Principles of Electronic Nanobiosensors

Unit 2: Settling Time

Lecture 2.2: Classical Sensors I

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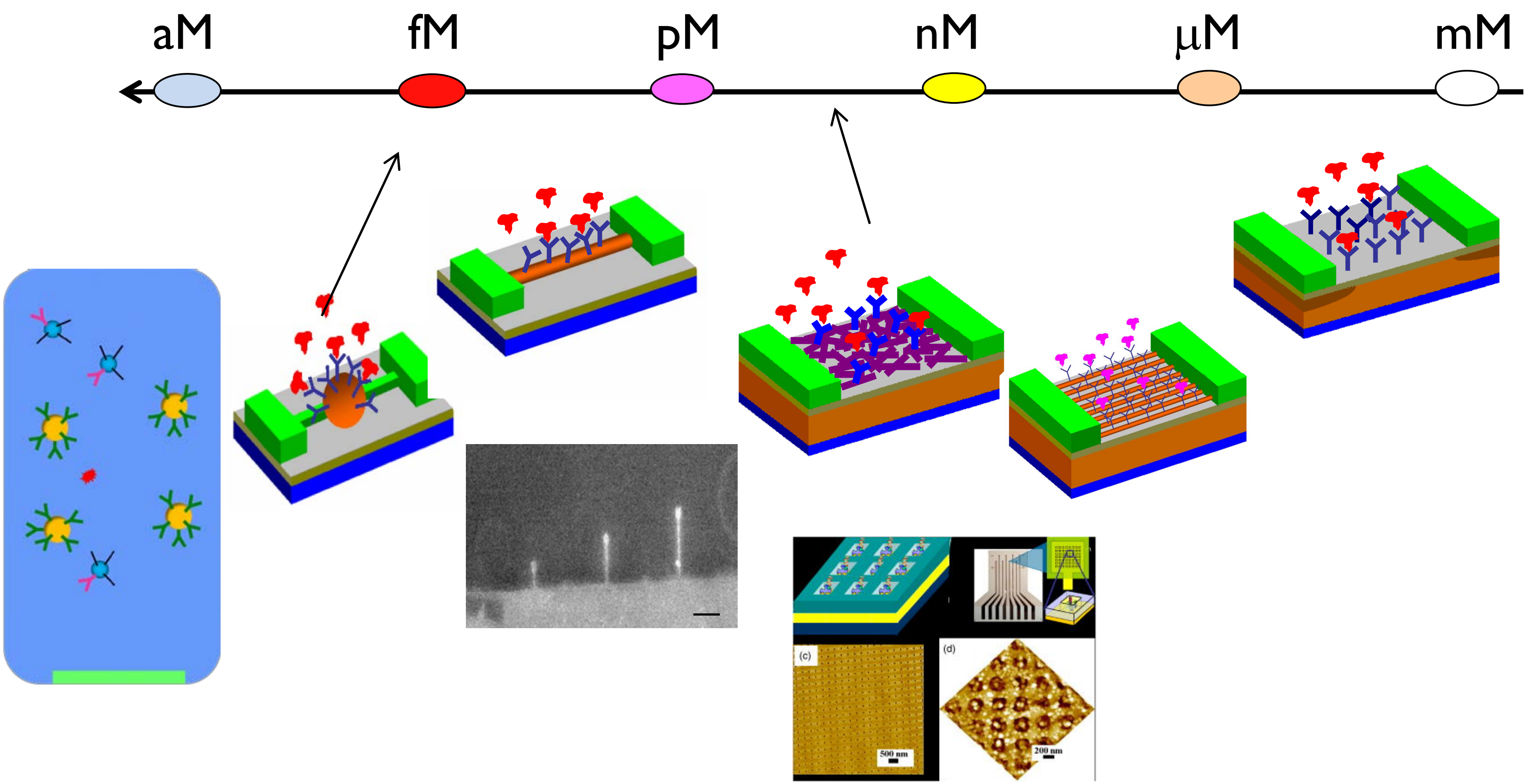
Outline

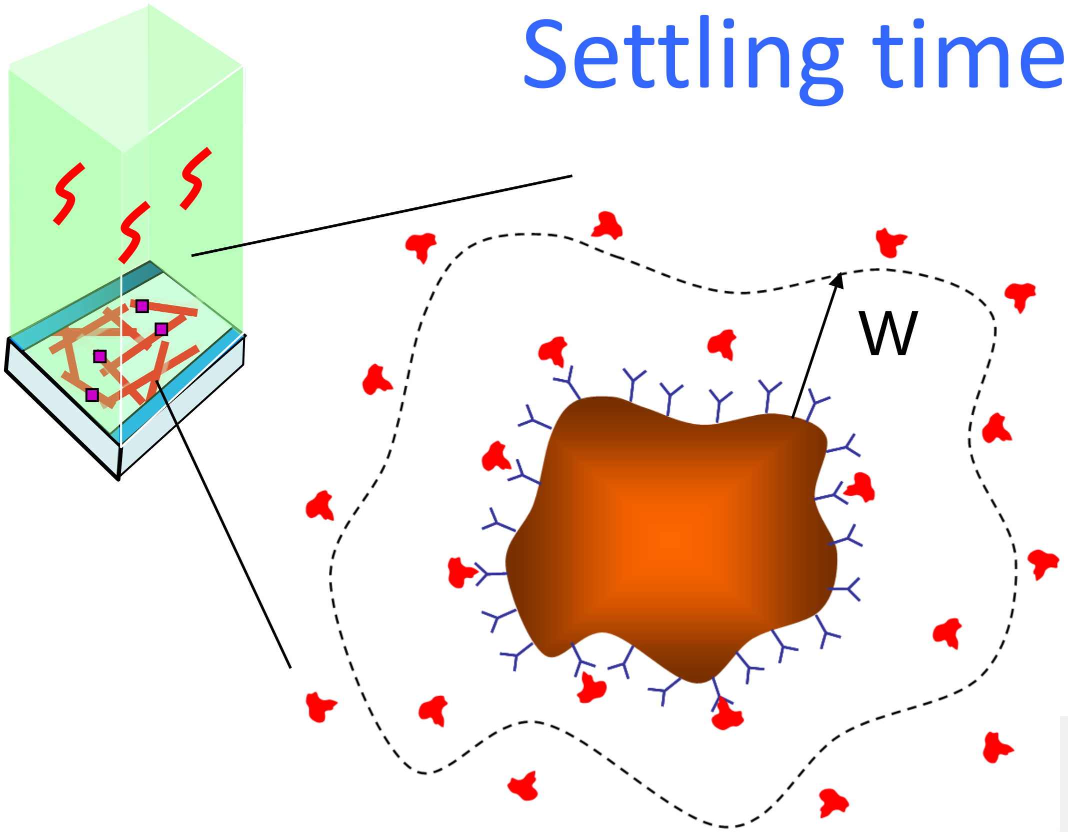
- Introduction
- So many sensors … How to classify them
- Geometry of diffusion defines response time
  - Approach based on 'diffusion triangle'
  - Approach based on 'diffusion capacitance'
- Conclusion

Nanobiosensors are highly sensitive



Is there something fundamental about the geometry?

Settling time for biosensors

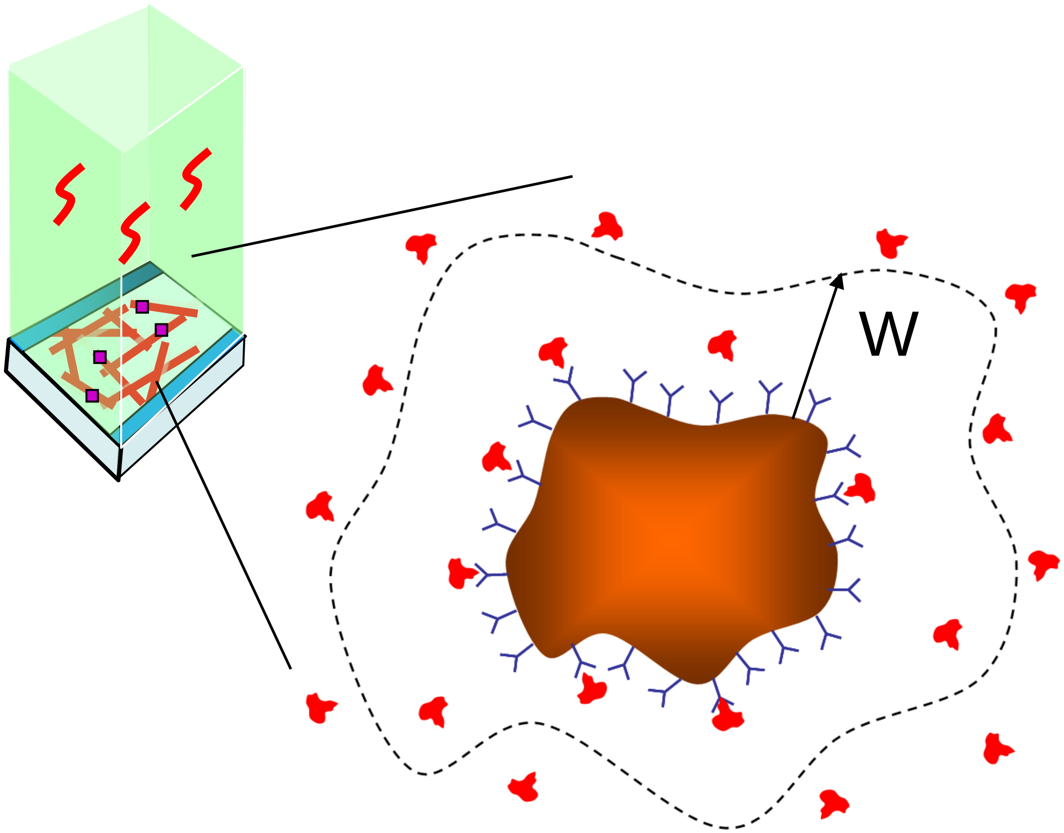


$$\frac{d\rho}{dt} = D\nabla^2 \rho$$

$$\frac{dN}{dt} = k_F (N_0 - N)\rho_s$$

The diffusion-capture problem is very challenging, especially for complex capture surfaces

Fractal geometry allows simple solution



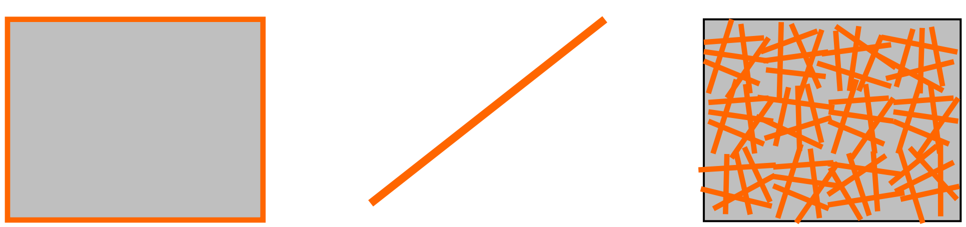
$$\frac{d\rho}{dt} = D\nabla^2\rho$$

$$\frac{dN}{dt} = k_F\left(N_0 - N\right)\rho_s$$

$$k_F \rightarrow \infty$$

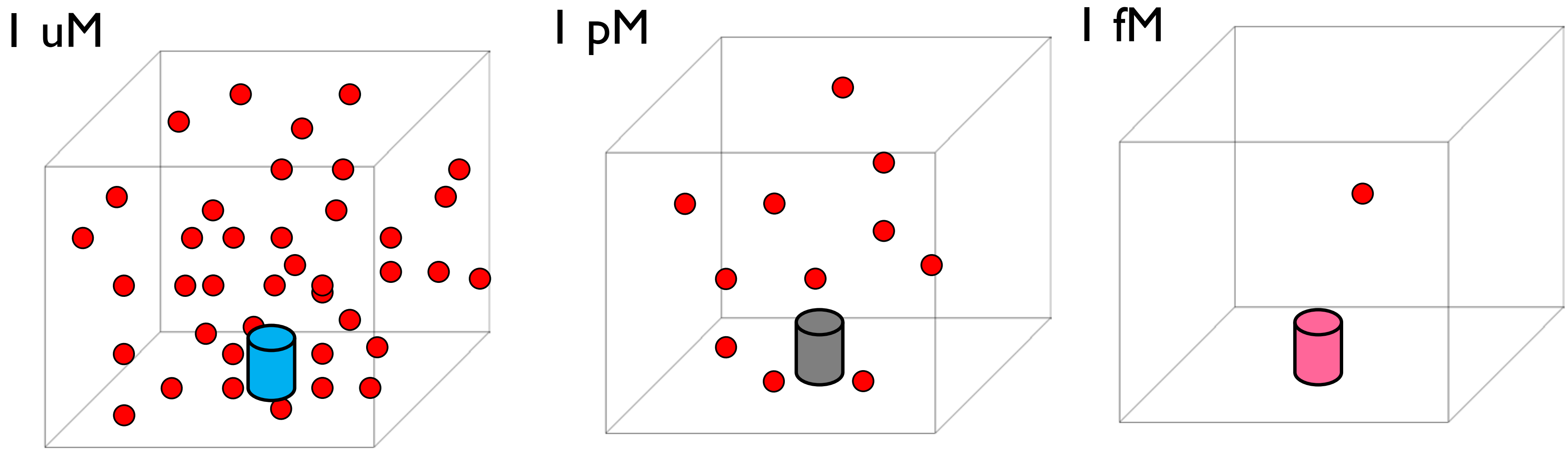
$$\rho_s \rightarrow 0$$

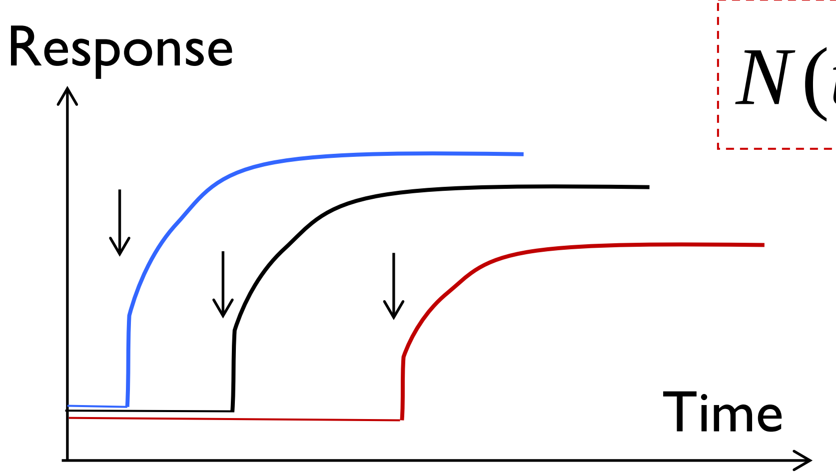
$$N\left(t\right) = \rho_0 t^{g\left(D_F\right)}$$



$D_F = 2$ $\quad$ $D_F = 1$ $\quad$ $1 < D_F < 2$

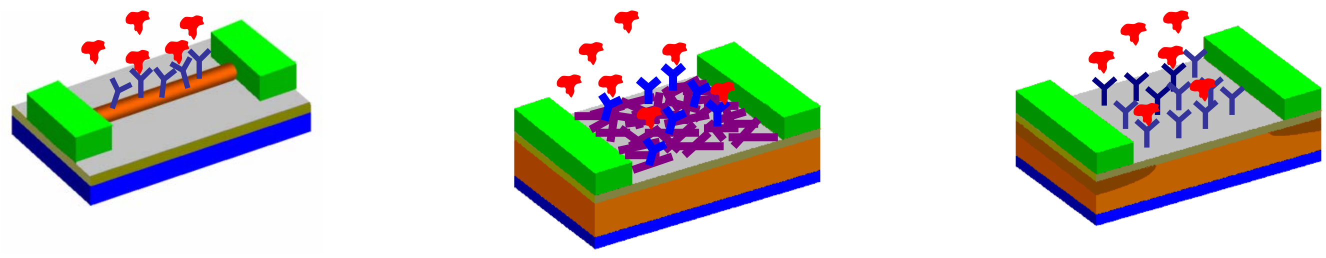
Response or settling time defined





$$N(t = t_s) \equiv N_s = \rho_0 \times t_s^{g(D_F)}$$

A fundamental relationship of biosensor



D=1 1<D<2 D=2

Fractal dimension

Limits of detection → $$\rho_0 = N_s \times t_s^{-\left(\frac{3-D_F}{2}\right)}$$ ← Settling (response) time

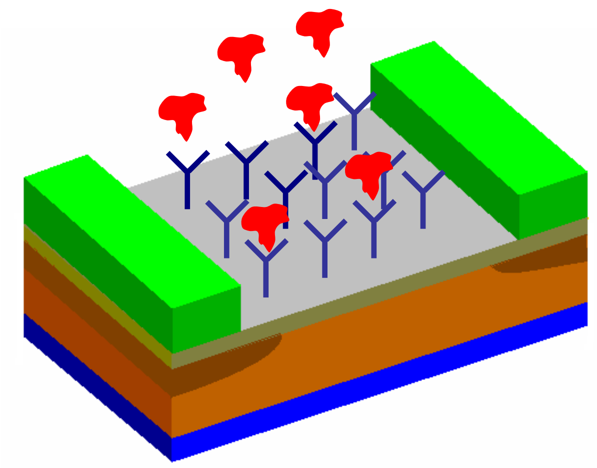
↑

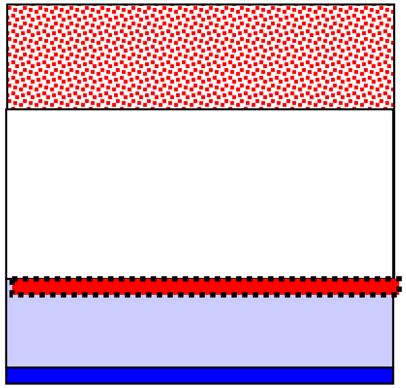
Minimum number of analyte (depends on transduction)

Like the Heisenberg relationship, but for a sensor …

Operation of a planar sensor

$$t, \rho_0$$





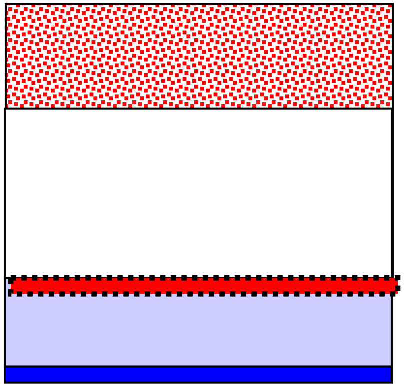
$$N(t) = ?$$

Exact solutions for a planar sensor

$$\frac{d\rho}{dt} = D\nabla^2 \rho$$

$$\frac{dN}{dt} = k_F (N_0 - N)\rho_s$$

$$k_F \rightarrow \infty, \rho_s = 0$$

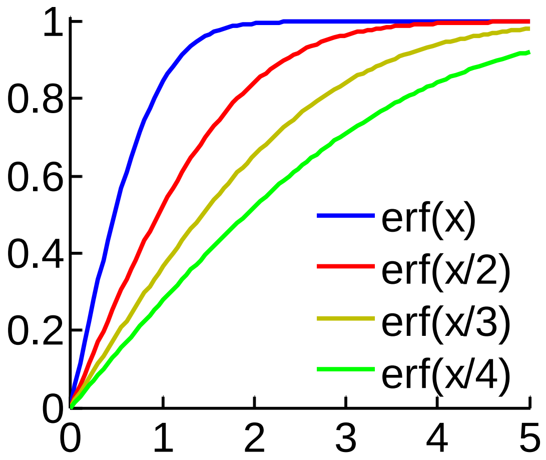


$$\rho(x,t) = \rho_0 \operatorname{erf}\left(x / 2\sqrt{Dt} \right)$$

Exact solutions for a planar sensor

$$\rho(x,t) = \rho_0 \ \mathrm{erf}\left(x/2\sqrt{Dt}\right)$$

$$\mathrm{erf}\,(x) \equiv \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy$$



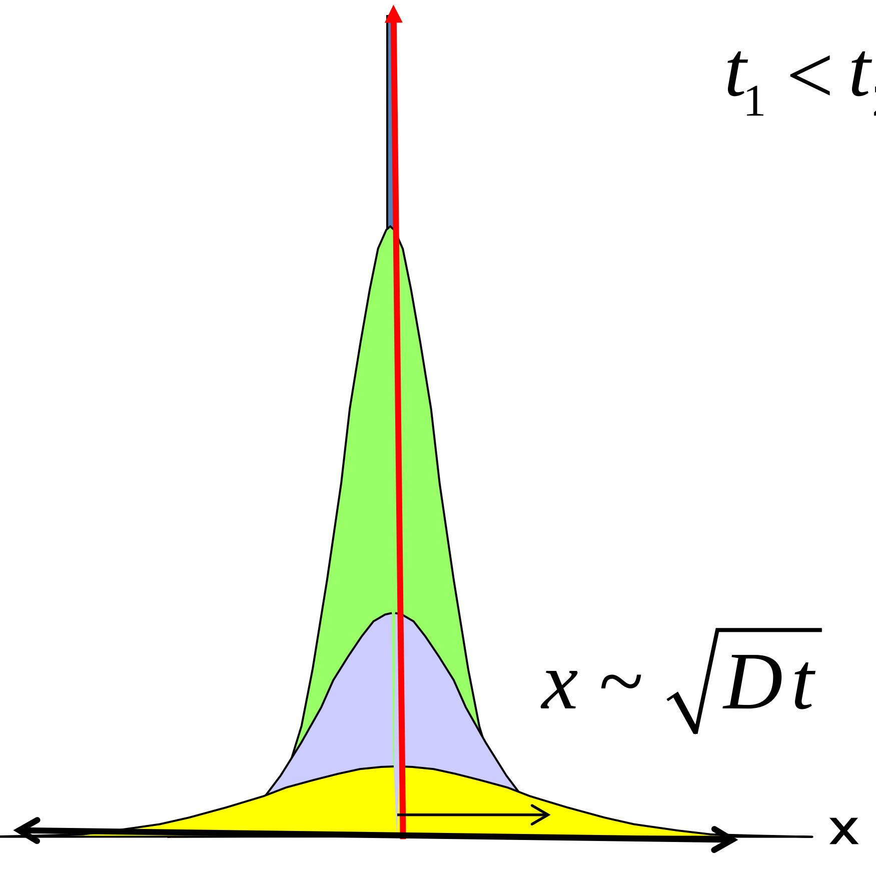
Particles captured

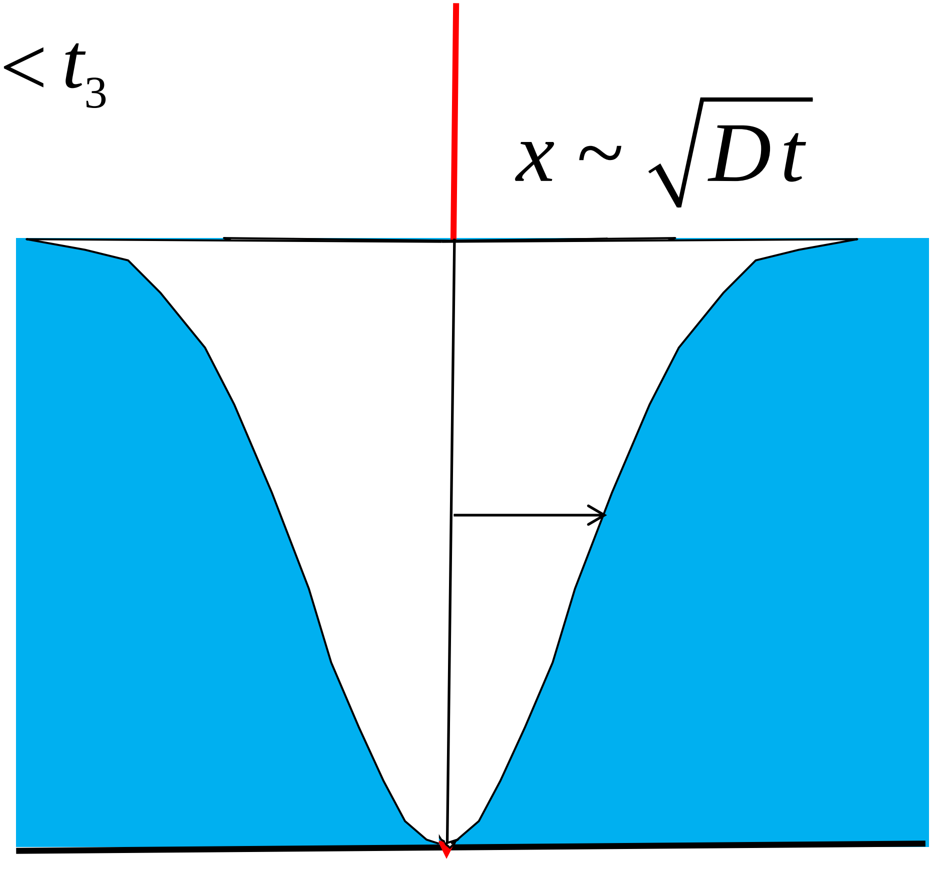
$$N(t) = \int_0^{\infty} \left[\rho_0 - \rho(x,t)\right] dx = \rho_0 \sqrt{\frac{4}{\pi}} \sqrt{Dt}$$

The concept of the diffusion distance

$$\frac{d\rho}{dt} = D\nabla^2\rho$$

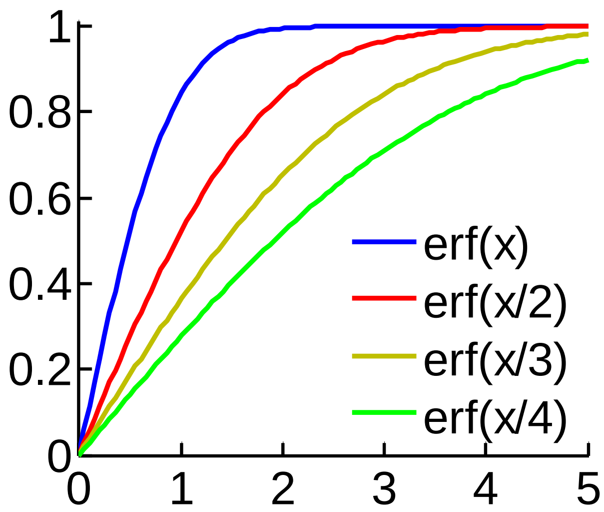
$$t_1 < t_2 < t_3$$





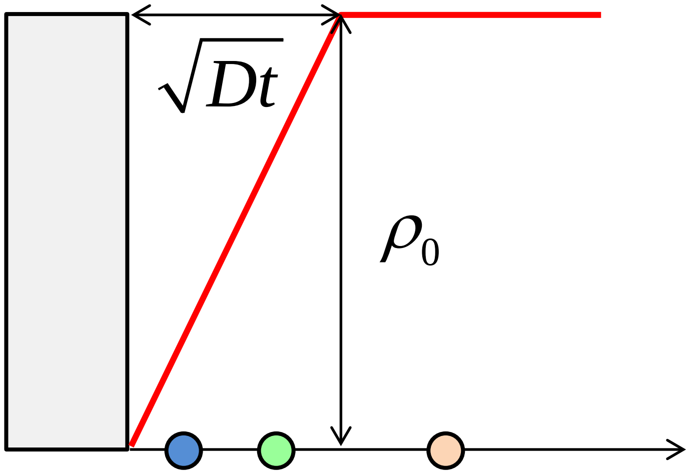
Approximate Solution in 1D: Diffusion Distance

Exact



$$N(t) = \sqrt{\frac{4}{\pi}} \times \rho_0 \times \sqrt{Dt}$$

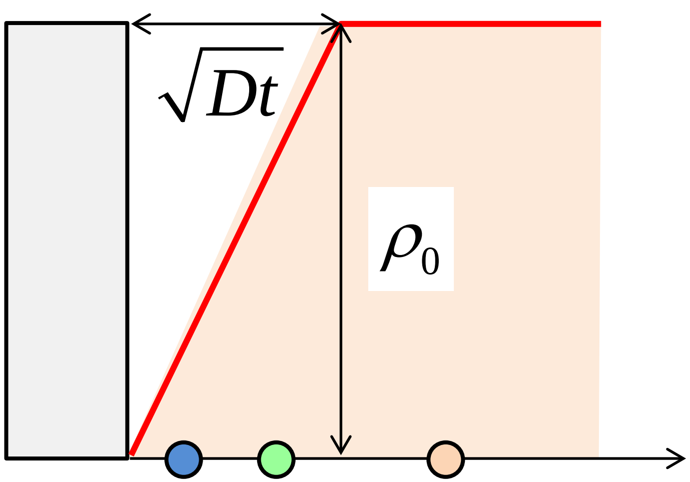
Approximate



$$N(t) \sim \frac{1}{2} \times \rho_0 \times \sqrt{Dt}$$

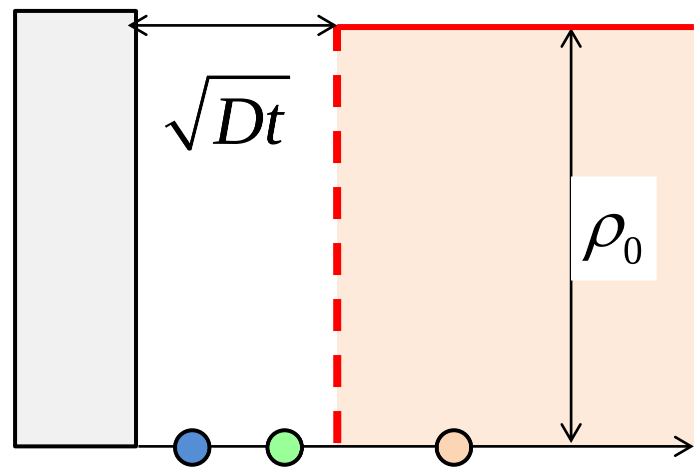
Approximate Solution in 1D: Diffusion Distance

Approximate



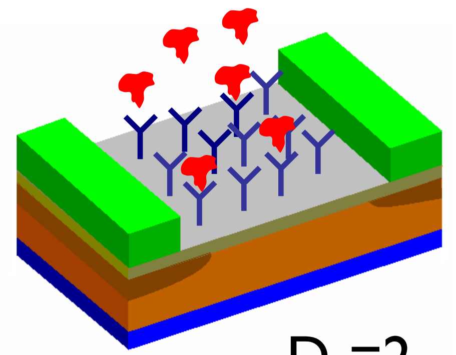
$$N(t) \sim \frac{1}{2} \times \rho_0 \times \sqrt{Dt}$$

Approximate

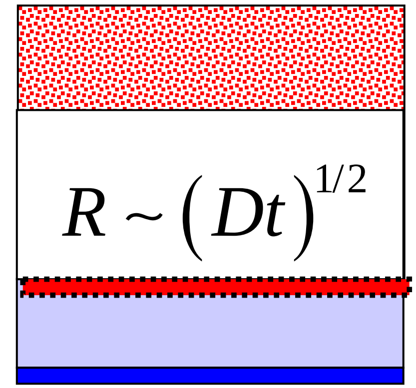


$$N(t) \sim \rho_0 \times \sqrt{Dt}$$

Response time of a planar sensor



$D_F=2$



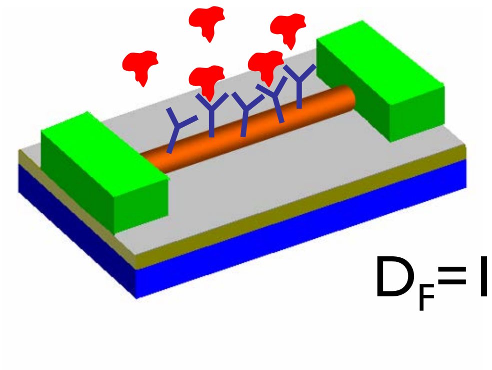
$$t_s \sim \frac{N_s^2}{D}\frac{1}{\rho_0^{\,2}}$$

$$N(t) \times A \sim \rho_0 \times R \times A$$
$$\sim \rho_0 \times \sqrt{Dt} \times A$$

$$N(t) \propto t^{1/2}$$

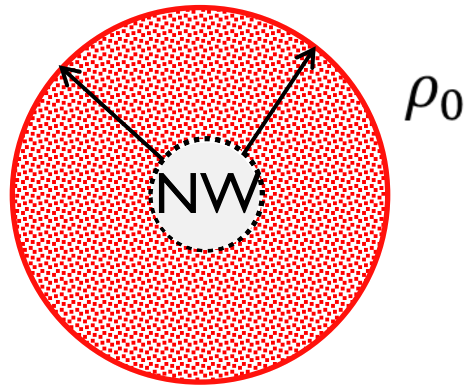
Diffusion slowdown

Response time of cylindrical sensor



$$t_s \sim \frac{N_s a}{D}\frac{1}{\rho_0}$$

$$R \sim \sqrt{Dt}$$



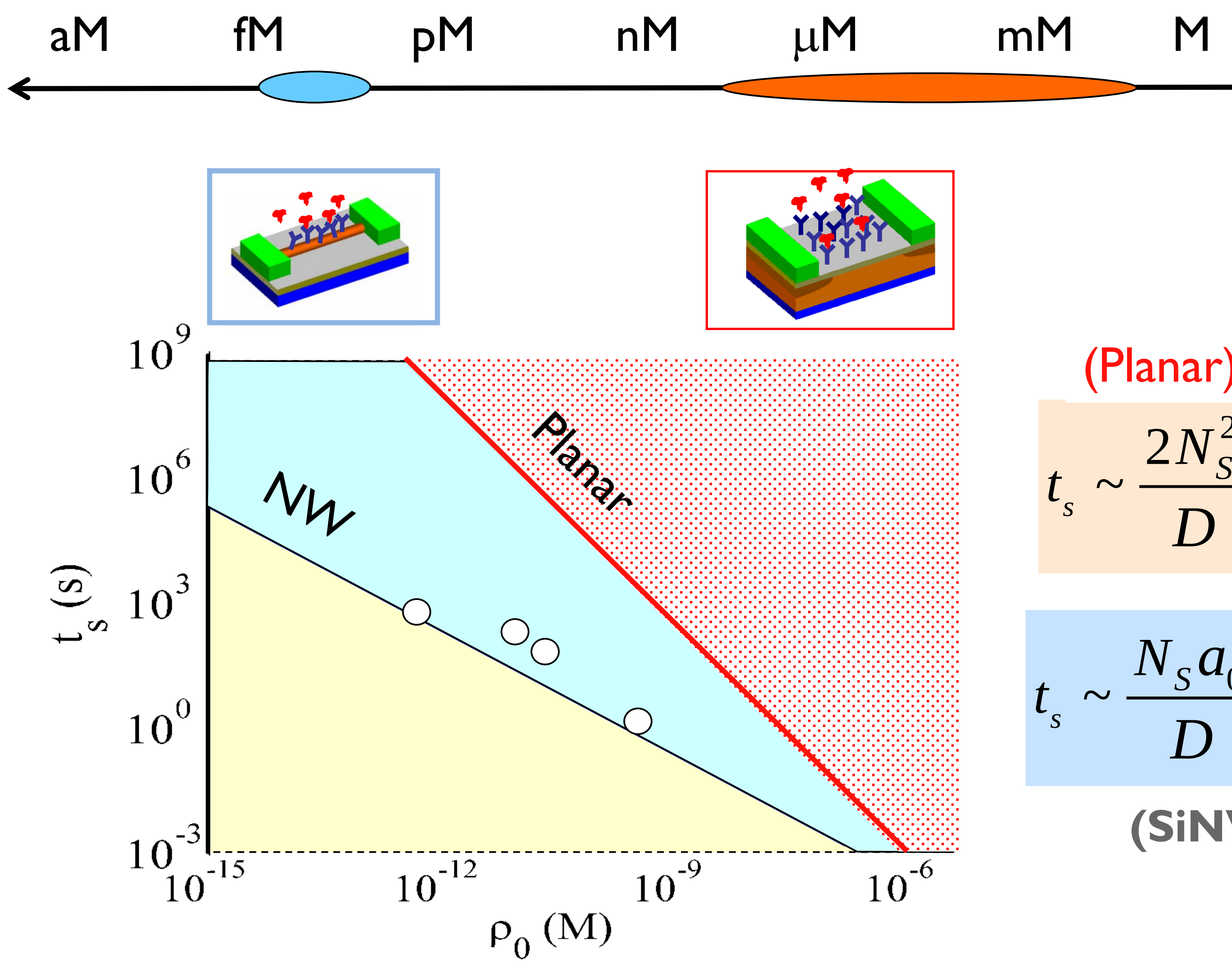
$$N(t)\times 2\pi a_0 \sim \rho_0 \times \pi\left(R^2 - a_0^2\right)$$

$$\sim \rho_0 \times \pi\left[\sqrt{Dt}\right]^2$$

$$N(t) \propto t^1$$

Diffusion slowdown absent?!

Geometry of diffusion/sensor response



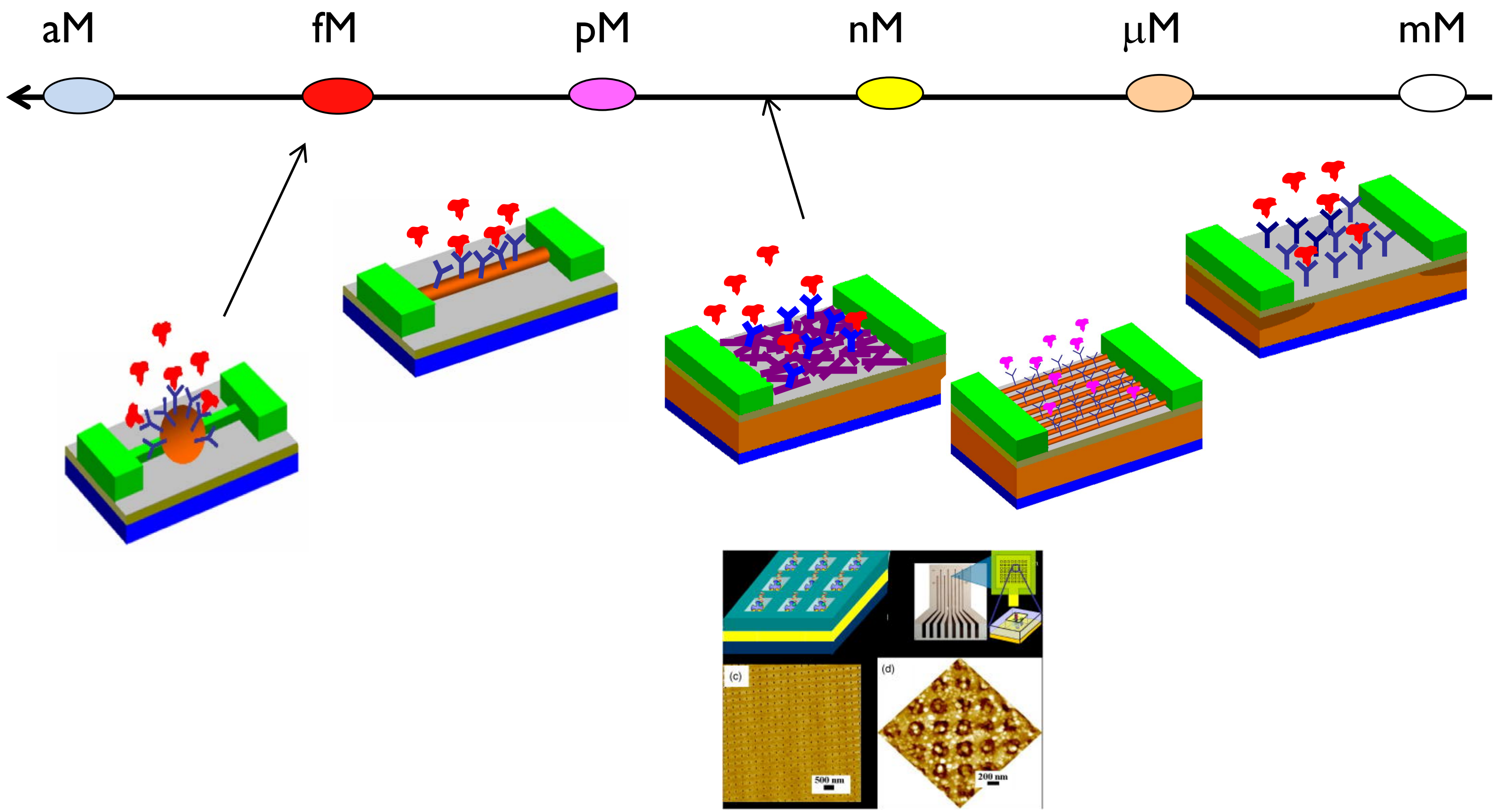
(Planar)

$$t_s \sim \frac{2N_S^2}{D}\frac{1}{\rho_0^2}$$

$$t_s \sim \frac{N_S a_0}{D}\frac{1}{\rho_0}$$

(SiNW)

A ‘Mendeleev table’ for biosensors



For other geometries, we need a slightly better technique based on ‘diffusion-equivalent capacitance …’