# Principles of Electronic Nanobiosensors

Unit 2: Settling Time

Lecture 2.8: First Passage and Narrow Escape Time I

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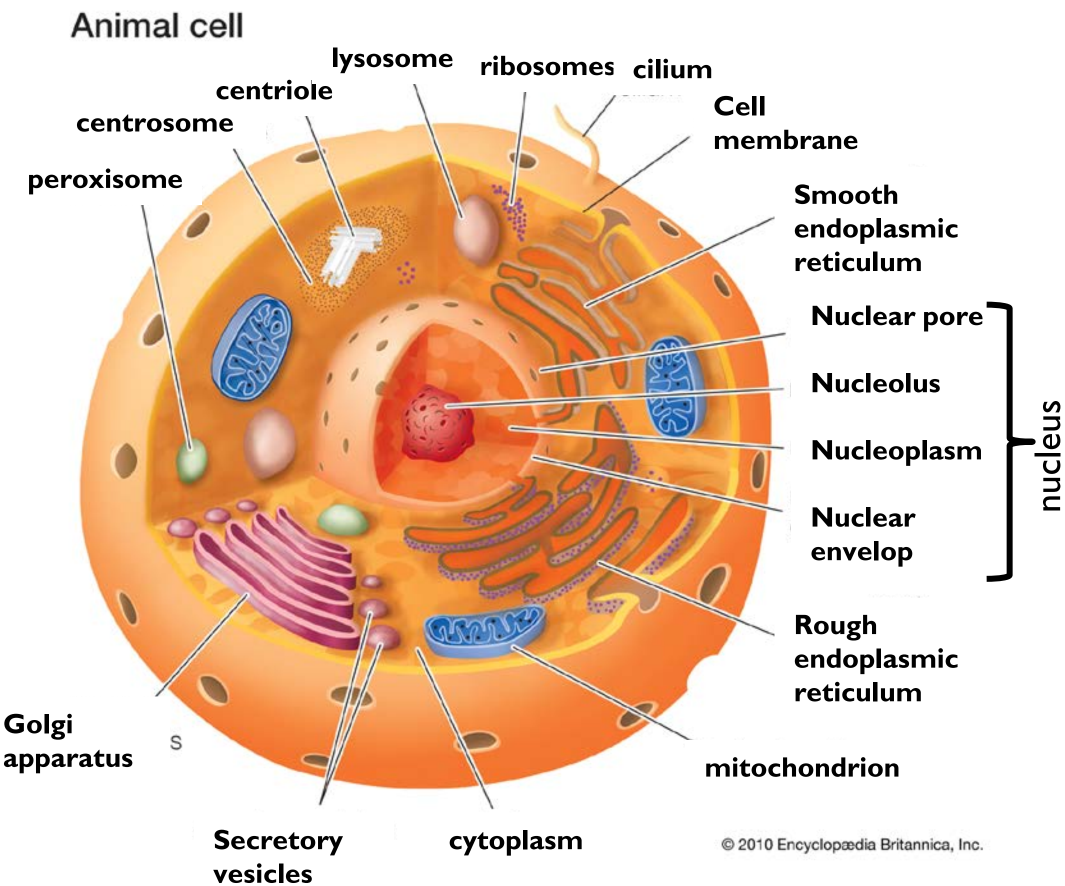
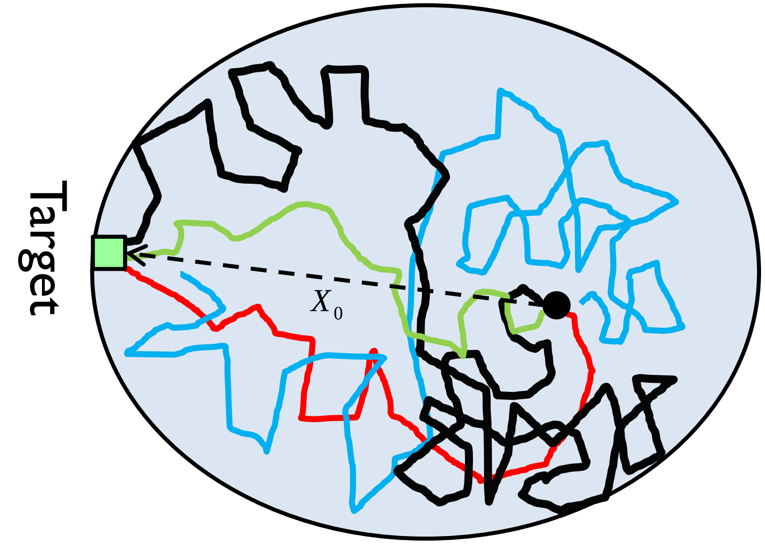
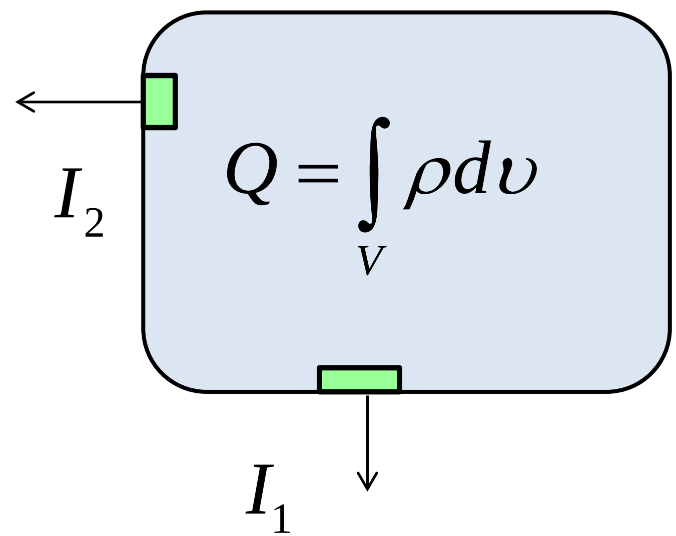
# Outline

- ‘Blind’ molecule escaping from a cell
- First passage time (FPT)
- First passage time (FPT): examples
- Narrow escape time (NET)
- Conclusion

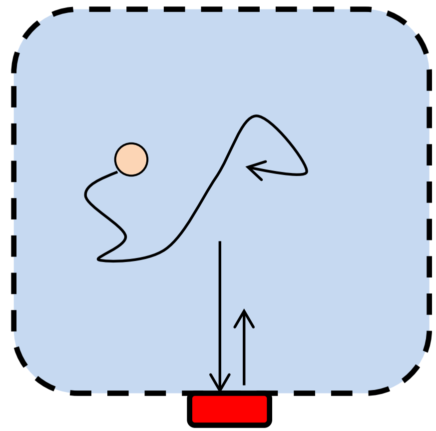
# On Cells and Sensors



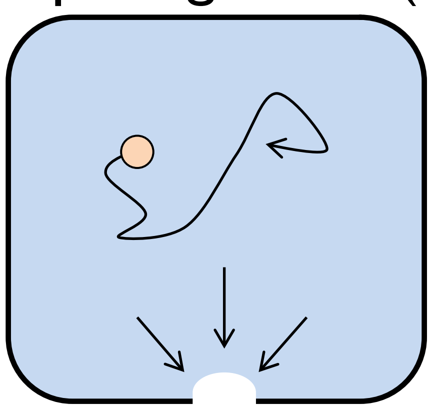



# Settling time, MFPT, and NET

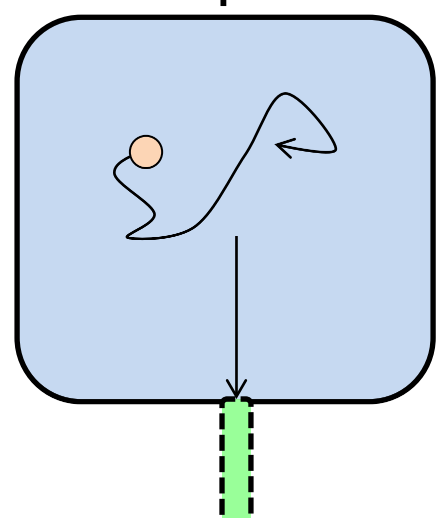


Settling time

First passage time (FPT)



Narrow escape time (NET)



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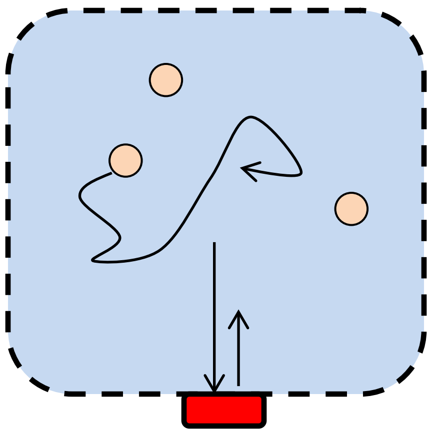
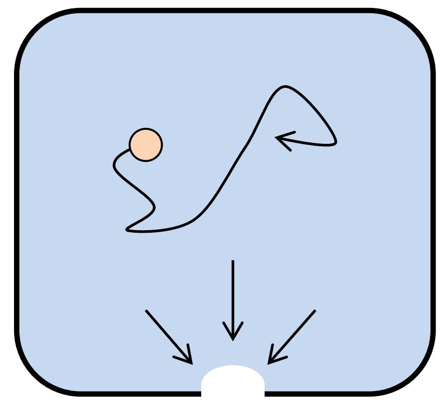
# Response Time vs. First Passage Time





$$N_T = \int_V \rho dV$$

$$\frac{d\rho}{dt} = D\nabla^2\rho$$

$$\frac{dN(t)}{dt} = k_F(N_0 - N)\rho_s - k_R N$$

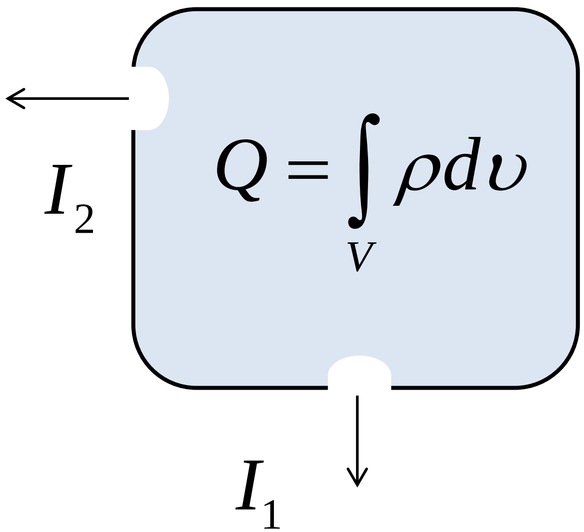
How long to capture $N_S$?

$$\frac{d\rho}{dt} = D\nabla^2\rho$$

$$\frac{dN_T}{dt} = F_{in} - F_{out} = -\frac{N_T}{\tau}$$

How long, on average, to escape?

# Charge Control: First Passage Time




$$\partial \int_V \rho\, d\upsilon \Big/ \partial t = dQ / dt$$

$$\int_V D\nabla^2 \rho\, d\upsilon = -\int_S J \bullet dS = -(I_1 + I_2)$$

$$dQ/dt = -Q/\tau = -(I_1 + I_2)$$

$$\tau = Q/(I_1 + I_2)$$

$$\partial \rho / \partial t = D\nabla^2 \rho$$

$$p = I_1/(I_1 + I_2)$$

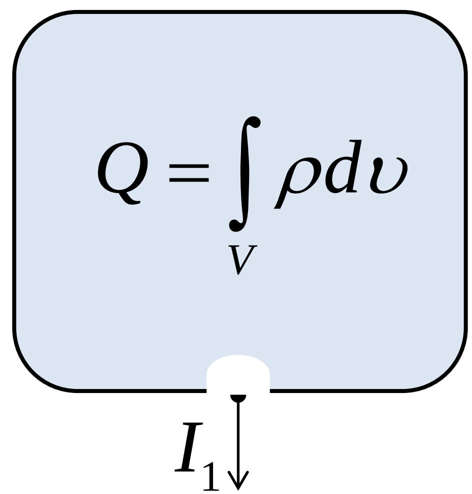
$$Q_1 = I_1 Q/(I_1 + I_2)$$

# MFPT and Diffusion Capacitance

$$\frac{dN_T}{dt} = -\frac{N_T}{\tau} \sim \cancel{I_{in}} - I_{out}$$

For large V… $N_T \approx V\rho_0$

$$\tau \sim N_T / I_{out} \sim V\rho_0 / I_1$$




$$I_1 = C_{D,SS}\left(\rho_0 - \rho_s\right) \sim C_{D,SS}\,\rho_0$$

$$\tau / V \rightarrow {C_{D,SS}}^{-1}$$

Remarkably simple, yet powerful result

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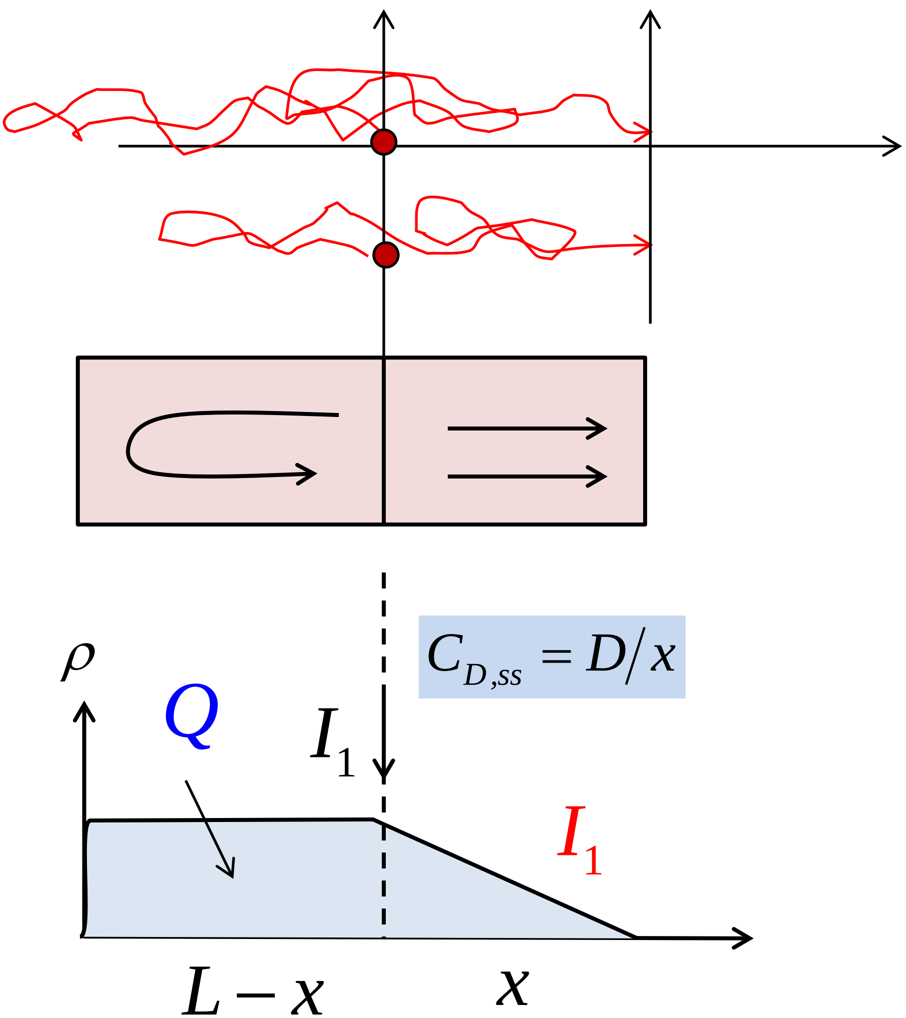
# Example 1: MFPT by Charge Control




$$I_1 = \left(D/x\right)\rho_0$$

$$Q = \rho_0\left(x/2\right) + \rho_0(L - x)$$

$$\boxed{\tau \sim Q/I_1}$$

$$\tau = \left(x^2/2D\right) + x(L - x)/D$$

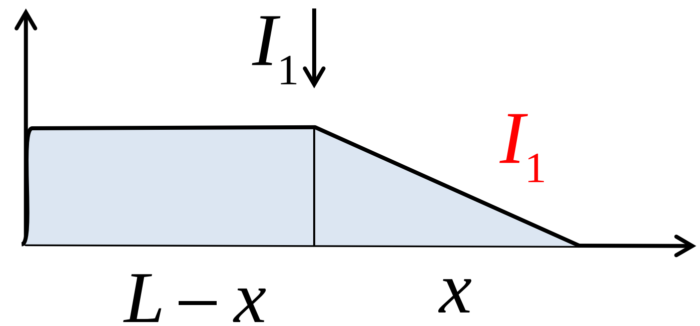
$$= (2Lx - x^2)/2D$$

$$\tau = (2Lx - x^2)/2D$$

# Example 1: MFPT in 1D by Capacitance

$$N_T \approx V\rho_0$$

$$\tau/V \to C_{D,SS}{}^{-1}$$



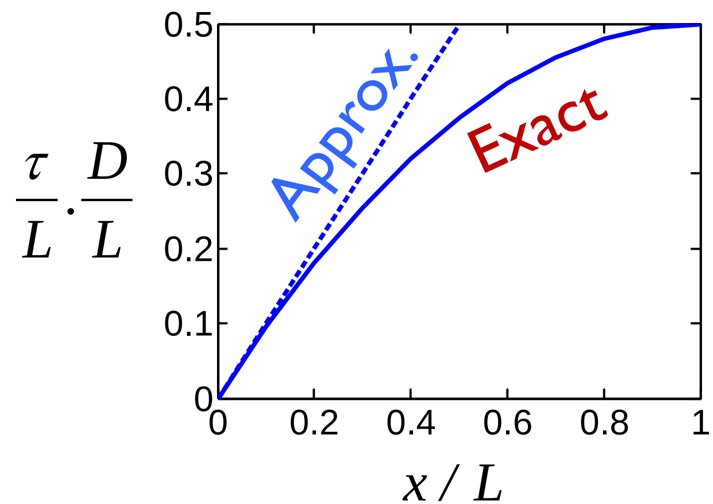

$$N_T = (\rho_0 x/2) + \rho_0 (L - x)$$

$$= \rho_0 L (1 - x/2L) \to \rho_0 L$$

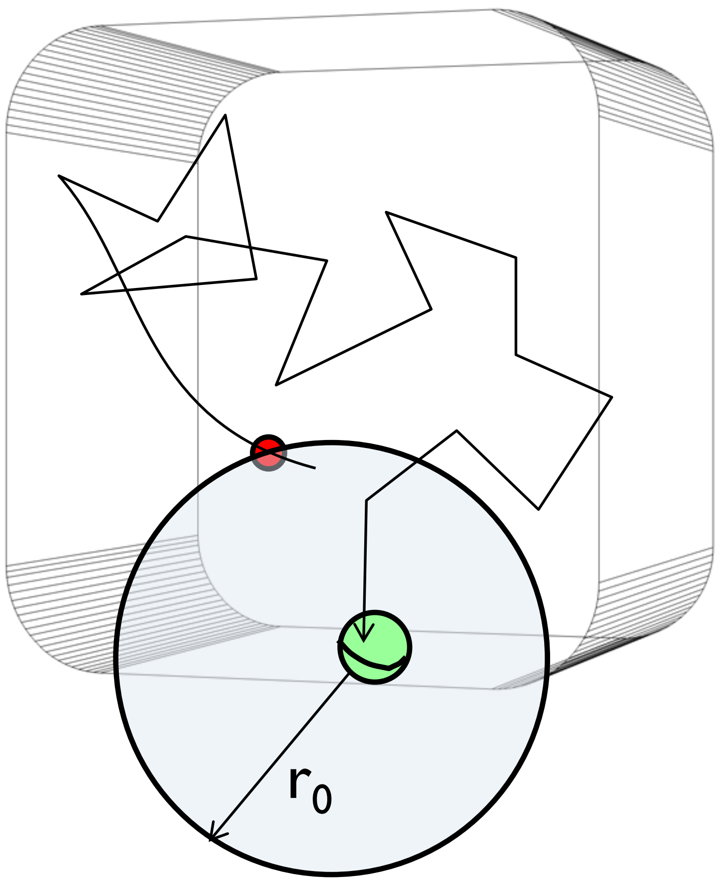
$$C_{D,ss} = D/x$$

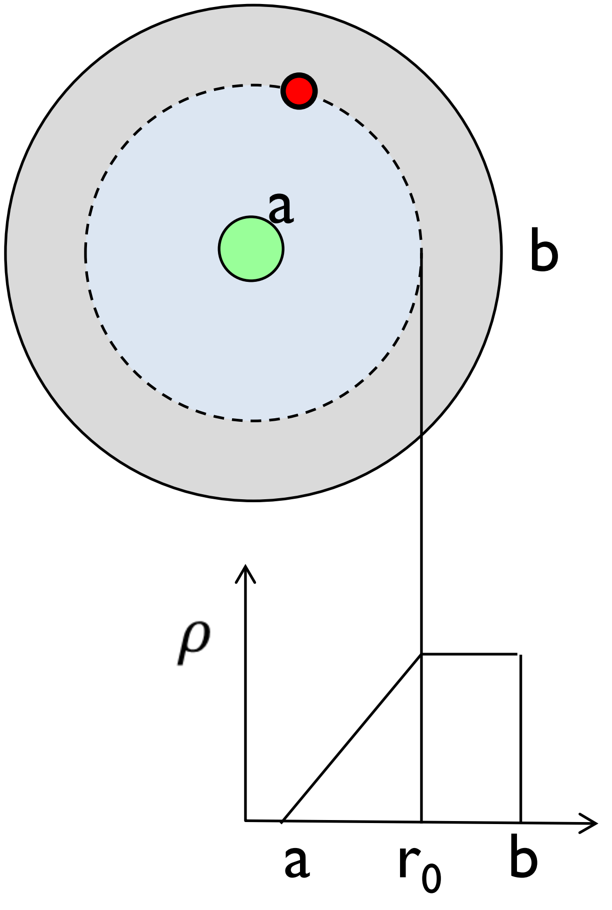
$$\Rightarrow \tau/L = D^{-1}x$$ Approximate

$$\tau/L = D^{-1}x(1 - x/2L)$$ Exact



# Example 2: MFPT in 3D



# Example 2: MFPT in 3D by Capacitance

$$C_{D,ss} = \frac{4\pi D}{\left(\frac{1}{a} - \frac{1}{r_0}\right)}$$

$$\tau / V \rightarrow C_{D,SS}{}^{-1}$$

$$\frac{\tau}{V} \approx \frac{1}{4\pi D}\left(\frac{1}{a} - \frac{1}{r_0}\right)$$ Approx.

Cell size

$$\frac{\tau}{V} = \frac{1}{4\pi D}\left(\frac{1}{a} - \frac{1}{r_0}\right) + \frac{\left(a^2 - r_0^2\right)}{8Db^3}$$ Exact