Principles of Electronic Nanobiosensors

Unit 3: Sensitivity

Lecture 3.2: Potentiometric Sensors:
Charge Screening for a Planar Sensor

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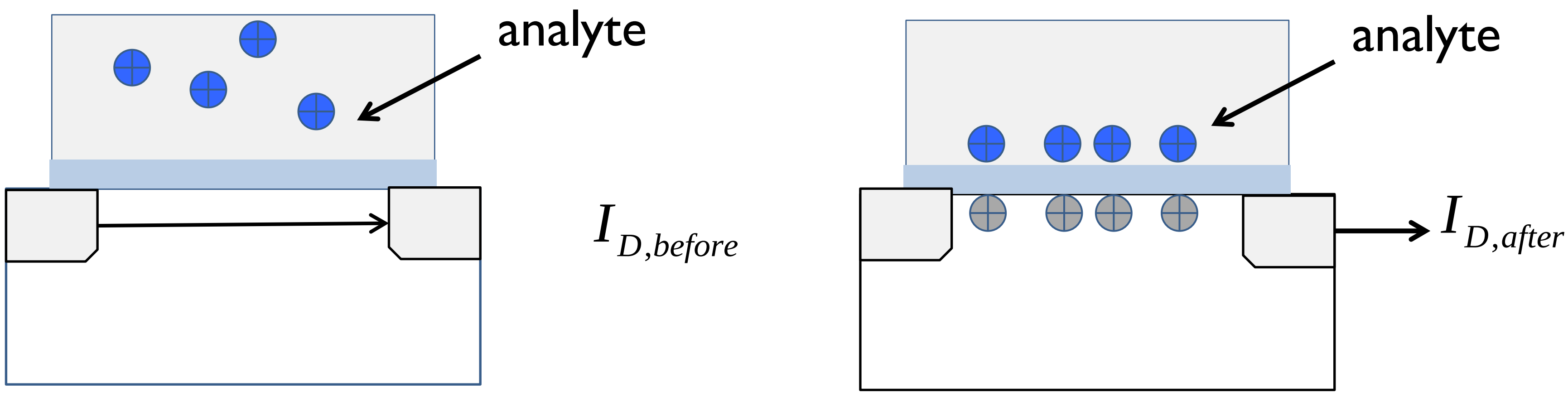
Outline

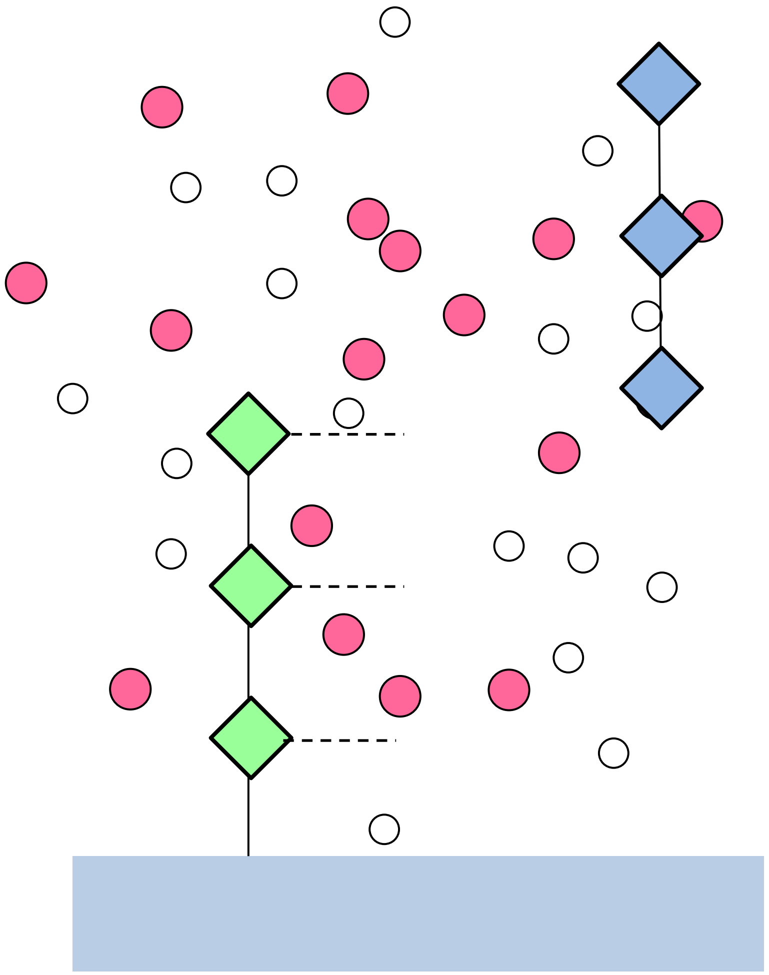
- Background: From a MOSFET to a biosensor
- Biocharges, salt and screening
- Debye theory response of a planar sensor
- Response of planar sensors at high salt concentration: Gouy-Chapman theory
- Conclusions
- Appendix: Derivation of Gouy-Chapman theory

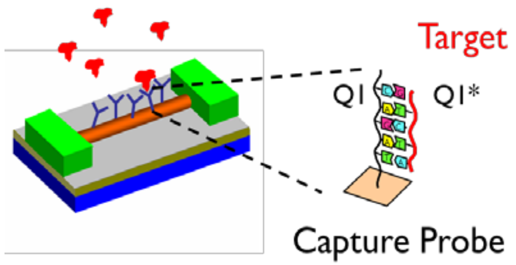
Sensitivity of a MOSFET-based potentiometric sensor

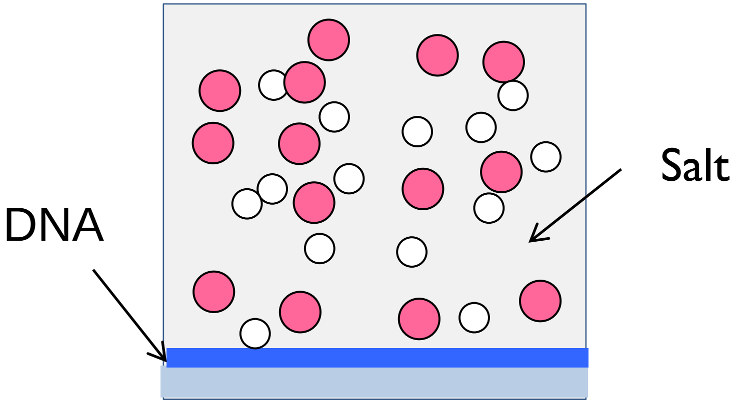


$$S \equiv \frac{I_{D,after} - I_{D,before}}{I_{D,before}} \equiv \frac{\Delta I_D}{I_D} \propto \rho_0$$

DNA binding and Salt screening

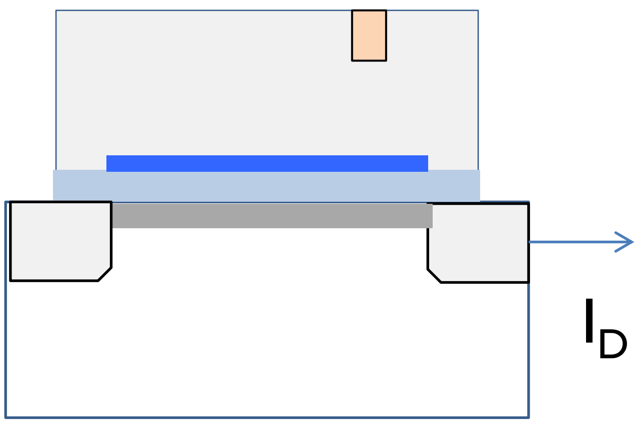


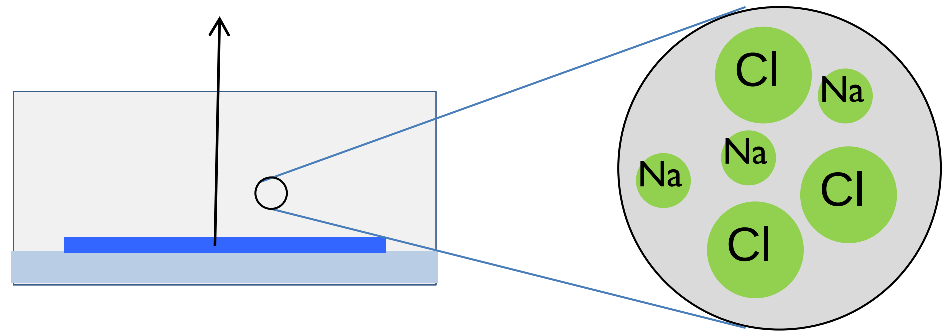




Salt and screening

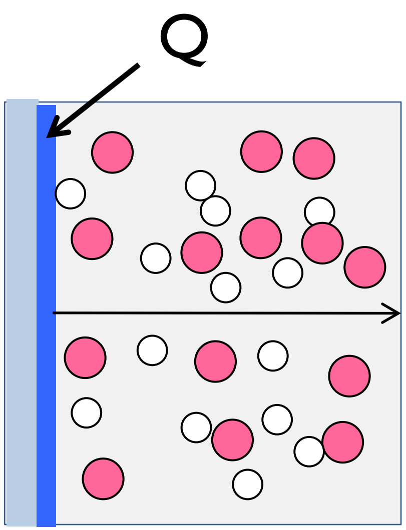
Salt is ionized in water.

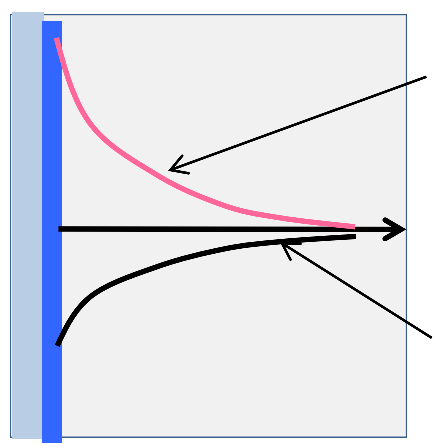




$$U = -\frac{q^2}{4\pi\varepsilon_0 r} \times \frac{1}{\kappa_w} \sim k_B T_L$$

Bjerrum length

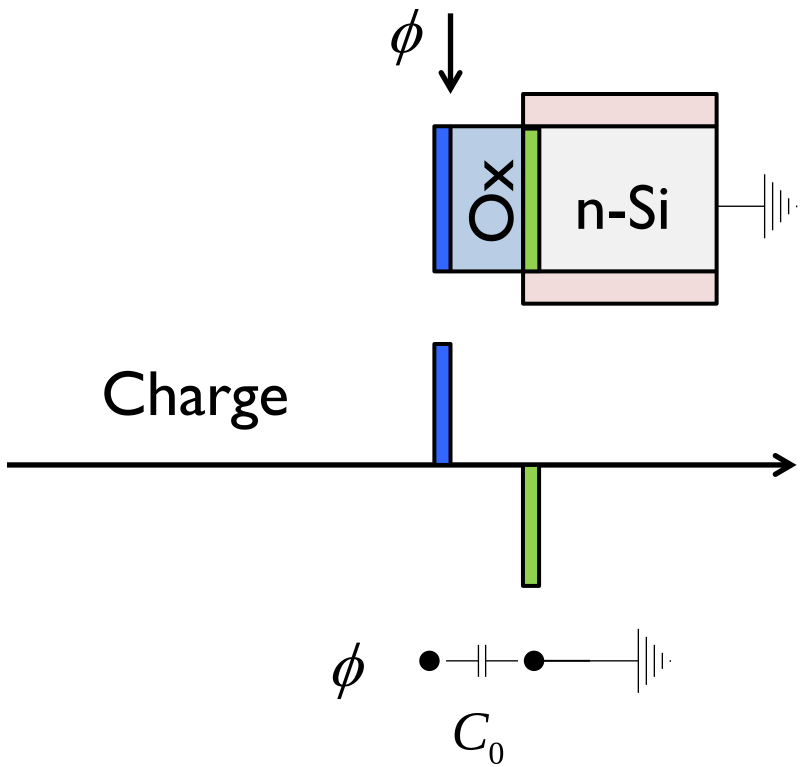




$$n^- \sim I_0 N_{av} e^{+q\psi/k_B T_L}$$

$$n^+ \sim I_0 N_{av} e^{-q\psi/k_B T_L}$$

How does a Ion-sensitive Field Effect Transistor work (accumulation)



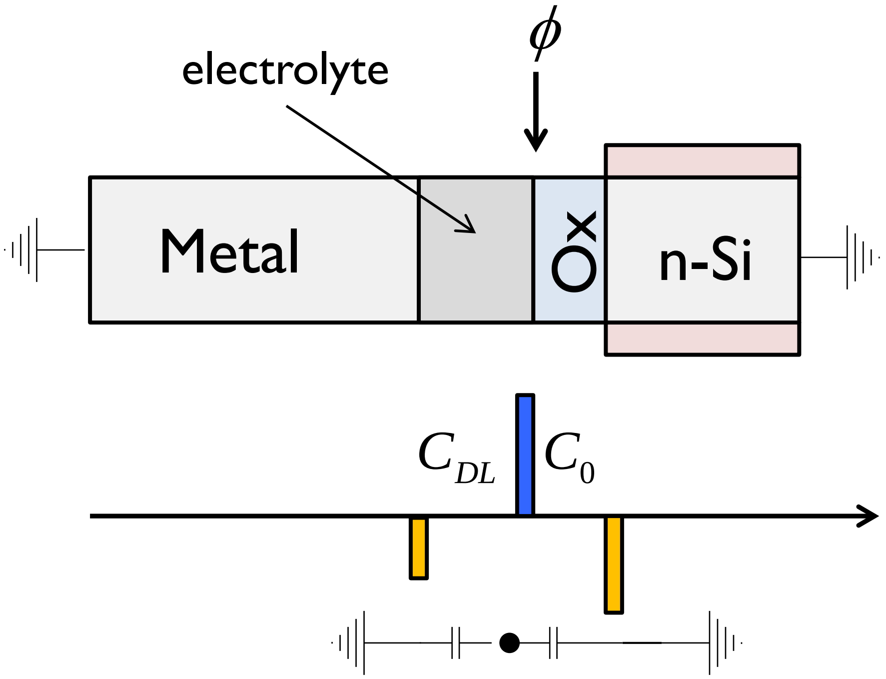
DNA, Protein, etc.

$$Q_{bio} = Q_{MOS} = C_0\phi$$

$$I = Q_{MOS}\upsilon = Q_{bio}\frac{\mu V_D}{L}$$

The response is proportional to the total charge of the biomolecule

ISFET in electrolyte (accumulation)



$$Q_{bio} = Q_L + Q_R = C_{DL}\phi + C_0\phi$$

$$\phi = \frac{Q_{bio}(\text{pH})}{C_{DL}(\text{salt}) + C_0}$$

$$Q_{MOS} = C_0 \frac{Q_{bio}}{C_{DL} + C_0}$$

$$I = Q_{MOS}\upsilon$$

Electrolyte reduces sensitivity

Calculating C_{DL}: screening in 1D

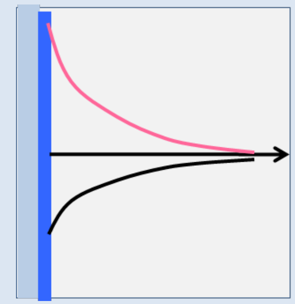
$$\nabla^2\psi = \frac{d}{dx}\left(\frac{d\psi}{dx}\right) = -\frac{\rho_{DL}}{\kappa_w\varepsilon_0}$$

$$= -\frac{zq}{\kappa_w\varepsilon_0}(n^+ - n^-)$$

$$= \frac{2zqI_0N_{av}}{\kappa_w\varepsilon_0}\sinh(zq\psi/k_BT)$$

$$\approx -\frac{\psi}{L_D^2} \quad (\text{if } \psi \sim k_BT_L)$$

$$\psi(x) = \psi_0 \times e^{-x/L_D}$$



$$\psi(0) = \psi_0, \quad \psi(\infty) = 0$$

$$n^- = I_0N_{av}e^{+zq\psi/k_BT_L}$$

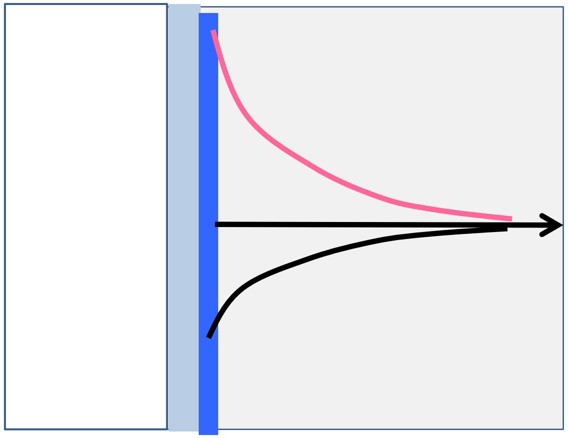
$$n^+ = I_0N_{av}e^{-zq\psi/k_BT_L}$$

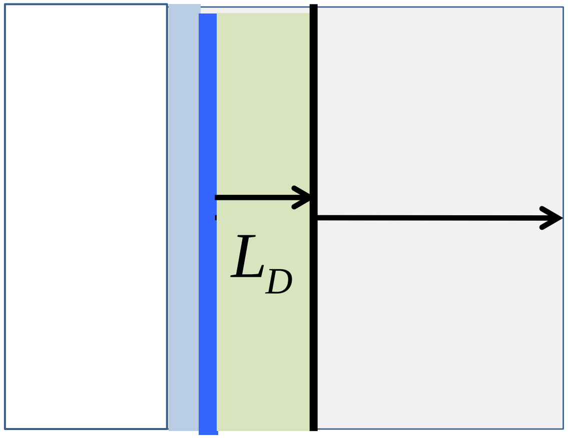
$$L_D^2 \equiv \frac{\kappa_w\varepsilon_0k_BT_L}{2z^2I_0N_{av}q^2}$$

Alam, Principles of Nanobiosensors, 2013

Centroid of DL charge & capacitance

$$\langle x \rangle = \frac{\int_0^\infty x \rho_{DL}(x) dx}{\int_0^\infty \rho_{DL}(x) dx} = L_D$$





$$\rho_{DL}(x) = -\frac{zq}{\kappa_w \varepsilon_0}(n^+ - n^-)$$

$$\approx -\frac{zq}{\kappa_w \varepsilon_0} \frac{zq}{k_B T_L} \times \psi$$

$$= -\frac{z^2 q^2}{\kappa_w \varepsilon_0 k_B T_L} \times \psi(0) e^{-\frac{x}{L_D}}$$

$$C_{DL} = \frac{k_w \varepsilon_0}{\langle x \rangle} = \frac{k_w \varepsilon_0}{L_D}$$

Screening in 1D

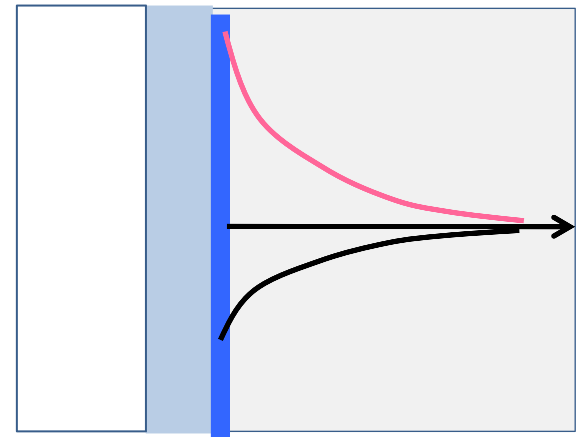
$$Q_{MOS} = \frac{C_{ox}}{C_{ox} + C_{DL}} Q_{bio}$$

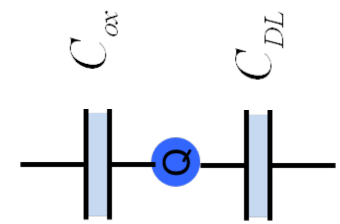
$$= \frac{k_{ox}\varepsilon_0 / T_{ox}}{k_{ox}\varepsilon_0 / T_{ox} + k_w\varepsilon_0 / L_D} Q_{bio}$$

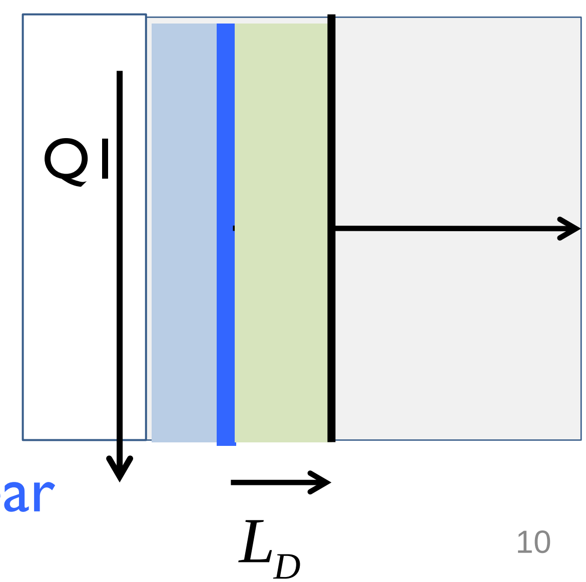
$$\rightarrow \frac{k_{ox}}{k_w} \frac{L_D}{T_{ox}} \rho_0 \quad (L_D \sim T_{ox})$$

$$L_D^2 = \kappa_w \varepsilon_0 k_B T_L / 2z^2 I_0 N_{av} q^2$$

$$S \propto Q_{MOS} \propto \rho_0 / \sqrt{I_0}$$







Increase in salt density decreases sensitivity

Response scaled & time-dependent, but still linear

At high salt concentration (Gouy-Chapman theory)

$$Q_{DL} \sim Q_0 \exp\left(\frac{zq\psi_0}{2k_B T_L}\right)$$

$$Q_0 \equiv \sqrt{2k_B T_L k_w \varepsilon_0 I_0 N_{avg}}$$

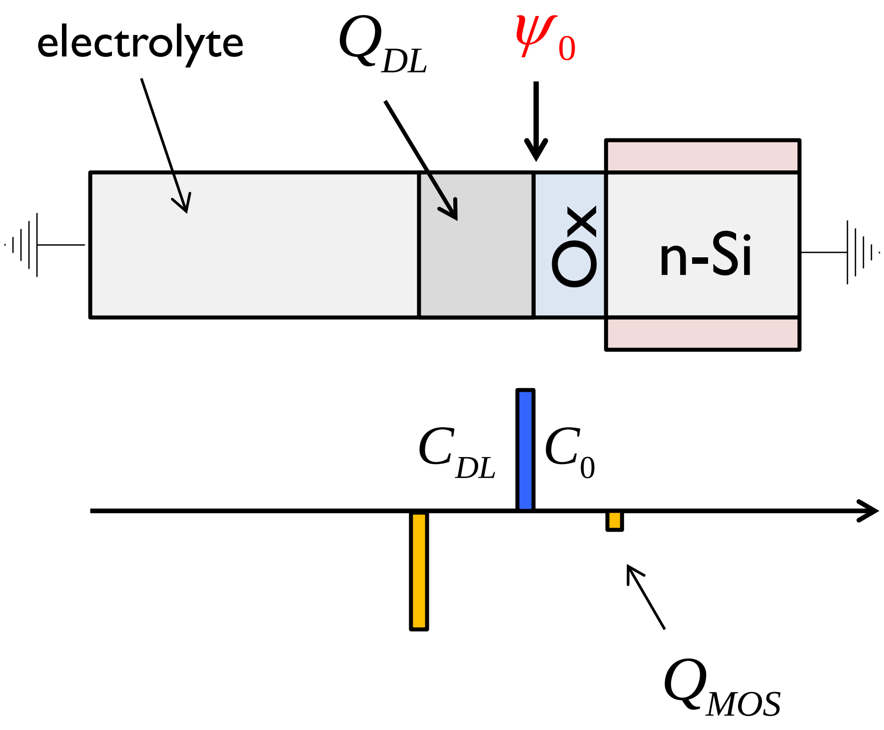
$$Q_{bio} = Q_{MOS} + Q_{DL}$$

$$Q_{bio} = C_{ox}\psi_0 + Q_{DL}$$

$$= C_{ox}\psi_0 + Q_0 \exp\left(\frac{zq\psi_0}{2k_B T_L}\right)$$

@ low salt concentration

$$Q_{DL} \propto C_{DL}\psi_0 = \frac{k_w \varepsilon_0}{L_D}\psi_0$$

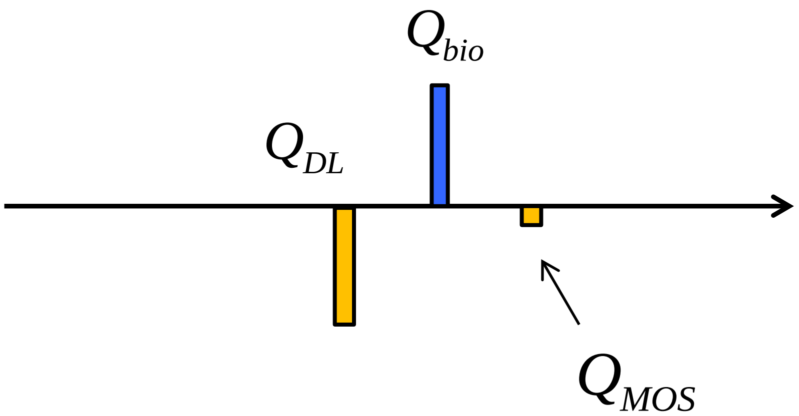


Strong screening (perturbative solution)

If MOS charge is small ....

$$Q_{bio} \approx Q_0 \exp\left(\frac{zq\psi_0}{2k_BT_L}\right)$$

$$\psi_0 = \frac{2k_BT_L}{zq}\ln\frac{Q_{bio}}{Q_0}$$

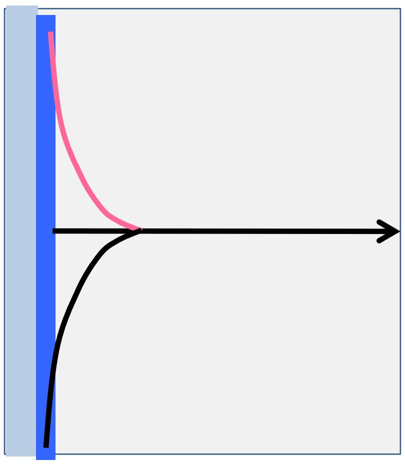


$$Q_{MOS} \approx C_{ox}\psi_0 \propto \frac{2k_BT_L}{zq}\ln\frac{Q_{bio}}{Q_0}$$

$$= \frac{k_BT_L}{q}\ln\frac{\sigma_T N(t)}{Q_0} = \frac{k_BT_L}{q}\ln\left(\frac{\sigma_T \times k\rho_0 \times t^{(3-D_F/2)}}{Q_0}\right)$$

Strong screening (perturbative solution)

$$Q_{MOS} \approx \frac{k_B T_L}{q} \ln\left(\frac{\sigma_T \times k \rho_0 \times t^{(3-D_F/2)}}{Q_0} \right)$$



$$S(t) \equiv \frac{\Delta I_D}{I_D} \propto Q_{MOS}$$

$$= c_1 \ln(\rho_0) + c_2 \frac{(3 - D_F)\ln(t)}{2} - c_3 \ln(I_0) + c_4$$

- $c_1 \ln(\rho_0)$ ↑ Response logarithmic in density, as observed experimentally
- $c_2 \frac{(3 - D_F)\ln(t)}{2}$ ↑ Increases with time, but logarithmically
- $c_3 \ln(I_0)$ ↑ Increasing salt decreases response

Conclusions

- Potentiometric sensors rely on charges of biomolecules for detection
- Many biomolecules can only survive at high –salt concentration
- The screening associated with the salt reduces charge reflected in the channel.
- The screening depends on a complex interplay of multiple variables.

Review questions

- If we bring a pair of DNA in water vs. in air, which configuration would have stronger repulsion?
- What is the role of salt in water in stabilizing the DNA pair? Can it play the same role in air?
- Why does increasing salt concentration reduce sensitivity? Explain physically.
- If we reduce the oxide thickness of a biosensor , would it increase or decrease the sensitivity? What about increasing the dielectric constant?

References

- Nair, Pradeep R., and Muhammad A. Alam. "Screening-limited response of nanobiosensors." *Nano letters* 8.5 (2008): 1281-1285.
- Nair, Pradeep R., and Muhammad A. Alam. "Design considerations of silicon nanowire biosensors." *Electron Devices, IEEE Transactions on* 54.12 (2007): 3400-3408.
- Vacic, Aleksandar, et al. "Determination of molecular configuration by debye length modulation." *Journal of the American Chemical Society* 133.35 (2011): 13886-13889.
- Baumgartner, Stefan, et al. "Predictive simulations and optimization of nanowire field-effect PSA sensors including screening." *Nanotechnology* 24.22 (2013): 225503.

Appendix

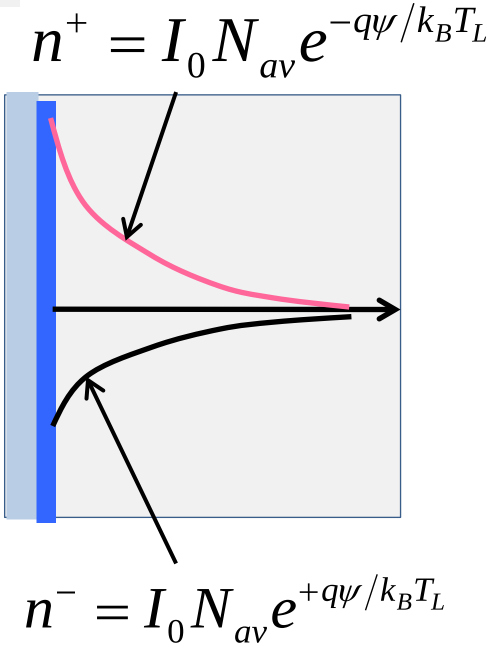
- Derivation of the Gouy-Chapman theory for high salt concentration

Strong screening: Gouy-Chapman model

Prove that: $$Q_{DL} = \sqrt{8k_B T_L k_w \varepsilon_0 I_0 N_{av}} \times \sinh\left(\frac{zq\psi_0}{k_B T_L}\right)$$

Boundary conditions: $\psi(0) = \psi_0, \quad \psi(\infty) = 0$

$$\frac{d^2\psi}{dx^2} = -\frac{\rho_{DL}}{\kappa_w \varepsilon_0} = -\frac{zq}{\kappa_w \varepsilon_0}(n^+ - n^-)$$



Recall that

$n^+ = I_0 N_{av} e^{-q\psi/k_B T_L}$ $\qquad n^- = I_0 N_{av} e^{+q\psi/k_B T_L}$

$$\frac{d^2\psi}{dx^2} = -\frac{\rho_{DL}}{\kappa_w \varepsilon_0} = -\frac{zq}{\kappa_w \varepsilon_0}(n^+ - n^-) = \frac{2zqI_0 N_{av}}{\kappa_w \varepsilon_0}\sinh(zq\psi/k_B T)$$

Gouy-Chapman model (2)

$$\frac{d^2\psi}{dx^2} = \frac{2zqI_0N_{av}}{\kappa_w\varepsilon_0}\sinh(zq\psi/k_BT)$$

$$2\frac{d\psi}{dx}\times\frac{d^2\psi}{dx^2} = \frac{d}{dx}\left(\frac{d\psi}{dx}\right)^2 = 2\frac{d\psi}{dx}\times\frac{2zqI_0N_{av}}{\kappa_w\varepsilon_0}\sinh\left(\frac{zq\psi}{k_BT}\right)$$

$$\int_0^\infty d\left(\frac{d\psi}{dx}\right)^2 = \int_{\psi_0}^0 \frac{4zqI_0N_{av}}{\kappa_w\varepsilon_0}\sinh\left(\frac{zq\psi}{k_BT}\right)d\psi$$

$$\cancel{\left(\frac{d\psi}{dx}\right)^2\Bigg|_{x=\infty}} - \left(\frac{d\psi}{dx}\right)^2\Bigg|_{x=0} = \frac{4I_0N_{av}k_BT}{\kappa_w\varepsilon_0}\left(\cosh\left(\frac{zq\psi_0}{k_BT}\right)-1\right)$$

Gouy-Chapman model (3)

$$\left(\frac{d\psi}{dx}\right)^2\Bigg|_{x=0} = \frac{4 I_0 N_{av} k_B T}{\kappa_w \varepsilon_0} \times 2 \sinh^2 \frac{z q \psi_0}{2 k_B T}$$

$$\left(\frac{d\psi}{dx}\right)\Bigg|_{x=0} = \sqrt{\frac{8 I_0 N_{av} k_B T}{\kappa_w \varepsilon_0}} \sinh \frac{z q \psi_0}{2 k_B T}$$

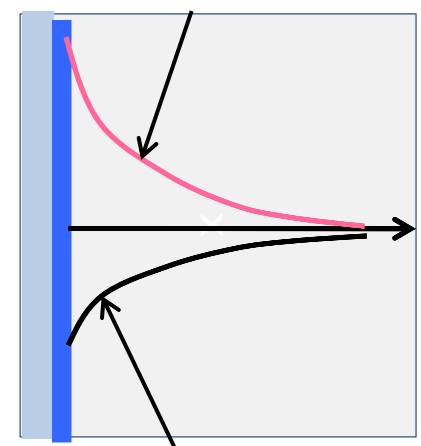
$$\frac{Q_{DL}}{k_w \varepsilon_0} = \mathcal{E} = \frac{d\psi}{dx}\Bigg|_{x=0}$$

Gauss's law @x=0

$$Q_{DL} = k_w \varepsilon_0 \sqrt{\frac{8 I_0 N_{av} k_B T}{\kappa_w \varepsilon_0}} \sinh \frac{z q \psi_0}{2 k_B T}$$

$$= \sqrt{8 k_B T_L k_w \varepsilon_0 I_0 N_{av}} \times \sinh\left(\frac{z q \psi_0}{2 k_B T_L}\right)$$

$$n^+ = I_0 N_{av} e^{-q\psi/k_B T_L}$$



$$n^- = I_0 N_{av} e^{+q\psi/k_B T_L}$$

Gouy-Chapman model (4)

$$Q_{DL} = k_w \varepsilon_0 \sqrt{\frac{8 I_0 N_{av} k_B T}{\kappa_w \varepsilon_0}} \sinh \frac{z q \psi_0}{2 k_B T}$$

$$= \sqrt{8 k_B T_L k_w \varepsilon_0 I_0 N_{av}} \times \sinh\left(\frac{z q \psi_0}{2 k_B T_L} \right)$$

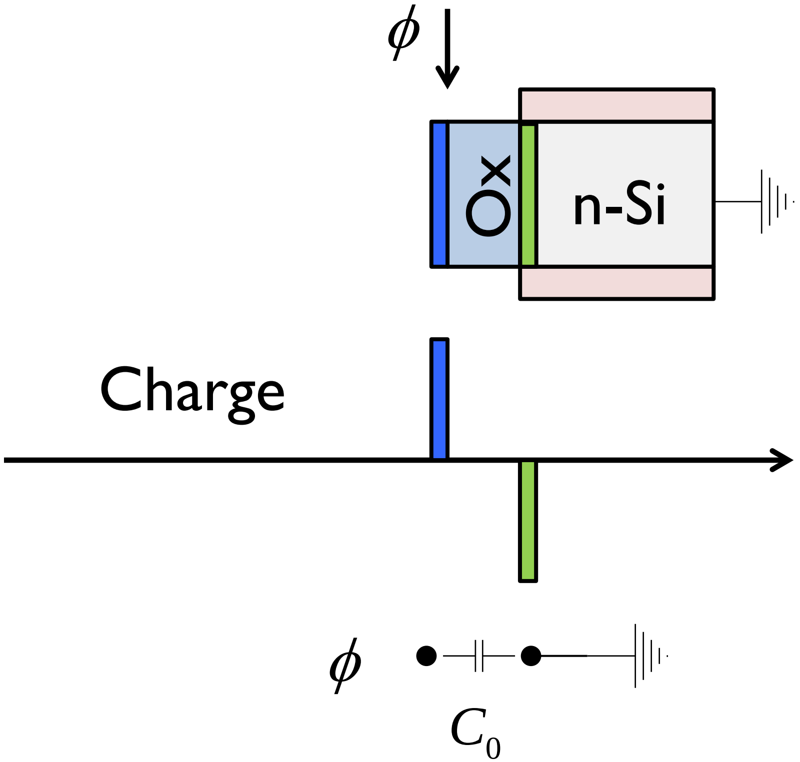
$$\approx Q_0 \times \exp \frac{z q \psi_0}{2 k_B T_L}$$

$$Q_0 \equiv \sqrt{2 k_B T_L k_w \varepsilon_0 I_0 N_{avg}}$$

Appendix

- Modes of operation of a field-effect transistor

How does a Ion-sensitive Field Effect Transistor work (accumulation)



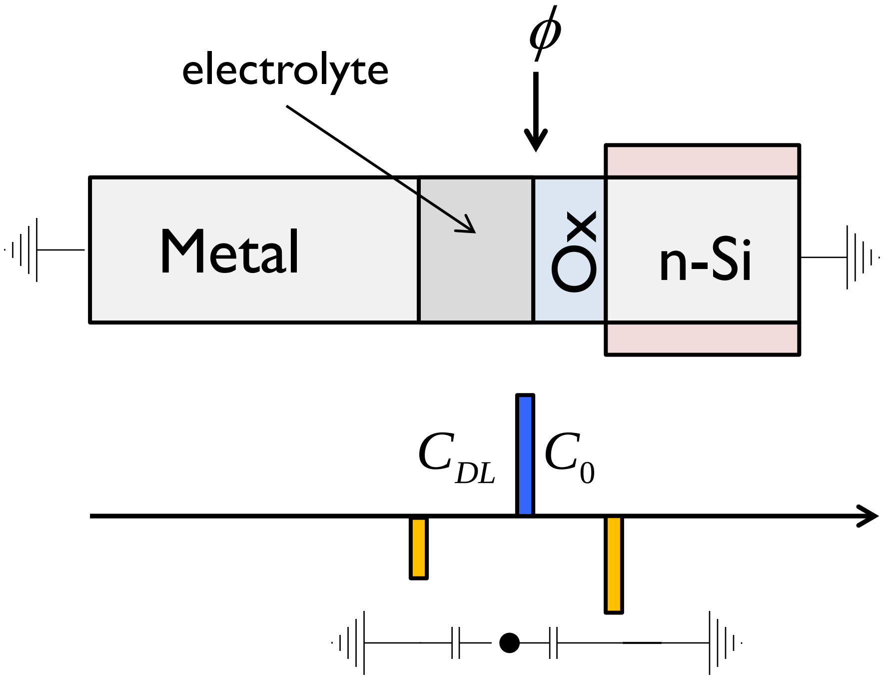
DNA, Protein, etc.

$$Q_{bio} = Q_{MOS} = C_0\phi$$

$$I = Q_{MOS}\upsilon = Q_{bio}\frac{\mu V_D}{L}$$

The role of charge can be viewed as shifting the flat-band voltage

ISFET in electrolyte (accumulation)



$$Q_{bio} = Q_L + Q_R = C_{DL}\phi + C_0\phi$$

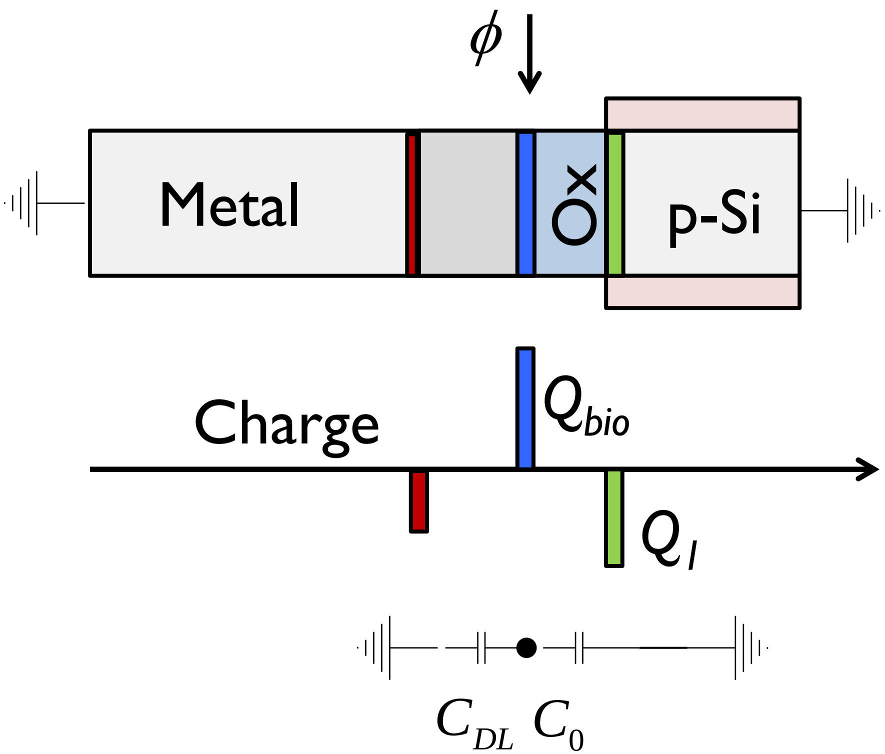
$$\phi = \frac{Q_{bio}(\text{pH})}{C_{DL}(\text{salt}) + C_0}$$

$$Q_{MOS} = C_0 \frac{Q_{bio}}{C_{DL} + C_0}$$

$$I = Q_{MOS}\upsilon$$

Electrolyte reduces sensitivity

How does an ISFET work (inversion)

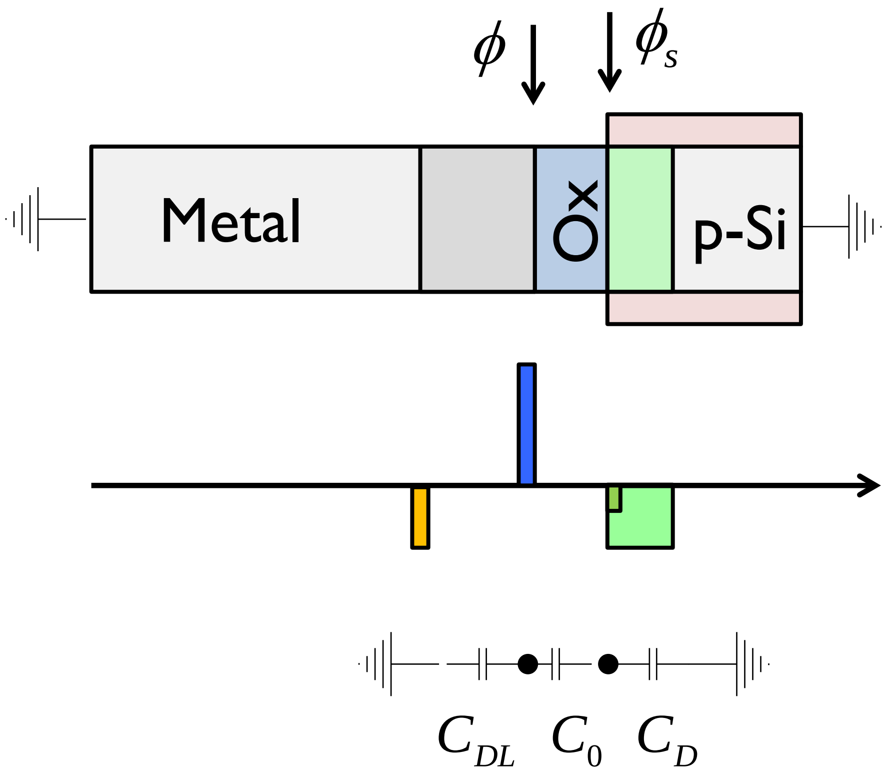


$$\phi = \frac{Q_{bio}}{C_{DL} + C_0}$$

$$Q_1 = C_0 \frac{Q}{C_{DL} + C_0}$$

$$\begin{aligned} Q_T &= C_O(V_G - V_T) + Q_{MOS} \\ &= C_O(V_G + \phi - V_T) \\ I_D &= Q_T \upsilon \\ \Delta I_D &= Q_{MOS} \upsilon \end{aligned}$$

How does an ISFET work (subthreshold)



$$m = \frac{C_0}{C_D + C_0}$$

$$\phi_s = \frac{V_G + \phi}{m}$$

$$\phi = \frac{Q_{bio}}{C_{DL} + C_0}$$

$$I = q\frac{n_i^2}{N_A} e^{\frac{q\phi_s}{k_B T}} \times \upsilon_D$$

$$= q\frac{n_i^2}{N_A} e^{\frac{q(V_G + \phi)}{mkT}} \times \upsilon_D$$

Although I have used ϕ as an effective gate voltage, it should be really be viewed as a change in flat-band voltage