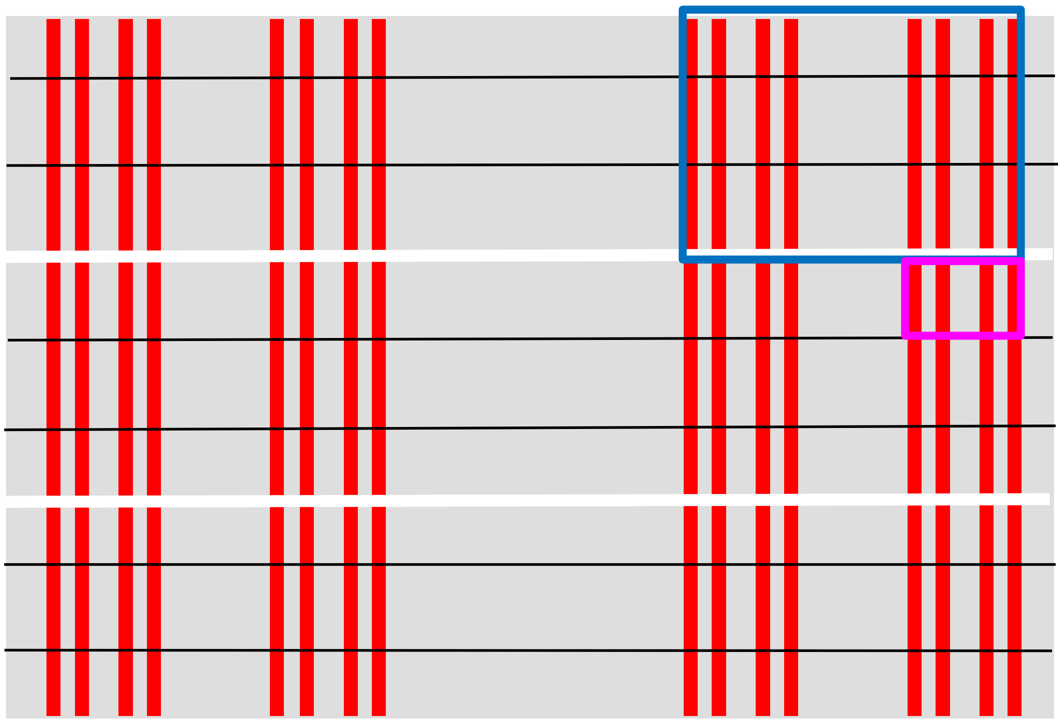
Dimension of quasi-2D Fractal



h	1/3
N	6

h	1/9
N	36

h	1/27
N	216

h	$1/3^n$
N	$3^n 2^n$

$$D_{F,2} = \frac{\log(N)}{\log(1/h)} = \frac{\log(3^n) + \log(2^n)}{\log(3^n)} = 1 + \frac{\log(2^n)}{\log(3^n)} = 1 + DF_x$$

In general, $D_{F,2} = DF_x + DF_y$

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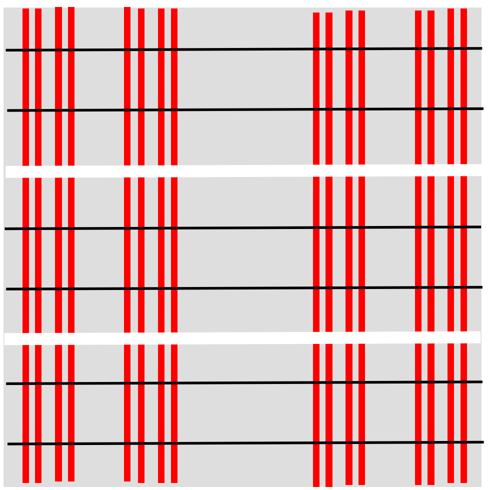
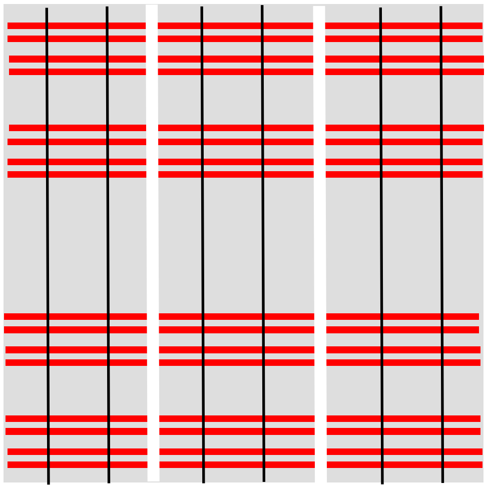
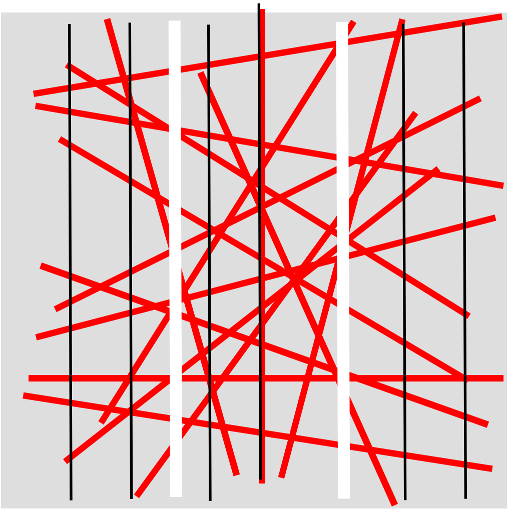
Same DF, but different geometry



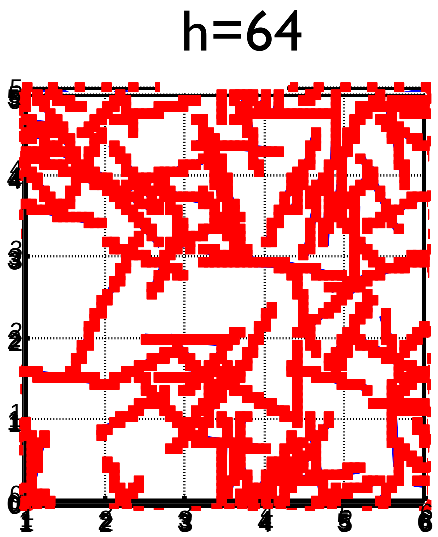


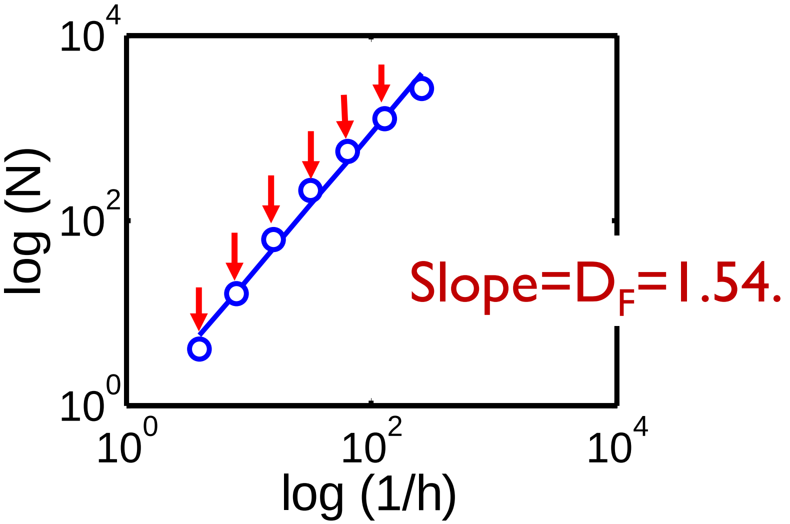


same dimension,
because DF=1+log(m)/log(n)

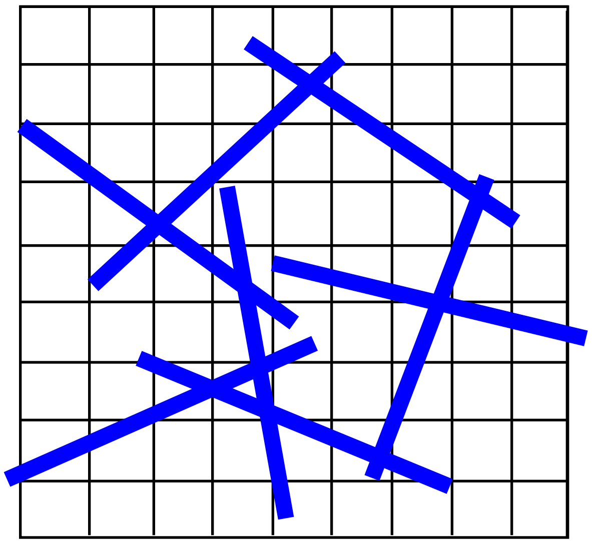
What about this irregular fractal?

Dimension of a irregular fractal surface

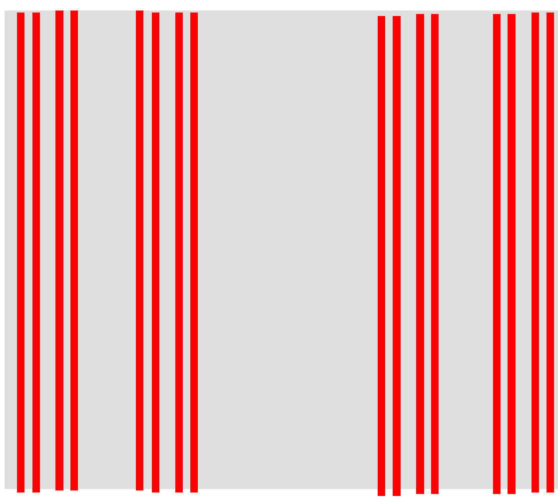




Irregular to regular surfaces



$D_{F,CT} = D_{F,stick}$



$D_{F,CT} = 1 + \log(m)/\log(n)$

For $D_{F,stick} = 1.5$

Let m=2, solve for n:

$\log(n) = \log(2)/(D_{F,stick} - 1)$

Result: n=4

Generation algorithm:

Take a line segment

Remove the fraction (n-2)/n

from its centre (result: ½)

repeat …

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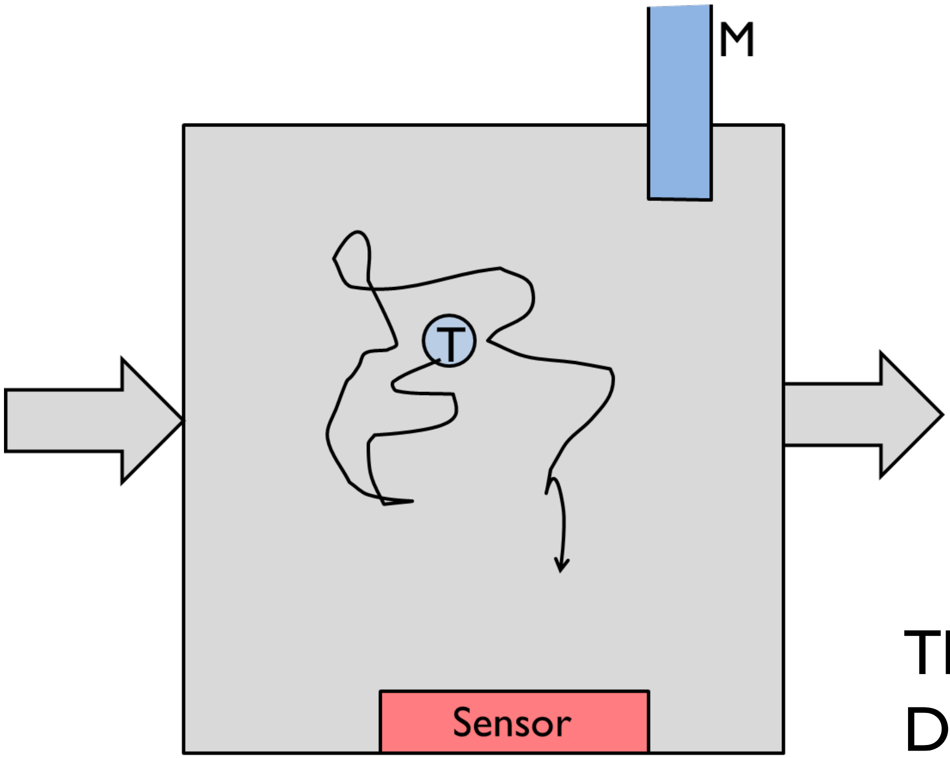
Settling time defines the fundamental limits of detection (Lectures 5-10)



Heisenberg principle

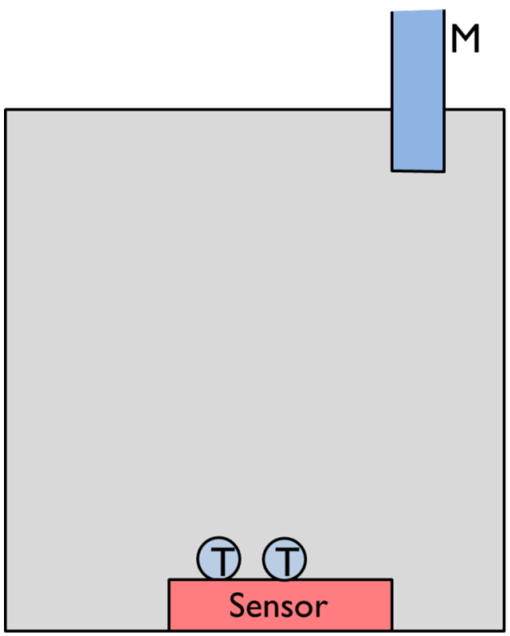
$$\Delta E \times \Delta t \sim \hbar$$

Settling time principle

$$\rho_0 \times t_s^{(3-D_F)/2} \sim c$$

The result is technology agnostic
Defined by the geometry of diffusion

Sensitivity defines transducer-specific limits of detection (Lectures 11-22)



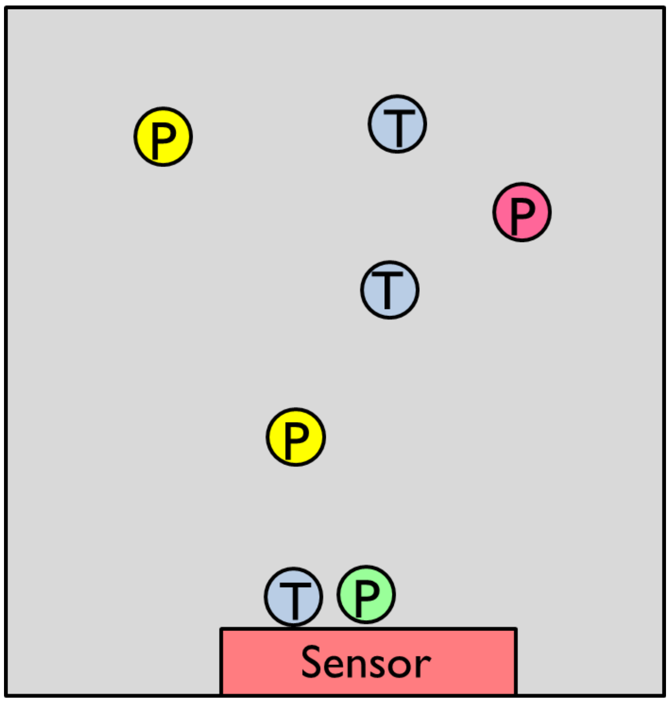
Mass, charge, and electron affinity – based biosensors

Noise limits of surrounding media

Geometry of the sensor

Defined by the geometry of screening

Selectivity defines the practical limits of detection (Lectures 22-30)



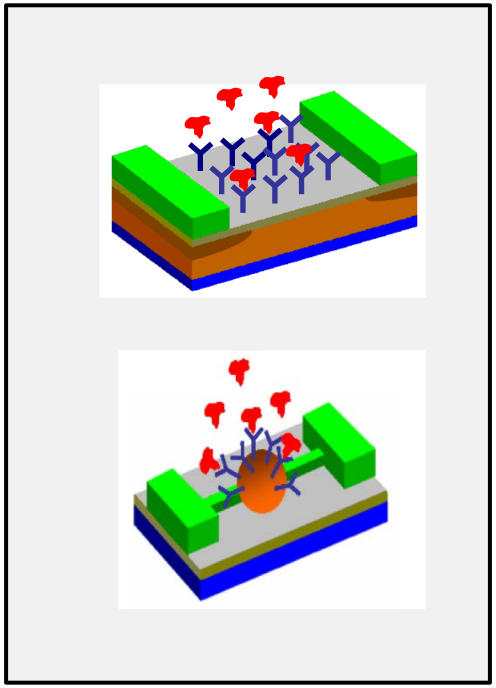
Selectivity defines the ability to differentiate between noise and signal

Related to fundamental issues of random sequential absorption

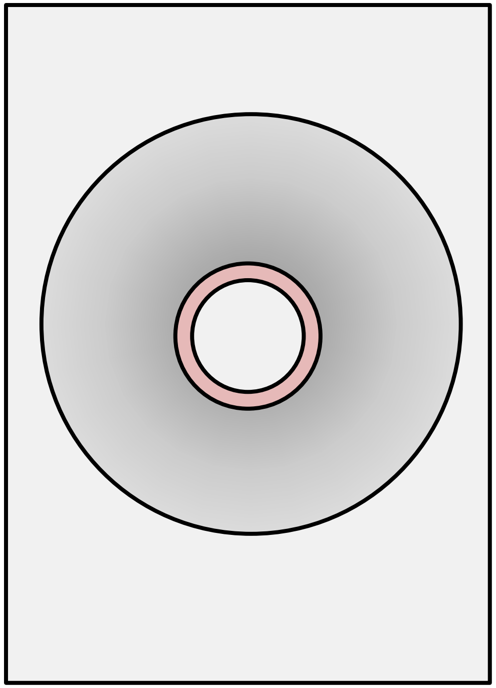
Technology-agnostic sensor metric

Geometry is the key for nanobiosensing

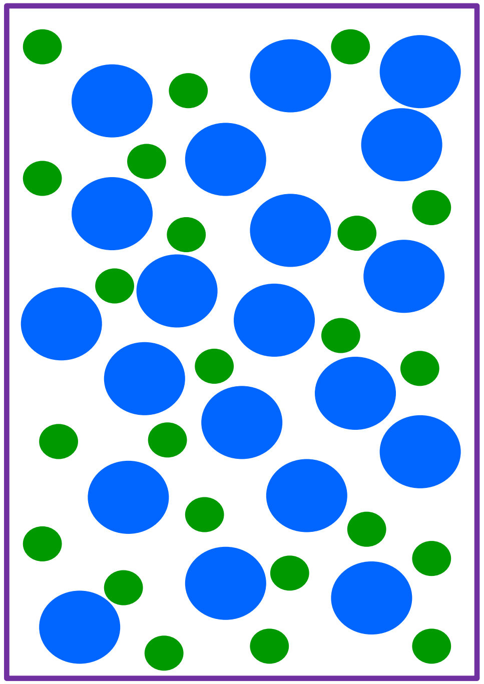
Geometry of diffusion for theory of selectivity



Geometry of screening for sensitivity



Geometry of random sequential adsorbtion for selectivity



Conclusions

- Fractal dimension characterizes the shape of a sensor surface quantitatively.
- Both regular or random surfaces may be characterized by fractals.
- A random surface may be converted into a regular surface for ease of analysis.
- The geometry of a sensor surface dictates a number of its properties, including settling time, sensitivity, and selectivity.