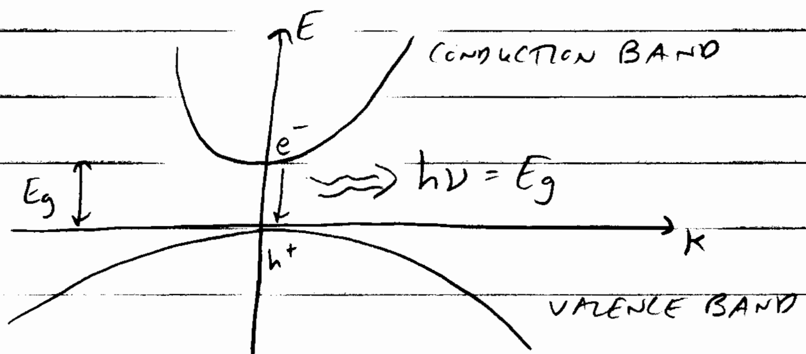
# LECTURE #24

## CLASSES OF MATERIALS:

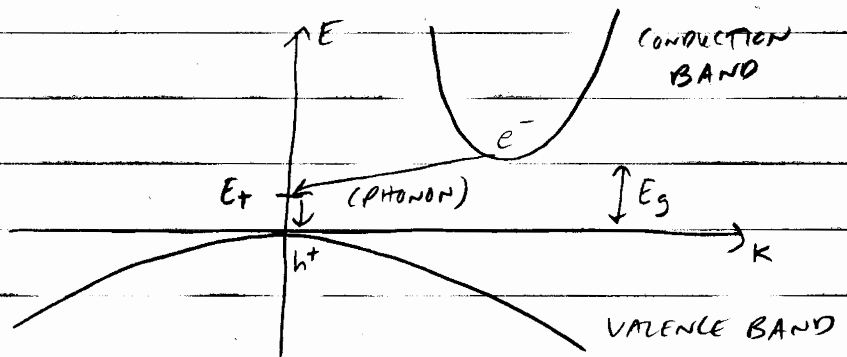
(1) METALS: $E_F$ WITHIN ONE BAND
(2) SEMIMETALS: $E_F$ WITHIN TWO OVERLAPPING BANDS
(3) INSULATORS: $E_F$ WITHIN LARGE BAND GAP (TYPICALLY > 2eV)
(4) SEMICONDUCTORS: $E_F$ WITHIN SMALL BAND GAP (TYPICALLY < 2eV)

FOR SEMICONDUCTORS, THERE ARE TWO SUBCLASSES OF BANDSTRUCTURE:

(A) DIRECT GAP:
(e.g., GaAs)




(B) INDIRECT GAP:
(e.g, Si)




* REAL SEMICONDUCTORS WILL HAVE MULTIPLE MINIMA IN THE E-K DIAGRAM THAT WILL VARY IN ENERGY AS A FUNCTION OF COMPOSITION.

SEE FIG. 3-6

IN MOST CIRCUMSTANCES, ELECTRONS IN THE CONDUCTION BAND WILL BE NEAR THE BOTTOM OF THE BAND. CONSEQUENTLY, THE E-K DIAGRAM IS APPROXIMATELY PARABOLIC WITH AN EFFECTIVE MASS ($m_n^*$)

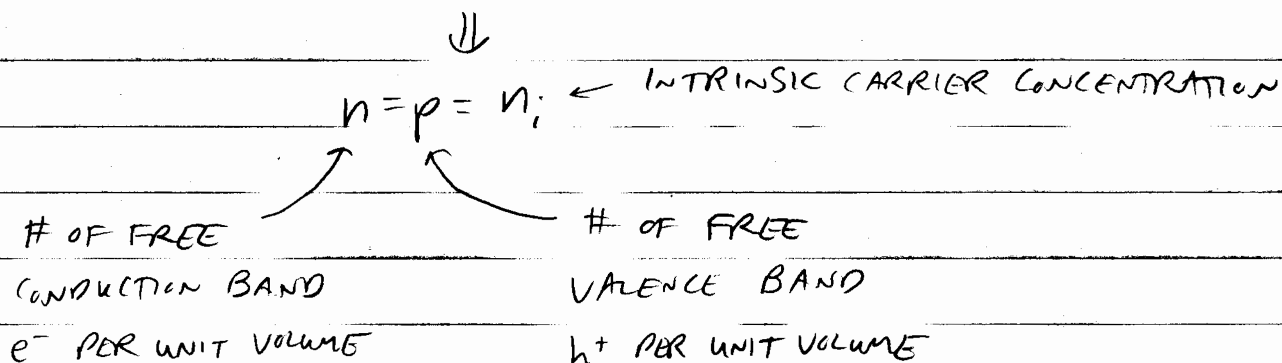
$$E = \frac{\hbar^2 k^2}{2m_n^*} + E_g$$

SIMILARLY, MISSING ELECTRONS IN THE VALENCE BAND WILL BE NEAR THE TOP OF THE BAND. THE NEGATIVE EFFECTIVE MASS AND NEGATIVE CHARGE IS EQUIVALENT TO POSITIVE EFFECTIVE MASS ($m_p^*$) AND POSITIVE CHARGE OF HOLES.

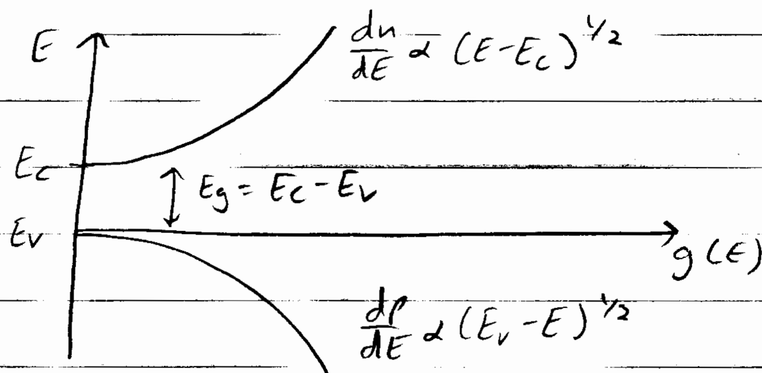
## INTRINSIC SEMICONDUCTORS:

FOR $T > 0K$, THE FERMI-DIRAC DISTRIBUTION IS THERMALLY BROADENED. EQUIVALENTLY, WE CAN INTERPRET THIS SITUATION AS ELECTRONS FROM THE VALENCE BAND BEING THERMALLY ACTIVATED TO THE CONDUCTION BAND. THIS THERMAL PROMOTION CREATES <u>TWO</u> FREE CHARGE CARRIERS (ONE ELECTRON AND ONE HOLE).

$\Downarrow$



SINCE ELECTRONS AND HOLES NEAR THE CONDUCTION AND VALENCE BANDEDGES RESPECTIVELY BEHAVE LIKE FREE PARTICLES WITH AN EFFECTIVE MASS $m^*$, THE DENSITY OF STATES IS THE SAME AS FREE PARTICLES WITH MASS $m^*$:




$$\frac{dn}{dE} = \frac{(2m_n^*)^{3/2}}{2\pi^2\hbar^3}(E-E_c)^{1/2}$$

$$\frac{dp}{dE} = \frac{(2m_p^*)^{3/2}}{2\pi^2\hbar^3}(E_v-E)^{1/2}$$

$$n = \int_{\substack{\text{CONDUCTION}\\\text{BAND}}} \frac{dn}{dE} f(E)\,dE$$

IF $E-E_F \gg K_BT$

$$f(E) = \frac{1}{1+e^{(E-E_F)/K_BT}} \approx e^{-(E-E_F)/K_BT}$$

↑ FERMI-DIRAC ↑ MAXWELL-BOLTZMANN

<u>NOTE</u>: NONDEGENERATE SEMICONDUCTORS BEHAVE LIKE CLASSICAL CASES OF DISTINGUISHABLE PARTICLES.

SINCE $f(E)$ EXPONENTIALLY DECREASES AS $E$ INCREASES, THE PRODUCT, $\frac{dn}{dE} f(E)$, IS CONCENTRATED NEAR THE BAND EDGE

**SEE FIG. 3.16**

THIS IMPLIES: (1) THE VALUE OF $\frac{dn}{dE}$ AWAY FROM THE BANDEDGE IS IRRELEVANT.

(2) THE VALUE OF THE ENERGY AT THE TOP OF THE BAND CAN BE ASSUMED TO BE ARBITRARILY LARGE.

$$\therefore n \approx \frac{(2m_n^*)^{3/2}}{2\pi^2 \hbar^3} \int_{E_c}^{\infty} (E-E_c)^{1/2} e^{-(E-E_F)/K_BT} dE$$

LET $x = (E-E_c)/K_BT$, $dx = \frac{1}{K_BT} dE$

$$\therefore n = \frac{(2m_n^* K_BT)^{3/2}}{2\pi^2 \hbar^3} e^{-(E_c-E_F)/K_BT} \underbrace{\int_0^{\infty} x^{1/2} e^{-x} dx}_{\sqrt{\pi}/2}$$

$$\therefore n = N_c e^{-(E_c-E_F)/K_BT} \quad \text{WHERE } N_c = 2\left(\frac{m_n^* K_BT}{2\pi \hbar^2}\right)^{3/2}$$

$N_c$ ← EFFECTIVE DENSITY OF STATES IN THE CONDUCTION BAND

<u>NOTE</u>: $n$ INCREASES AS $E_F$ MOVES NEAR $E_c$.

FOR HOLES, $p = \int_{\text{VALENCE BAND}} \frac{dp}{dE}(1-f(E))\, dE$

NOTE: $1 - f(E) = 1 - \frac{1}{1+e^{(E-E_F)/k_BT}} = \frac{e^{(E-E_F)/k_BT}}{1+e^{(E-E_F)/k_BT}}$

$$= \frac{1}{1+e^{(E_F-E)/k_BT}} \approx e^{-(E_F-E)/k_BT}$$

← IF $E_F - E \gg k_BT$

$$\therefore p \approx \frac{(2m_p^*)^{3/2}}{2\pi^2\hbar^3} \int_{-\infty}^{E_V} (E_V - E)^{1/2} e^{-(E_F-E)/k_BT} dE$$

LET $x = (E_V - E)/k_BT$, $dx = -\frac{1}{k_BT} dE$

$$\therefore p = \frac{(2m_p^* k_BT)^{3/2}}{2\pi^2\hbar^3} e^{-(E_F-E_V)/k_BT} \underbrace{\int_0^\infty x^{1/2} e^{-x} dx}_{\sqrt{\pi}/2}$$

$\therefore p = N_V e^{-(E_F-E_V)/k_BT}$ WHERE $N_V = 2\left(\frac{m_p^* k_BT}{2\pi\hbar^2}\right)^{3/2}$

← EFFECTIVE DENSITY OF STATES IN THE VALENCE BAND

NOTE: $p$ INCREASES AS $E_F$ MOVES NEAR $E_V$.

FOR INTRINSIC SEMICONDUCTORS, LET $E_F = E_i$

SINCE $n = p = n_i$, $N_c e^{-(E_c-E_i)/k_BT} = N_V e^{-(E_i-E_V)/k_BT}$

$$e^{2E_i/k_BT} = \frac{N_V}{N_c} e^{(E_V+E_c)/k_BT}$$

$$\frac{2E_i}{k_BT} = \frac{E_V+E_c}{k_BT} + \frac{3}{2}\ln\frac{m_p^*}{m_n^*}$$

$$E_i = \frac{E_V}{2} + \frac{E_C}{2} + \frac{3}{4} K_B T \ln\left(\frac{m_p^*}{m_n^*}\right)$$

$$= E_V + \frac{E_C - E_V}{2} + \frac{3}{4} K_B T \ln\left(\frac{m_p^*}{m_n^*}\right)$$

$$\therefore \boxed{E_i = E_V + \frac{E_g}{2} + \frac{3}{4} K_B T \ln\left(\frac{m_p^*}{m_n^*}\right)}$$

→ THE FERMI LEVEL IS NEARLY IN THE MIDDLE OF THE BAND GAP FOR INTRINSIC SEMICONDUCTORS

SINCE $n = p = n_i \Rightarrow np = n_i^2$ ← LAW OF MASS ACTION

$$\therefore n_i^2 = N_c e^{-(E_c - E_F)/K_B T} N_V e^{-(E_F - E_V)/K_B T}$$

$$n_i^2 = N_c N_V e^{-E_g/K_B T}$$

$$\therefore \boxed{n_i = \sqrt{N_c N_V}\, e^{-E_g/2K_B T}}$$

→ THE INTRINSIC CARRIER CONCENTRATION INCREASES EXPONENTIALLY WITH TEMPERATURE

SEE FIG. 3.17

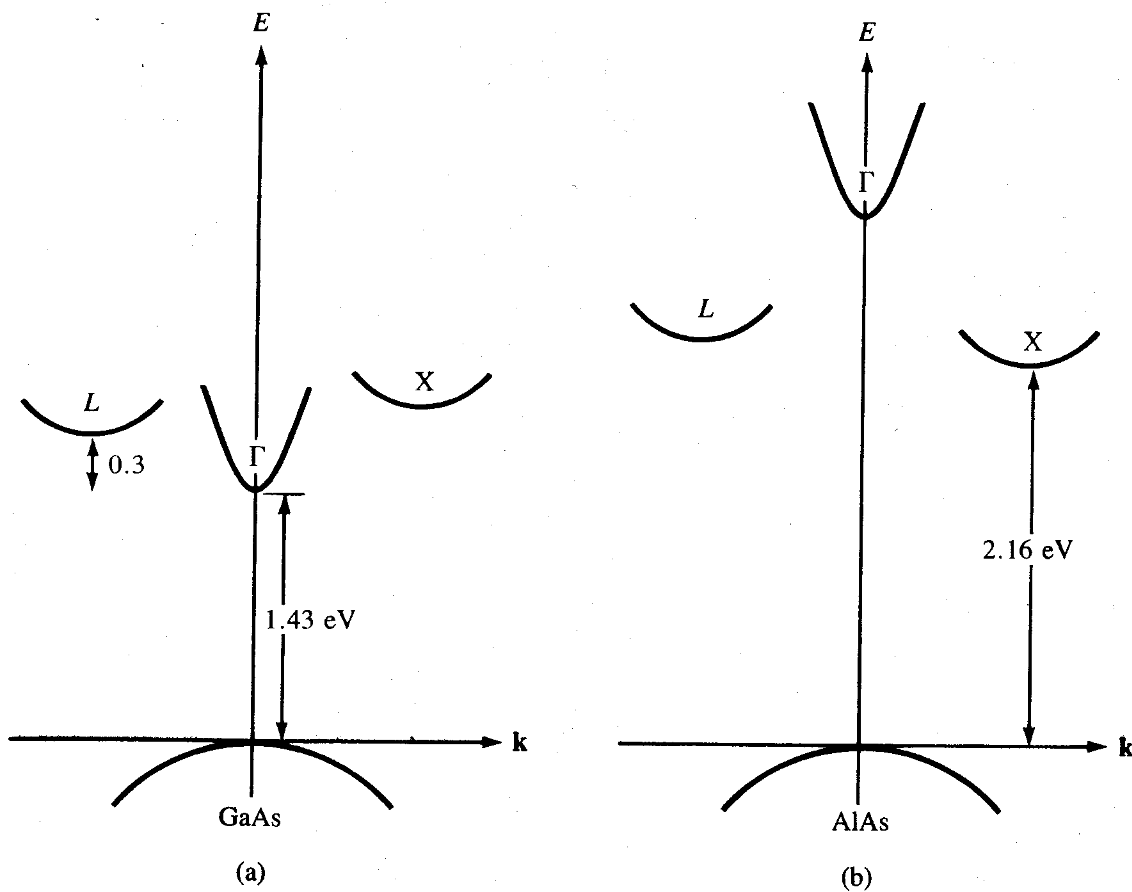



**Figure 3-6**
Variation of direct and indirect conduction bands in AlGaAs as a function of composition: (a) the $(E, \mathbf{k})$ diagram for GaAs, showing three minima in the conduction band; (b) AlAs band diagram;

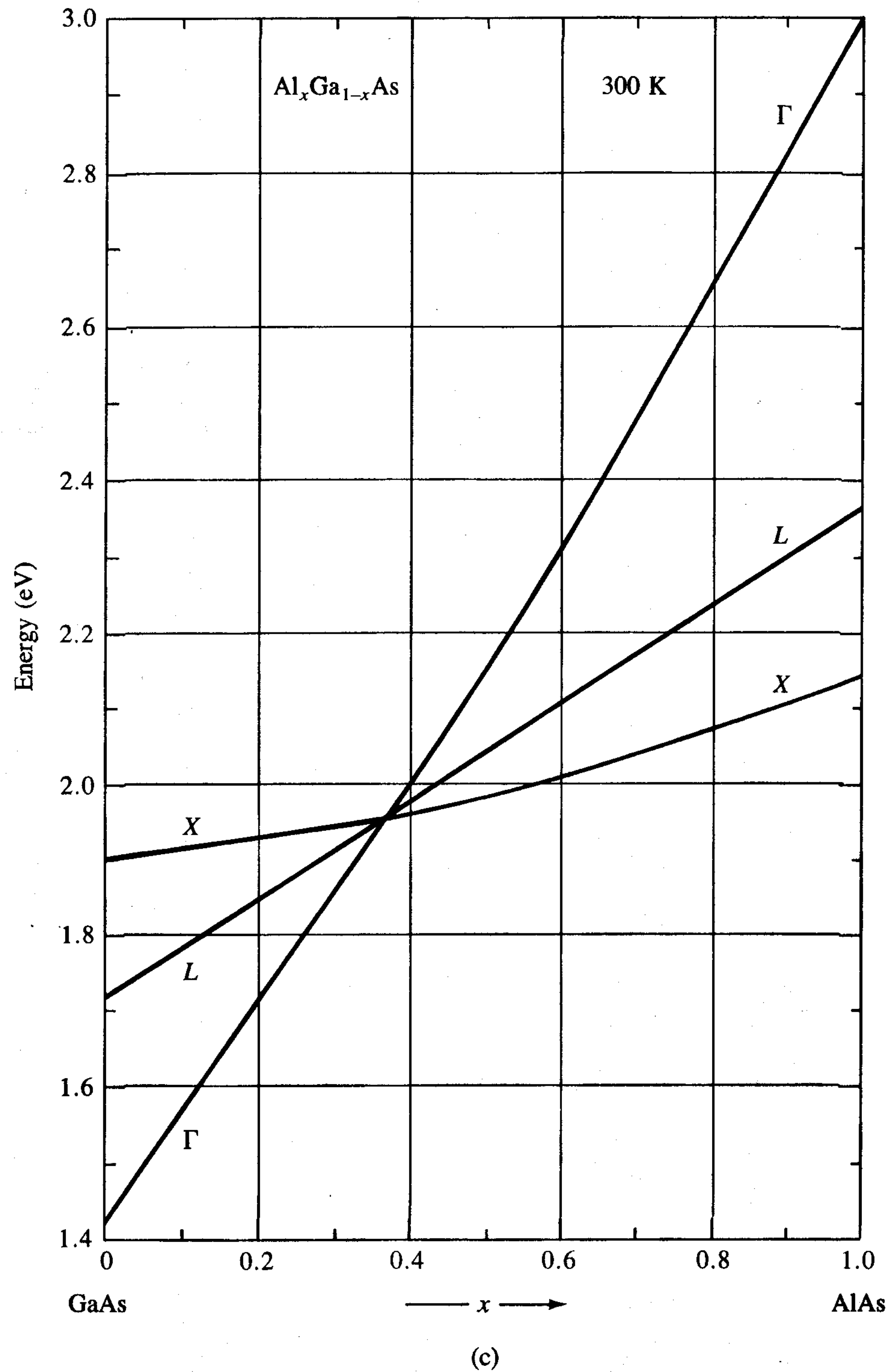
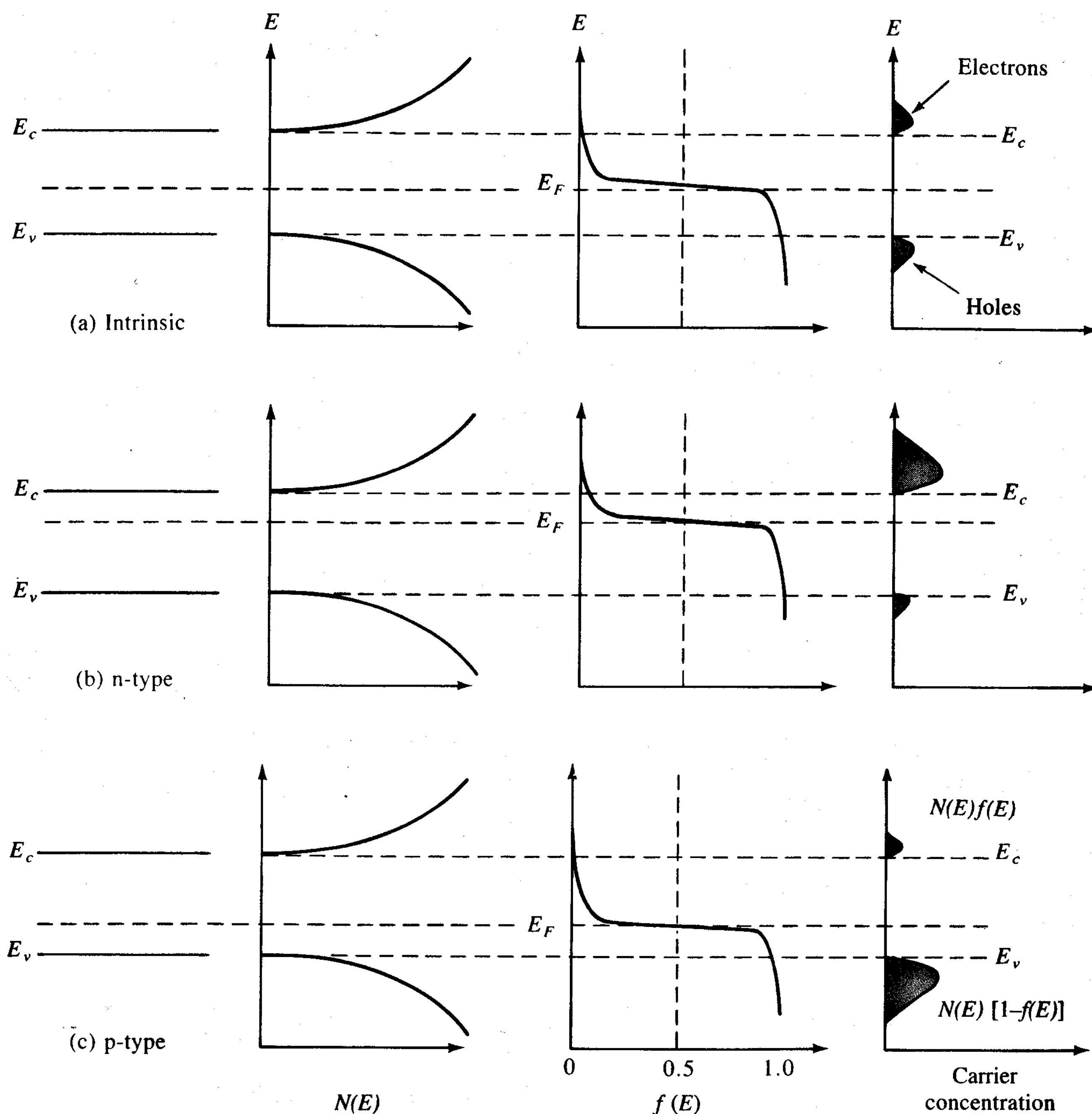



**Figure 3-6 (cont.)** (c) positions of the three conduction band minima in $Al_xGa_{1-x}As$ as $x$ varies over the range of compositions from GaAs ($x = 0$) to AlAs ($x = 1$). The smallest band gap, $E_g$ (shown in color), follows the direct Γ band to $x = 0.38$, and then follows the indirect X band.




Schematic band diagram, density of states, Fermi–Dirac distribution, and the carrier concentrations for (a) intrinsic, (b) n-type, and (c) p-type semiconductors at thermal equilibrium. **Figure 3-16**

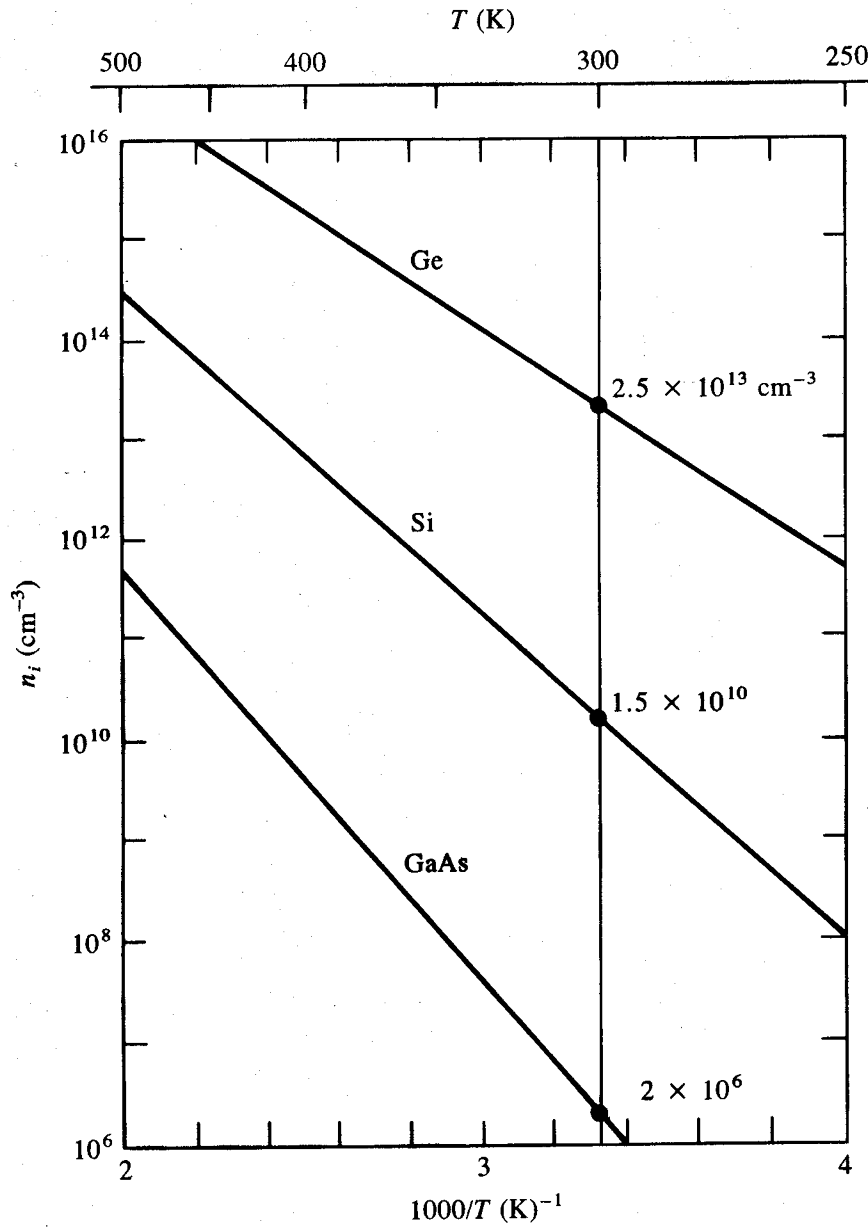



**Figure 3-17** Intrinsic carrier concentration for Ge, Si, and GaAs as a function of inverse temperature. The room temperature values are marked for reference.