

Principles of Electronic Nanobiosensors

Unit 2: Settling Time

Lecture 2.5: Beating the Limits – Barcode Sensors

By Muhammad A. Alam

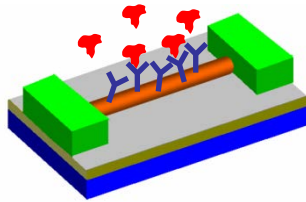
Professor of Electrical and Computer Engineering

Purdue University

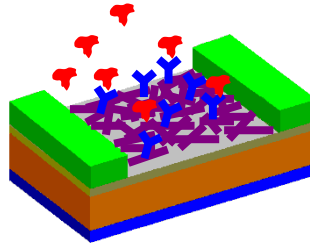
alam@purdue.edu



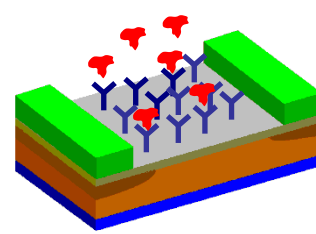
A 'fundamental' relationship of biosensor



$D=1$



$1 < D < 2$



$D=2$

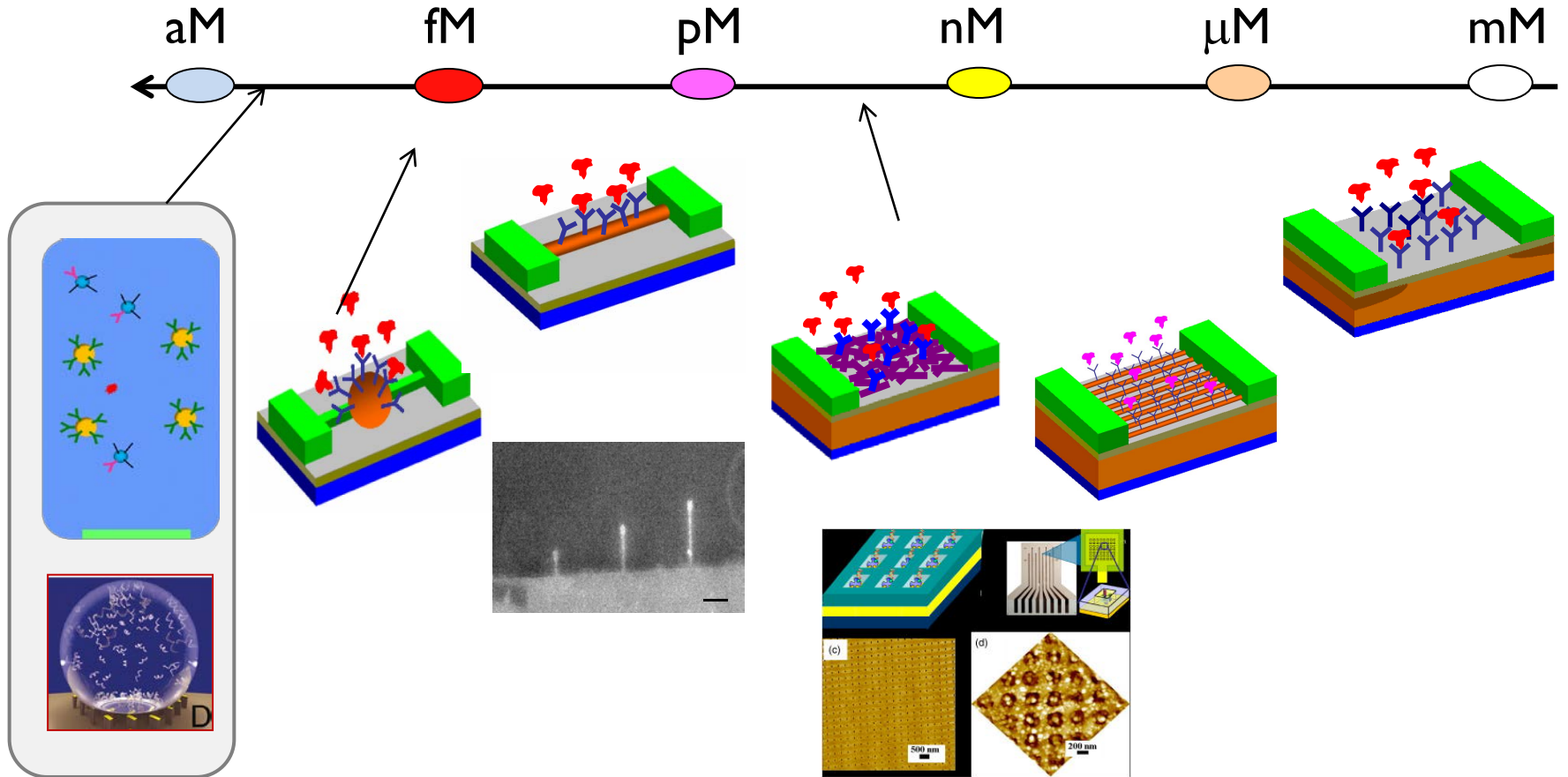
$$\rho_0 = N_s \times t_s^{-\left(\frac{3-D_F}{2}\right)}$$

... not as fundamental as the uncertainty principle!

Outline

- Three approaches to beat the diffusion limit
- Technique of distributed sensors: Biobarcode
- Physics of biobarcode operation
- Enhancement of detection limits by biobarcode and closely related approaches
- Conclusion

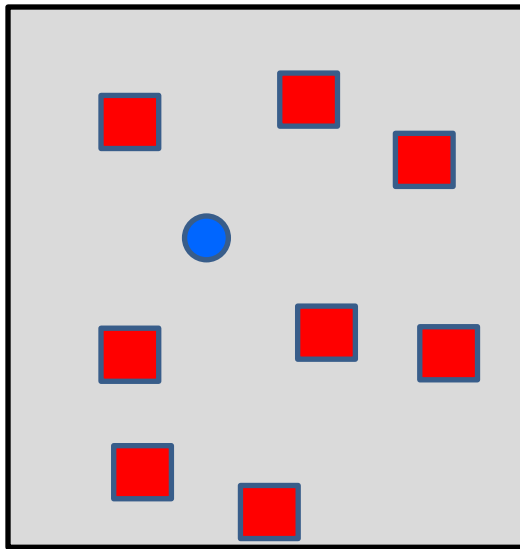
A 'Mendeleev table' for biosensors



Strategies to beat the diffusion limit

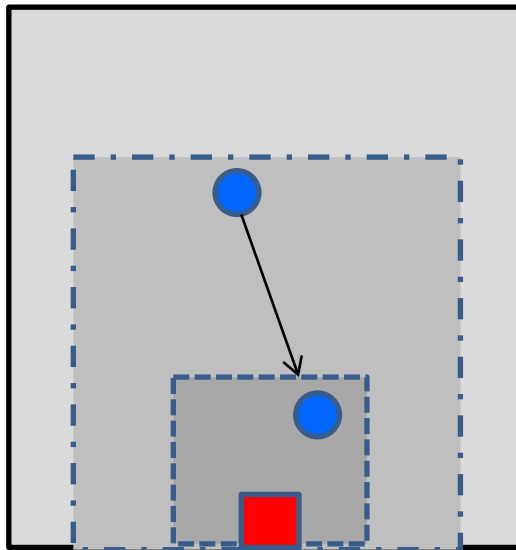
$$\tau \sim L^2 / D$$

Fragment
the space



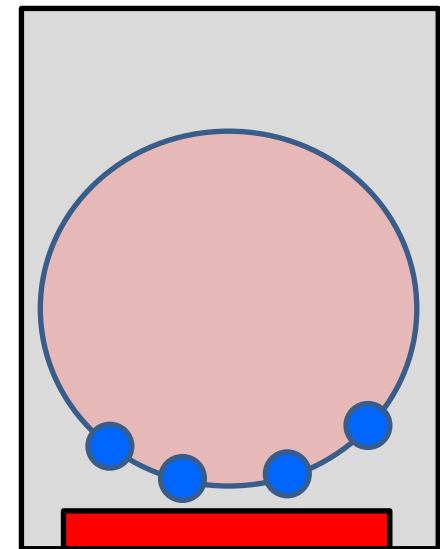
Magnetic
biobarcode

Reduce
the space



Droplet
evaporation

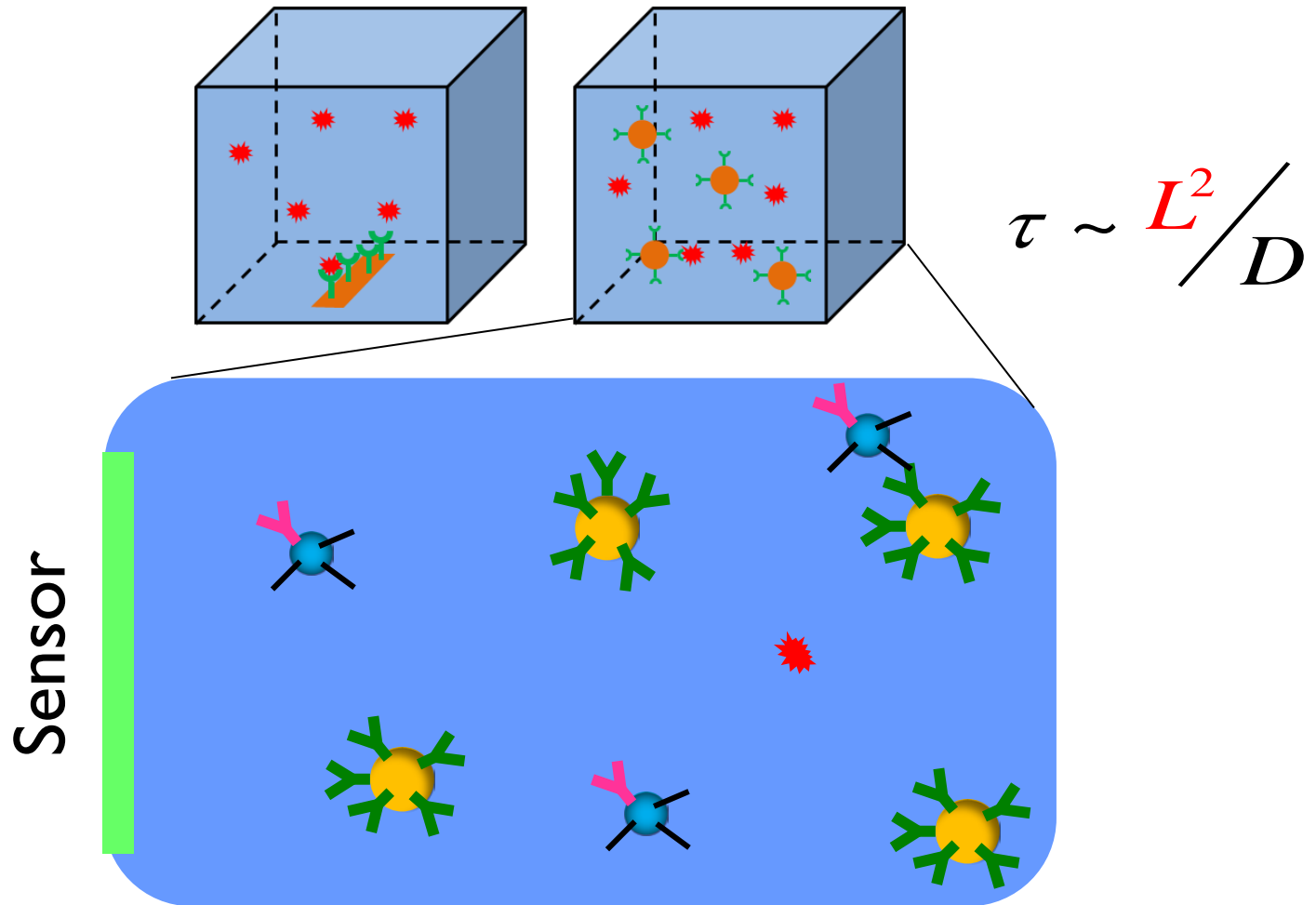
Generate
locally



Ion torrent
approach

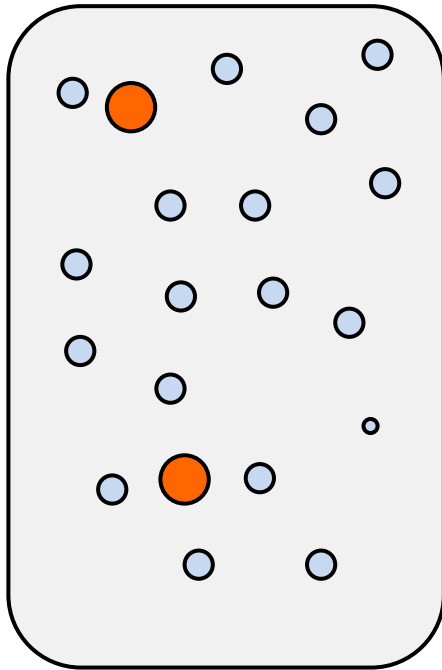
All can achieve sub-fM detection in reasonable time

Magnetic nanoparticle barcode sensor



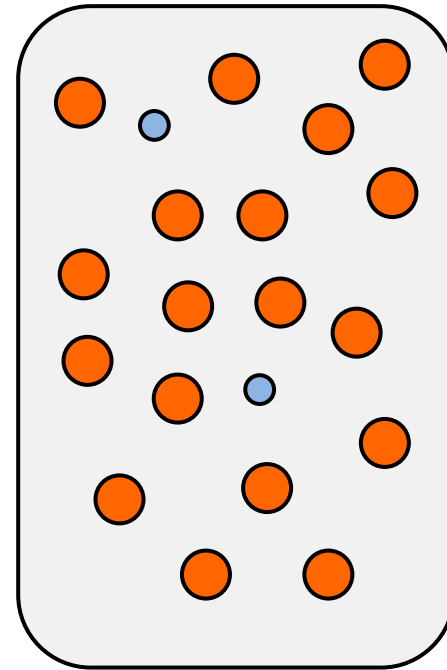
In sum, a police-thief story!

Analytical solution: two limits



$$t_s = \frac{N_s}{4\pi D a_0 \rho_T} \quad (\text{Ia})$$

$$(\rho_{MP} \leq \rho_T, N_s \geq 1)$$

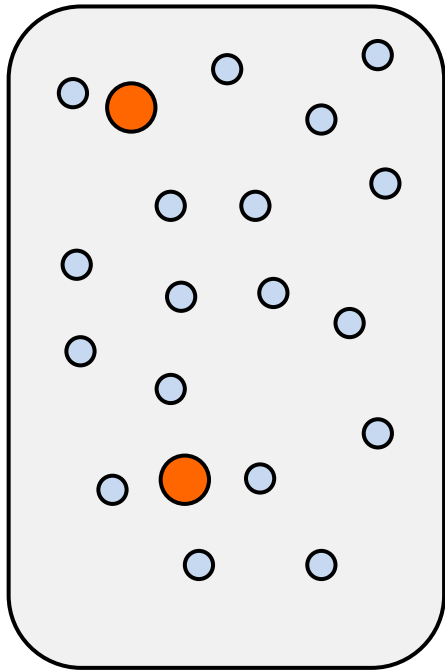


$$t_s = \frac{N_s}{4\pi D a_0 \rho_{MP}} \quad (\text{Ib})$$

$$(\rho_{MP} \geq \rho_T, N_s = 1)$$

Analytical solution ($\rho_T < \rho_{MP}$)

Capture probes are widely separated

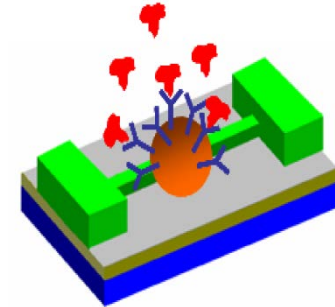


$$\frac{\partial \rho}{\partial t} = D \nabla^2 \rho$$

$$\rho_s = 0$$

$$N(t) = C_{D(t)} \rho t$$

$$C_{D(t)} = \frac{D}{a_0^{-1} - (\sqrt{Dt})^{-1}}$$



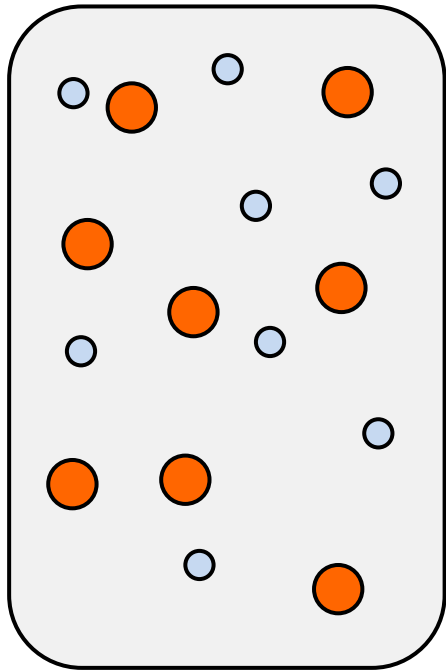
$$N(t) = 4\pi D a_0 \rho t$$

$$t_s = N_s / 4\pi D a_0 \rho$$

No different than a spherical sensor

Analytical solution ($\rho_T = \rho_{MP}$)

Each MP captures on average one target particles, $N_S = 1$



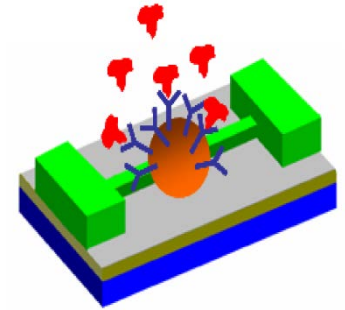
$$\frac{\partial \rho}{\partial t} = D \nabla^2 \rho$$

$$\rho = 0$$

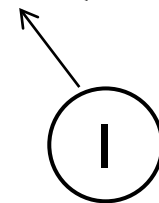
$$N(t) = C_{D(t)} \rho t$$

$$C_{D(t)} = \frac{D}{a_0^{-1} - (\sqrt{Dt})^{-1}}$$

$$N(t) = 4\pi D a_0 \rho t$$

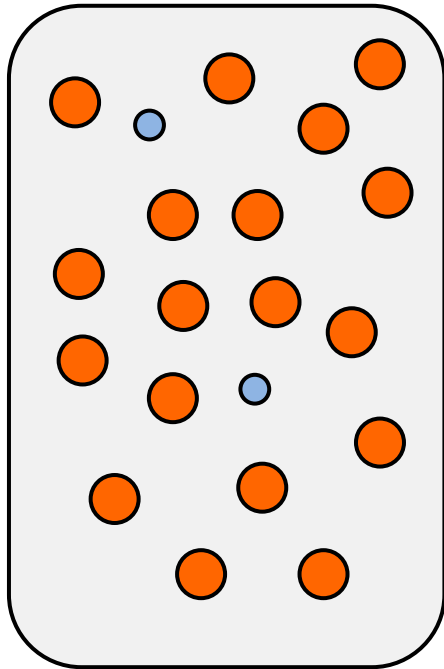


$$t_S = N_S / 4\pi D a_0 \rho$$



Analytical solution ($\rho_T > \rho_{MP}$)

Each probe captures at most 1 target particle



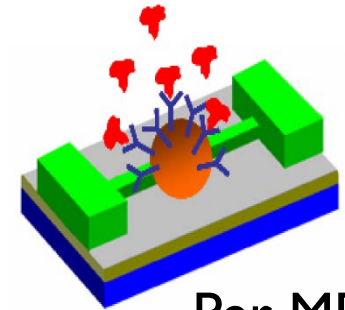
$$\frac{\partial \rho}{\partial t} = D \nabla^2 \rho - \frac{\rho}{\tau} \leftarrow \text{Captured by spherical probes}$$

$$\rho_s = 0$$

$$N(t) = 4\pi D a_0 \rho t$$

$$R_1 = N(t)/t = 4\pi D a_0 \rho$$

$$R = 4\pi D a_0 \rho \rho_{MP} \equiv \rho / \tau$$



Per MP

$$\tau = \frac{1}{4\pi D a_0} \frac{1}{\rho_{MP}}$$

Analytical solution: transient solution

$$\frac{\partial \rho}{\partial t} = D \nabla^2 \rho - \frac{\rho}{\tau} \quad \tau \equiv \frac{1}{4\pi D a_0} \frac{1}{\rho_{MP}}$$

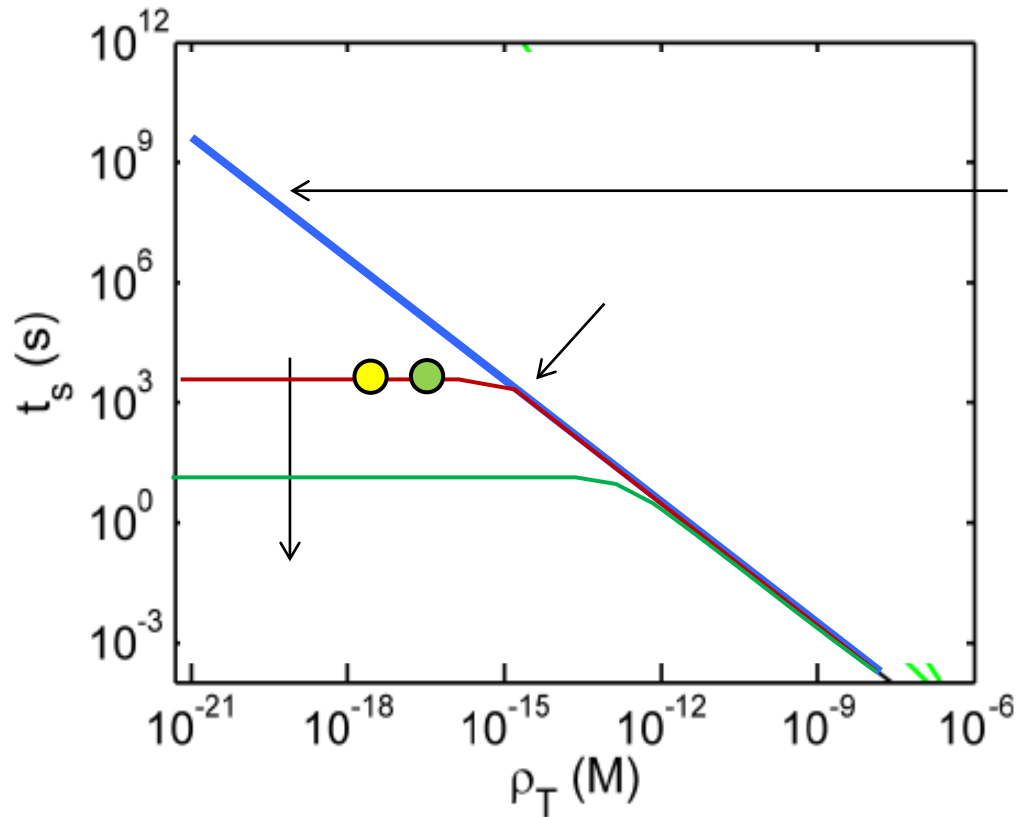
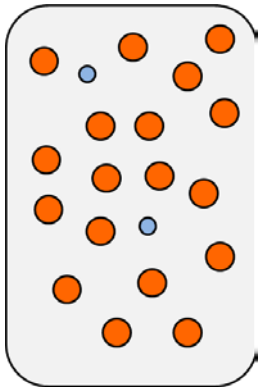
$$\rho(r, t) = A t^{-3/2} e^{-t/\tau} e^{-\left(\frac{r^2}{4Dt}\right)}$$

$$S(t) = \frac{\int_0^{\infty} \rho(r, t) 4\pi r^2 dr}{\int_0^{\infty} \rho(r, t=0) 4\pi r^2 dr} = \frac{e^{-t/\tau}}{\tau}$$

Analytical solution for barcode sensor

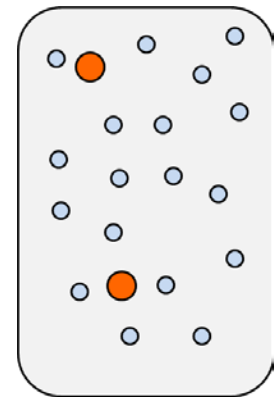
Eq. Ib

$$t_s \propto \rho_{MP}^{-1}$$

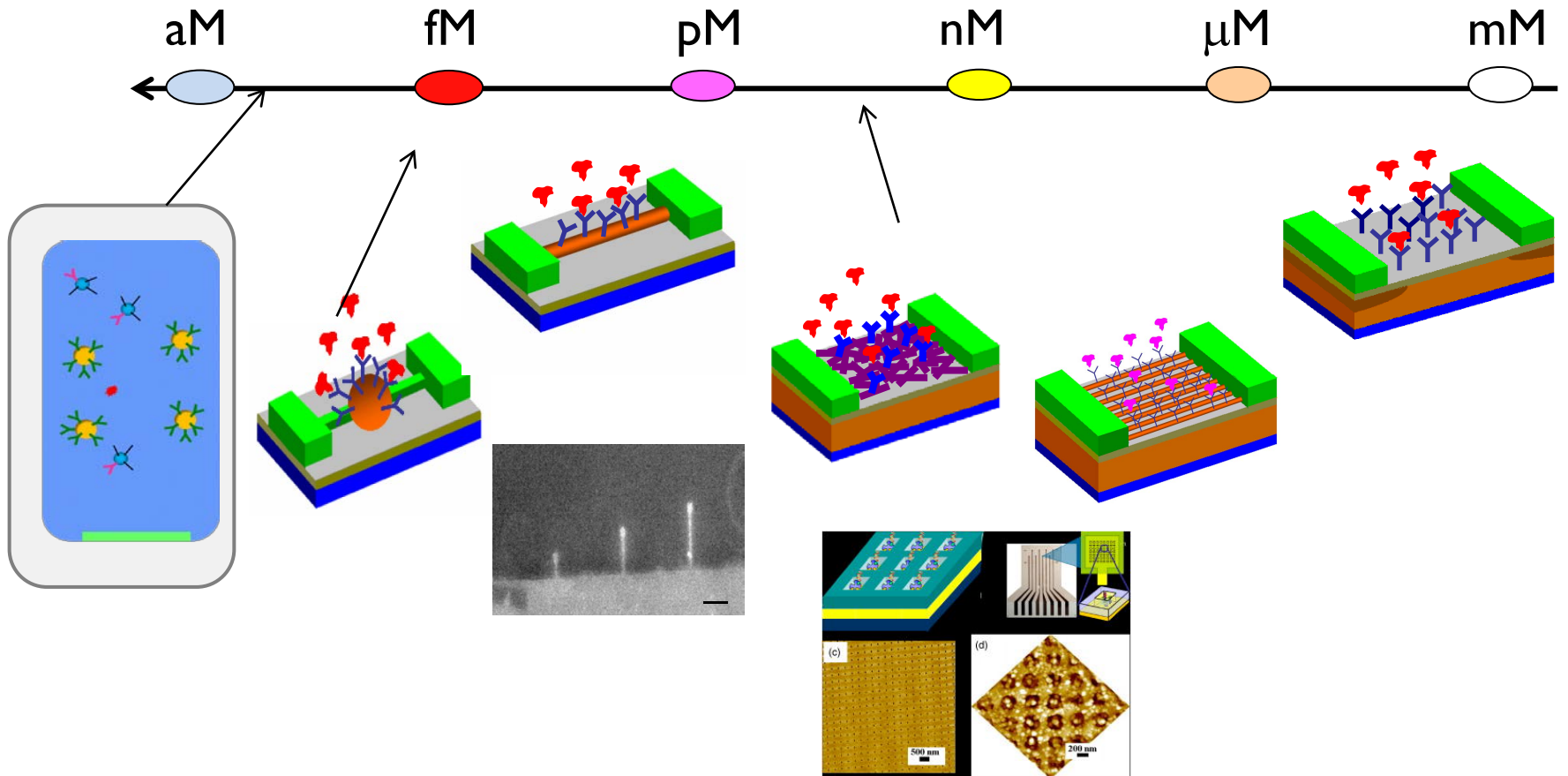


Eq. Ia

$$t_s \propto \rho_T^{-1}$$

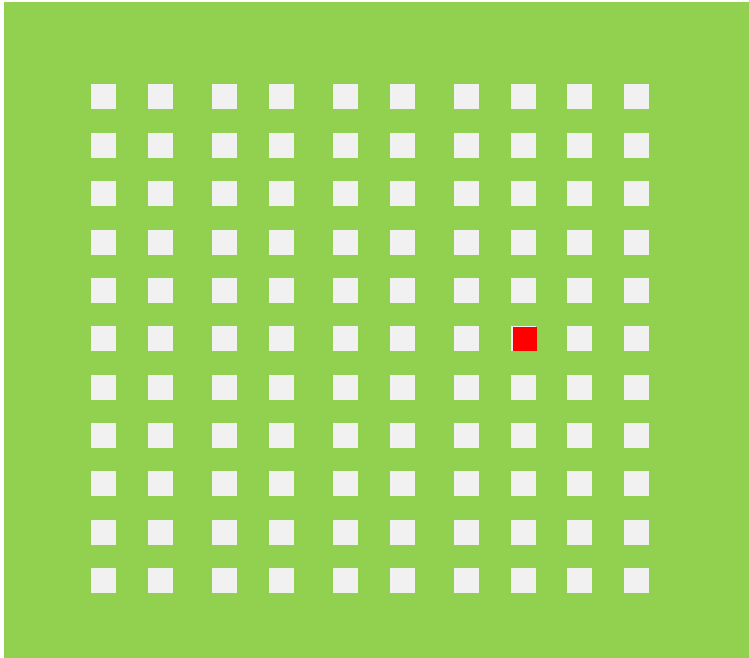


A 'Mendeleev table' for biosensors



Biobarcode sensors 'beat' the diffusion limit by fragmenting the space

Sensor array: fragmenting sensor volume



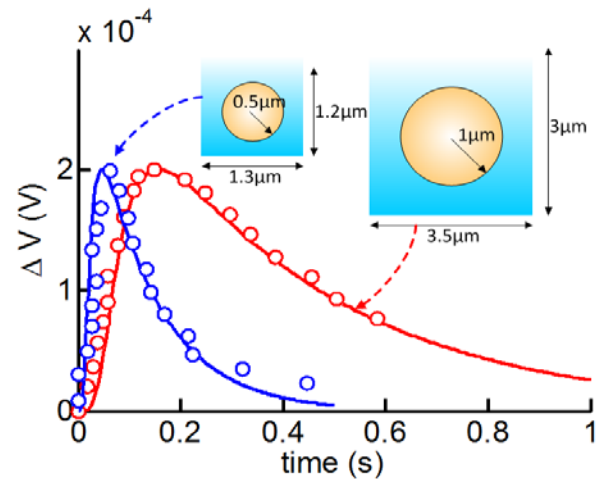
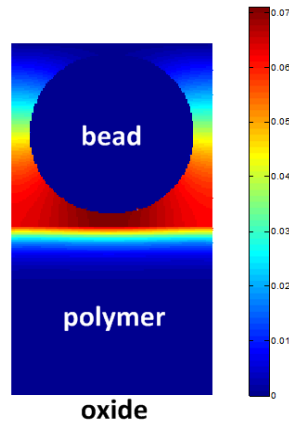
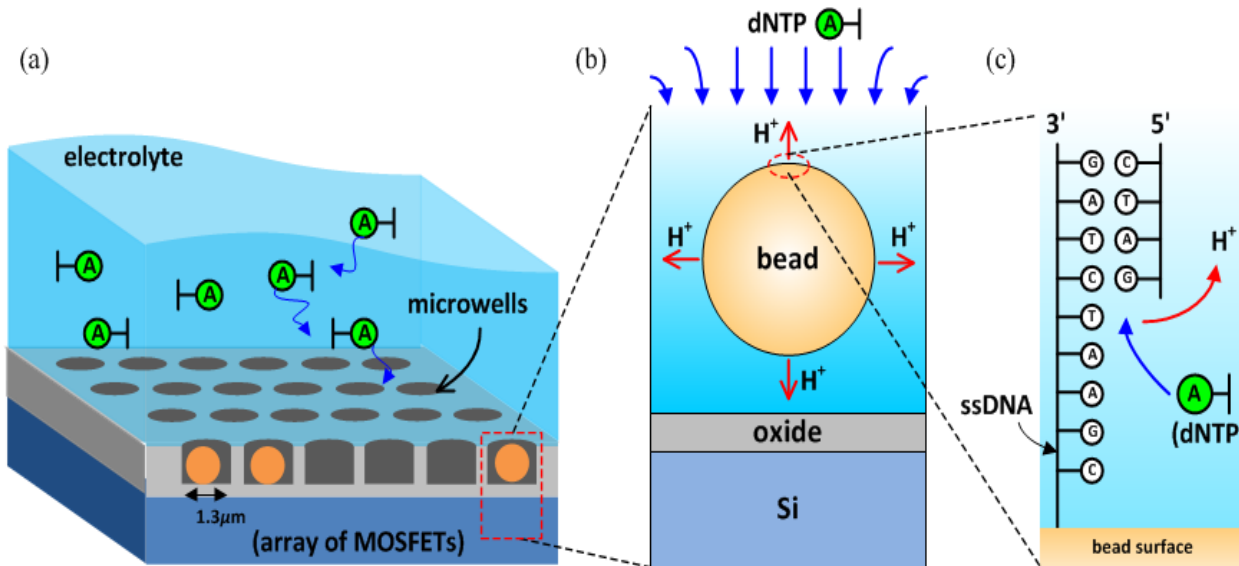
Advantage

- much greater area
- redundancy
- multiple analytes
- *etc.*

Disadvantage

- “Dead Space” competition (i.e., adsorption betⁿ sensors)
- cost of multiplexing (potentially \$100k ‘s)
- loss of signal-to-noise
- complexity, power use, etc.

Local generation/fast diffusion



$$\tau \sim L^2 / D$$

Conclusions

- Biobarcode approach is still defined by diffusion limits – it just reduces diffusion time by using many probes.
- Biobarcode does not sense anything. It just catches the molecules. Sensing is done in a later step using amperometric or potentiometric methods.
- Using multiple sensors to detect the single analyte is equivalent to distributing probes in solution. Therefore, one anticipates similar gain in settling time.
- Both approaches increase cost and processing time, but could be necessary for detection at ultra-low concentration.