Principles of Electronic Nanobiosensors

Unit 3: Sensitivity

Lecture 3.11: Cantilever-based Sensors: Static Response

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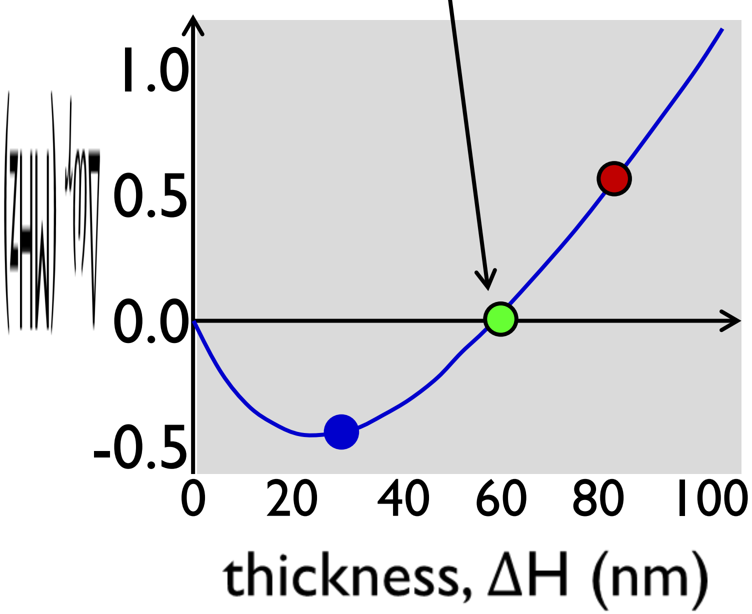
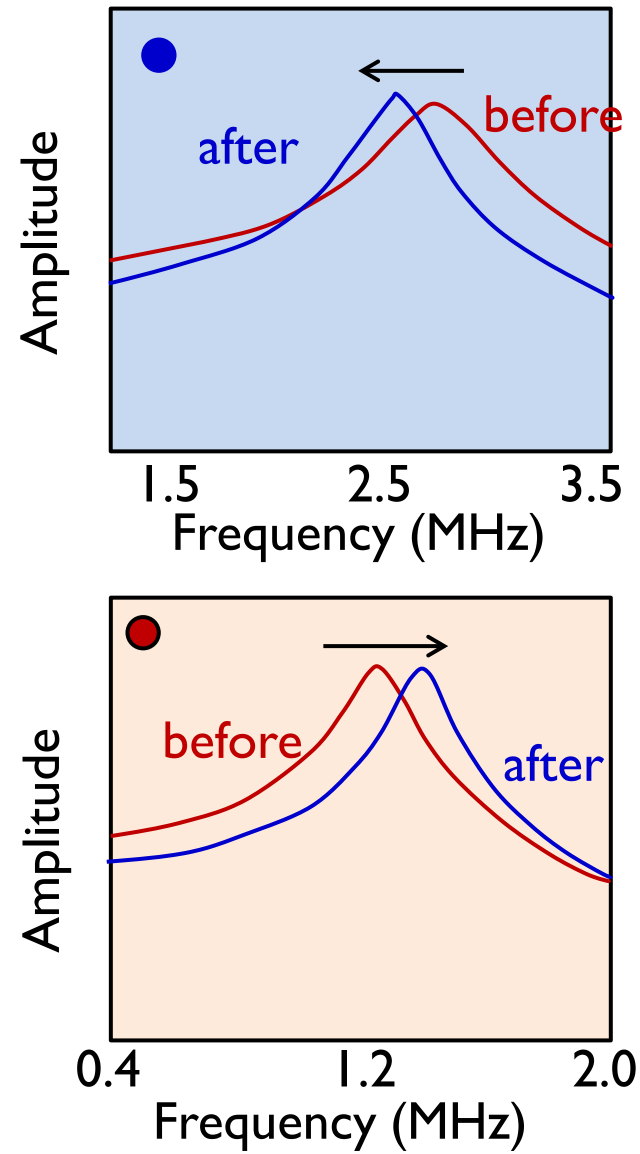
Outline

- Review : dynamic biosensing
- Static biosensing
  - Challenges of mass-based sensing
  - Nonlinear biosensing with electrostatic actuation
- Conclusion

Reversal of frequency at nanoscale

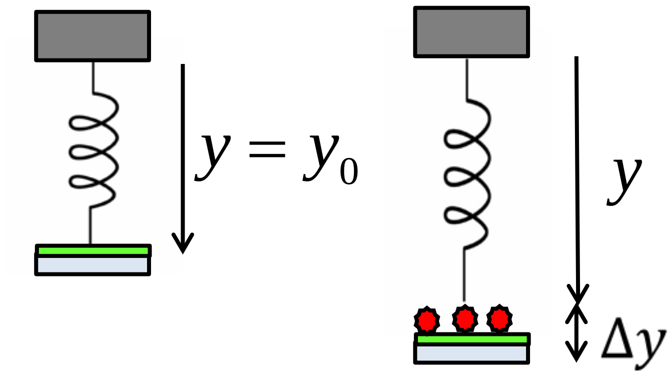
$$\frac{dH_c}{H} = \sqrt{\frac{E_b \rho_a}{E_a \rho_b}} - 1$$



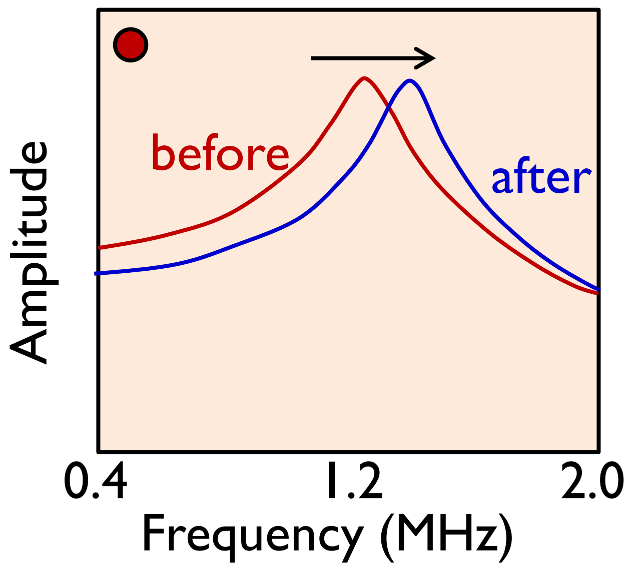


balance between
mass and spring effects

Broadening of frequency response



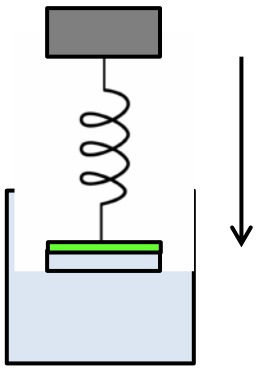
$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$



Response is broaden, and the peak is displaced.

Frequency change due to damping (in water) in response to Transient Input

$$m\frac{d^2y}{dt^2}+\gamma\frac{dy}{dt}+ky=F_{ext}=0$$



Solution:

$$y=Ae^{-\alpha\omega_0 t}\sin(\omega' t+\phi)$$

$$\alpha=\frac{\gamma}{2m\omega_0}\equiv\frac{1}{2Q} \qquad \omega'=\left(\frac{k}{m}-\frac{\gamma^2}{4m^2}\right)^{\frac{1}{2}}$$

The response is damped and the natural frequency changed.

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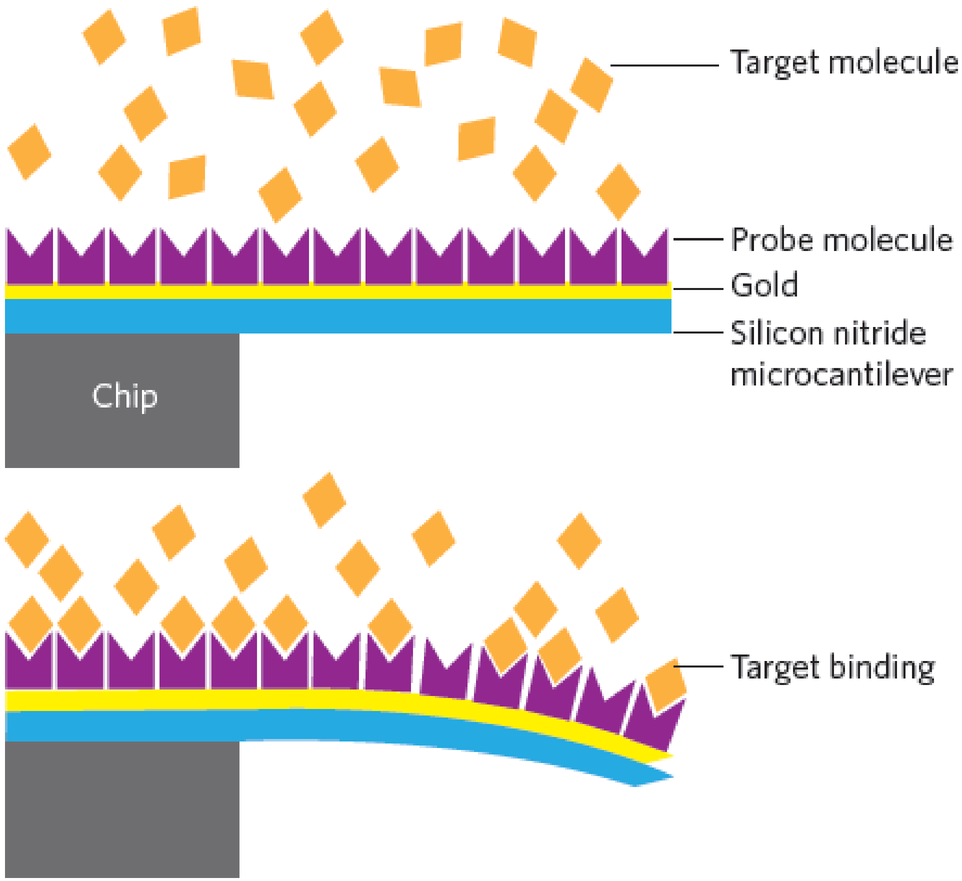
Static deflection can be measured



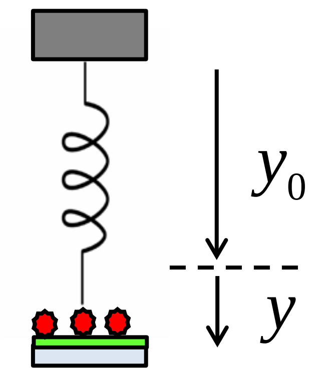
Mass increase
Surface stress
Spring constant

Static detection: At your supermarket

$$m\frac{d^2y}{dt^2} + \gamma\frac{dy}{dt} + ky = F = mg$$

$$k \equiv \alpha_2 \frac{EI}{L^3}$$

$$ky = F = mg$$



$$\frac{\Delta k}{k_0} + \frac{\Delta y}{y_0} = \frac{\Delta m}{m_0}$$

$$\frac{\Delta y}{y_0} = \frac{\Delta m}{m_0} - \frac{\Delta k}{k_0}$$

the mass change vs. the change in stiffness …

An Example: Mass-induced deflection

Si beam: L=5 um, W=1.5um H=25nm, Density=2330 Kg/m^3

Protein: L=5 um, W=1.5um H=50nm, Density=1220 Kg/m^3

$$\frac{\Delta y}{y_b} = \frac{\Delta m}{m_b} = \frac{\rho_T \Delta HWL}{\rho_b HWL} = \frac{1220 \times 50}{2330 \times 25} \sim 1$$

Great sensitivity

$$y = \frac{m}{k} g = \frac{6.3 \times 10^{-17}\ \mathrm{Kg}}{0.0152\ \mathrm{N/m}} \times 9.8 \frac{\mathrm{m}}{\mathrm{s}^2} = 40.6\ \mathrm{fm}$$

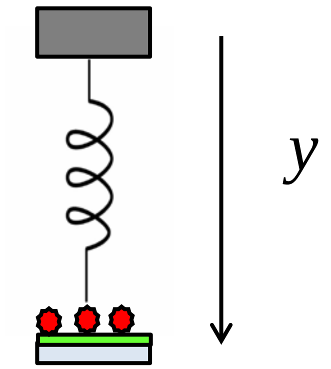
Unmeasurable!

If we could somehow soften the spring …. !

Static vs. dynamic response

$$y = A\cos(\omega t)$$

$$a \equiv \frac{d^2 y}{dx^2} = \omega^2 A$$



$$a = \omega^2 A$$

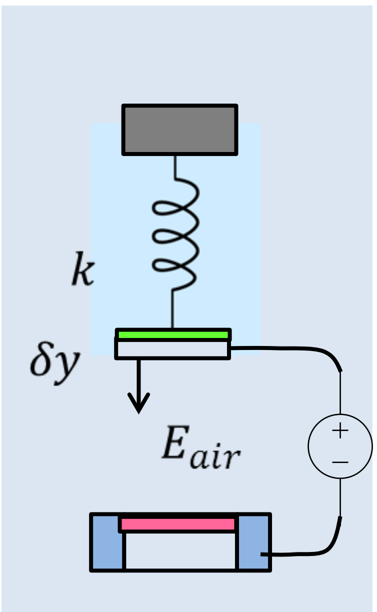
$$= (2\pi \times 2.4 \times 10^6)^2 \times 10^{-9}\,\mathrm{m}$$

$$= 2.27 \times 10^5\,\mathrm{m \bullet s^{-2}}$$

$$\sim 23{,}000\,g$$

High acceleration allowed measurement of small mass (essence of centrifuge operation)

Displacement with an static force



$$k \equiv \frac{\alpha_2 EI}{L^3}$$

Force balance equation

$$m\frac{d^2y}{dt^2} + \gamma\frac{dy}{dt} + k(y_0 - y) = F_{ext} + F_{elec}$$

At Steady state: $k(y_0 - y) = F_{ext} + F_{elec}$

Electrical force:

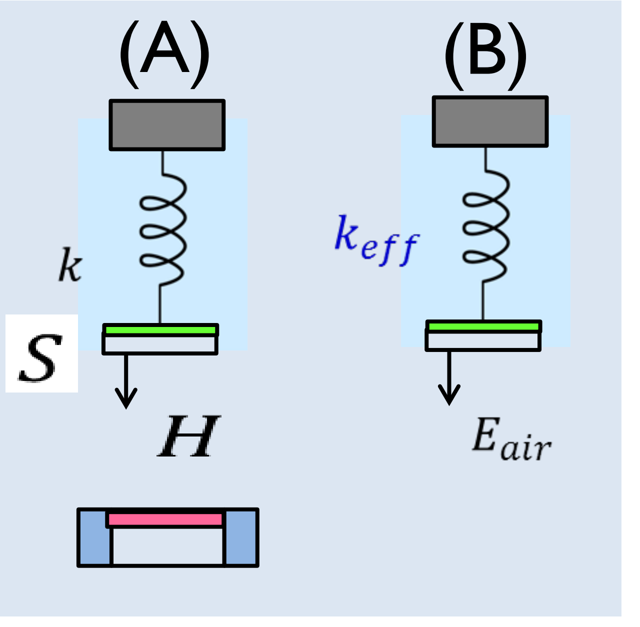
$$E = \frac{1}{2}CV^2$$

$$F_{elec} = \frac{dE}{dy} = -\frac{1}{2}V^2\frac{dC}{dy}$$

$$C = \frac{\varepsilon_0 A}{y}$$

$$C = \frac{\varepsilon A}{y}$$

Spring-softening



Equilibrium condition:

$$k(y_0 - y) = -\frac{d}{dy}\left(\frac{CV^2}{2}\right) = \frac{V^2 \varepsilon A}{2y^2}$$

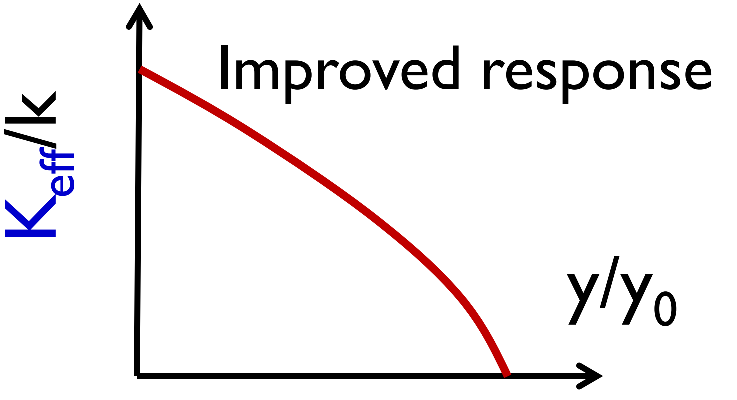
As if the spring has weakened ..

$$F_{ext} \equiv k(y_0 - y) + \frac{d}{dy}\left(\frac{CV^2}{2}\right)$$

Effective spring constant:

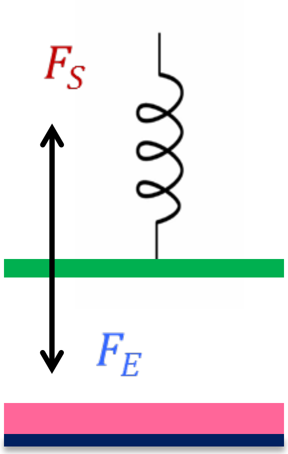
$$k_{eff} = -\frac{dF_{ext}}{dy} = k - \frac{2V^2 \varepsilon A}{2y^3}$$

$$k_{eff} = k - \frac{2}{y} k\left(y_0 - y\right) = k\left(3 - \frac{2y_0}{y}\right)$$



Alam, Principles of Nanobiosensors, 2013

Pull-in instability



$$E = \frac{1}{2} C V^2$$

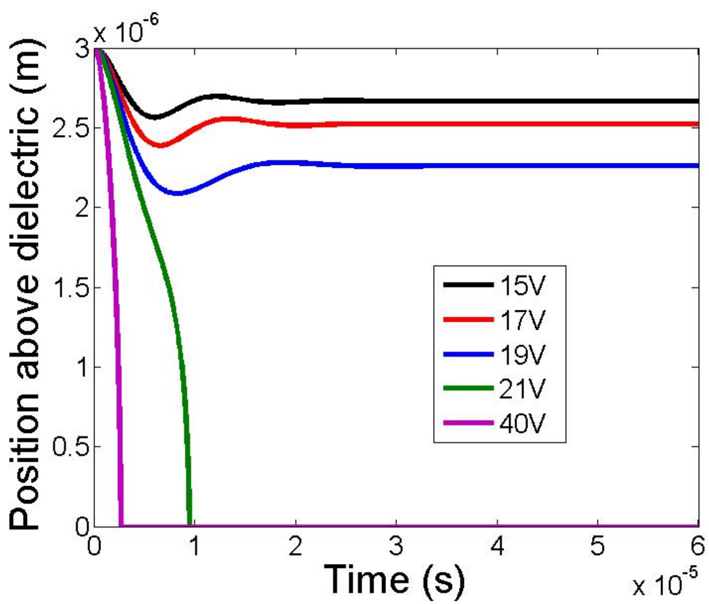
$$F_{elec} = \frac{dE}{dy} = -\frac{1}{2} V^2 \frac{dC}{dy}$$

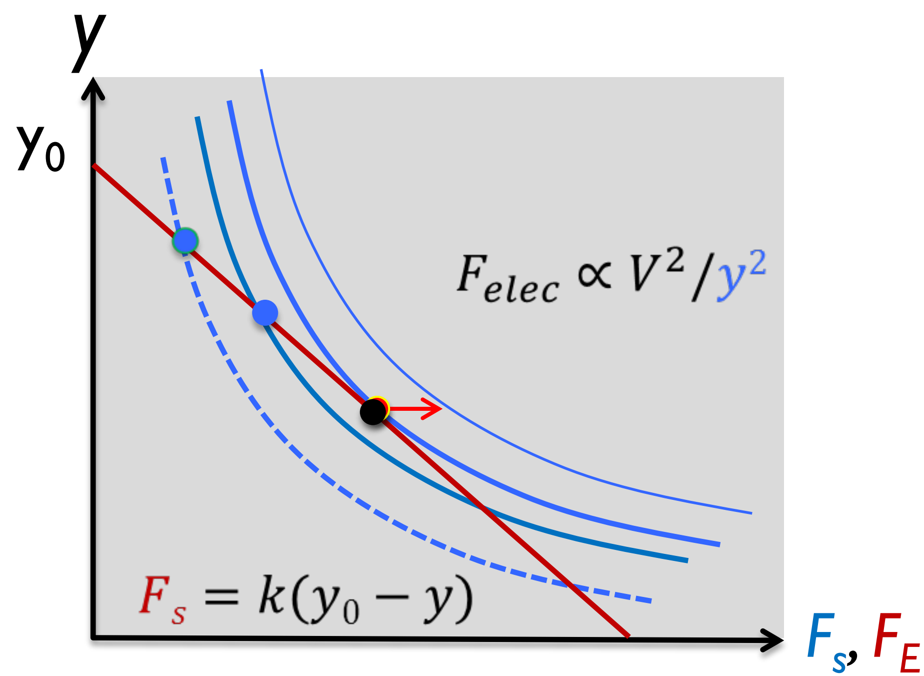
$$C = \frac{\varepsilon_0 A}{y}$$

$$F_{elec} = F_s$$

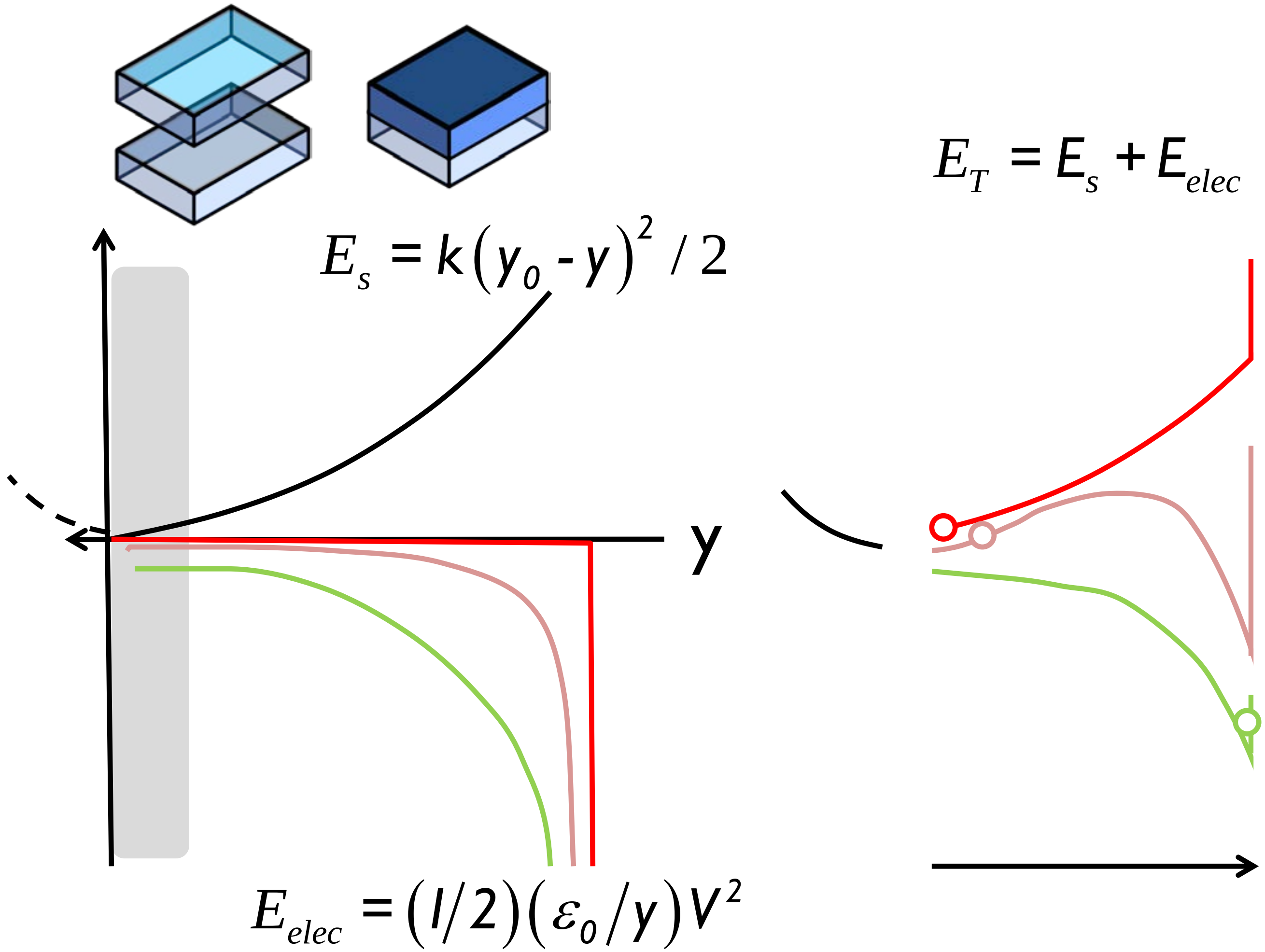
$$\frac{dF_{elec}}{dy} = \frac{dF_S}{dy}$$

$$y^* = \frac{2}{3} y_0$$





Energy Landscape of Spring weakening



Effective spring constant changes due to changes in energy landscape

Conclusions

- Dynamic biosensing is widely used. Frequency reversal and fluid damping at the nanoscale make things complicated.
- Pure mass-based static deflection is difficult to measure.
- Spring-softening can be achieved by changes in material properties and cantilever geometry, but electrostatic actuation provides a simpler approach.

References

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Homework: Resonance in CNT

1 The following example is taken from Zhong et al. NL, 2010. A CNT is suspended over a ridge defined by two SiO2 pillers. Assume the following parameters for CNT: d=2nm, L=3 micron, E=1 TPa, mass density, mu=5x10e-15 (kg/m), moment of inertia (I) = 7x10e-37, \epsilon=8.854e-12, and height y_0=1 um.

(a) Find the effective spring constant, k, for the ssytem. Show that the resonant frequency is given by ~11 MHz.
(b) If k changes by 10% due to biomolecule absorption, find the corresponding change in frequency.
(c) Derive an expression for spring hardening to show f=f_0+ A V_gate^2, where A ~ C \sqrt (1/ mu E I). Recall that C = 2\pi \epsilon_0 L/ ln (2 y_0/d).