LECTURE #27

SPATIALLY VARYING CARRIER CONCENTRATIONS:

WHEN THERE IS A GRADIENT IN THE FREE CARRIER CONCENTRATION, THERE WILL BE NET MOTION OF CARRIERS FROM REGIONS OF HIGH CONCENTRATION TO REGIONS OF LOW CONCENTRATION → SEE FIG. 4-12

$\Downarrow$

DIFFUSION CURRENT

CONCENTRATION (cm^{-3})

FICK'S FIRST LAW: (1-D) $$\phi = -D\frac{dC}{dx}$$

DIFFUSION COEFFICIENT (cm^2/s)

FLUX DENSITY ($\frac{1}{m^2 \cdot s}$)

$$\therefore J_n = -(-e)D_n\frac{dn}{dx} = eD_n\frac{dn}{dx}$$

$$J_p = -(e)D_p\frac{dp}{dx} = -eD_p\frac{dp}{dx}$$

DIFFUSION CURRENT

IN THE PRESENCE OF AN ELECTRIC FIELD AND CONCENTRATION GRADIENTS,

$$\boxed{\begin{aligned} J_n(x) &= \underbrace{e\mu_n n(x)\mathcal{E}(x)}_{\text{DRIFT}} + \underbrace{eD_n\frac{dn(x)}{dx}}_{\text{DIFFUSION}} \\ J_p(x) &= \underbrace{e\mu_p p(x)\mathcal{E}(x)}_{\text{DRIFT}} - \underbrace{eD_p\frac{dp(x)}{dx}}_{\text{DIFFUSION}} \\ J_{TOTAL}(x) &= J_n(x) + J_p(x) \end{aligned}}$$

→ SEE FIG. 4-14

<u>NOTE</u>: DRIFT CURRENT IS PROPORTIONAL TO THE NUMBER OF ELECTRONS AND/OR HOLES

$\Downarrow$

MAJORITY CARRIERS DOMINATE DRIFT CURRENT

<u>HOWEVER</u>, DIFFUSION CURRENT IS PROPORTIONAL TO THE DERIVATIVE OF THE NUMBER OF ELECTRONS AND/OR HOLES

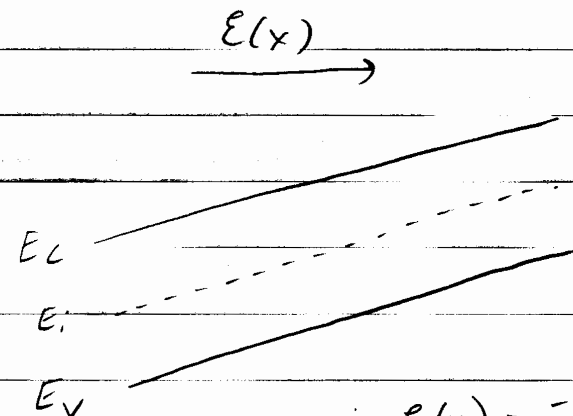
$\Downarrow$

MINORITY CARRIERS CAN CONTRIBUTE SIGNIFICANTLY TO CURRENT FLOW

<u>NOTE</u>: SINCE $\mathcal{E}(x) = -\frac{dV(x)}{dx}$ AND

$E(x) = -eV(x)$ FOR ELECTRONS,

THE ENERGY BAND DIAGRAM IN THE PRESENCE OF AN ELECTRIC FIELD IS:



$$\therefore \mathcal{E}(x) = \frac{-dV(x)}{dx} = -\frac{d}{dx}\left[\frac{E_i(x)}{-e}\right] = \frac{1}{e}\frac{dE_i}{dx}$$

$\Rightarrow$ ELECTRONS DRIFT DOWNHILL IN THE BAND DIAGRAM

AT EQUILIBRIUM, NO NET CURRENT FLOWS IN A SEMICONDUCTOR.

$\Downarrow$

ANY FLUCTUATION THAT WOULD INITIATE A DIFFUSION CURRENT ALSO SETS UP AN ELECTRIC FIELD THAT REDISTRIBUTES CARRIERS BY DRIFT

$\Downarrow$

GRADIENTS IN CONCENTRATION LEAD TO BUILT-IN FIELDS

FOR EXAMPLE, CONSIDER HOLES:

$$J_p(x) = e\mu_p\, p(x)\, \mathcal{E}(x) - eD_p \frac{dp(x)}{dx} = 0$$

$$\therefore \mathcal{E}(x) = \frac{D_p}{\mu_p} \frac{1}{p(x)} \frac{dp(x)}{dx}$$

<u>RECALL</u>: $p = n_i\, e^{(E_i - E_F)/k_B T}$

$$\therefore \frac{dp}{dx} = n_i\, e^{(E_i - E_F)/k_B T} \left[\frac{dE_i}{dx} - \frac{dE_F}{dx} \right] \frac{1}{k_B T}$$

$$\therefore \mathcal{E}(x) = \frac{D_p}{\mu_p k_B T} \Big(\underbrace{\frac{dE_i}{dx}}_{e\mathcal{E}(x)} - \frac{dE_F}{dx} \Big)$$

ZERO IN EQUILIBRIUM

$$\therefore \frac{D_p}{\mu_p} = \frac{k_B T}{e}$$

THE SAME RESULT IS OBTAINED FOR ELECTRONS

$$\therefore \boxed{\frac{D}{\mu} = \frac{k_B T}{e}}$$

<u>EINSTEIN RELATION</u>

THUS FAR, WE HAVE TREATED ELECTRONS AND HOLES AS INDEPENDENT CARRIERS THAT UNDERGO CHARGE TRANSPORT WITHIN THE CONDUCTION BAND AND VALENCE BAND RESPECTIVELY.

HOWEVER, INTER-BAND ELECTRON-HOLE RECOMBINATION / GENERATION CAN OCCUR, WHICH LEAD TO GRADIENTS IN CARRIER CONCENTRATIONS AND THUS DIFFUSION CURRENTS.

THE NET RECOMBINATION / GENERATION RATE WILL BE NONZERO WHEN THE CARRIER CONCENTRATION DEVIATES FROM EQUILIBRIUM VALUES.

$\therefore$ ASSUMING FIRST-ORDER KINETICS,

$$\left(\frac{dn}{dt}\right)_{g-r} = -\frac{(n-n^\circ)}{\tau_n^{g-r}} = -\frac{\Delta n}{\tau_n^{g-r}}$$

$$\left(\frac{dp}{dt}\right)_{g-r} = -\frac{(p-p^\circ)}{\tau_p^{g-r}} = -\frac{\Delta p}{\tau_p^{g-r}}$$

<u>NOTE</u>: THE RECOMBINATION TIMES (τ_n^{g-r}, τ_p^{g-r}) ARE DIFFERENT FROM THE COLLISION TIMES (τ_n^{coll}, τ_p^{coll})

<u>RECOMBINATION</u>: <u>INTER</u>-BAND PROCESS (RELATIVELY UNLIKELY)

<u>COLLISION</u>: <u>INTRA</u>-BAND (MUCH MORE LIKELY)

$$\therefore \underbrace{\tau^{coll}}_{\sim 1\,ps} \ll \underbrace{\tau^{g-r}}_{\sim 1\,\mu s}$$

CONSIDER HOLE CURRENT ENTERING AND LEAVING A VOLUME $\Delta x A$

← CROSS-SECTIONAL AREA

$$\left.\frac{\partial p}{\partial t}\right|_{x \to x+\Delta x} = \frac{1}{e} \frac{J_p(x) - J_p(x+\Delta x)}{\Delta x} - \frac{\Delta p}{\tau_p^{g-r}}$$

AS $\Delta x \to 0$,

$$\boxed{\frac{\partial p}{\partial t} = -\frac{1}{e} \frac{\partial J_p}{\partial x} - \frac{\Delta p}{\tau_p^{g-r}}}$$

CONTINUITY EQUATION FOR HOLES

SIMILARLY, FOR ELECTRONS (WITH CHARGE $-e$),

$$\boxed{\frac{\partial n}{\partial t} = \frac{1}{e} \frac{\partial J_n}{\partial x} - \frac{\Delta n}{\tau_n^{g-r}}}$$

CONTINUITY EQUATION FOR ELECTRONS

WHEN THE CURRENT IS CARRIED STRICTLY BY DIFFUSION, $J_n = e D_n \frac{\partial n}{\partial x}$, $J_p = -e D_p \frac{\partial p}{\partial x}$

FURTHER ASSUMING THAT n^0 AND p^0 ARE INDEPENDENT OF TIME AND SPACE,

$$\boxed{\frac{\partial \Delta n}{\partial t} = D_n \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_n^{g-r}}}$$

DIFFUSION EQUATION FOR ELECTRONS

$$\boxed{\frac{\partial \Delta p}{\partial t} = D_p \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{\tau_p^{g-r}}}$$

DIFFUSION EQUATION FOR HOLES

EXAMPLE: STEADY STATE CARRIER INJECTION

i.e., $\frac{\partial \Delta n}{\partial t} = 0$ AND $\frac{\partial \Delta p}{\partial t} = 0$

$$\therefore \quad \frac{\partial^2 \Delta n}{\partial x^2} = \frac{\Delta n}{D_n \tau_n^{g-r}} = \frac{\Delta n}{L_n^2}$$

WHERE $L_n = \sqrt{D_n \tau_n^{g-r}}$ (ELECTRON DIFFUSION LENGTH)

SIMILARLY, $\frac{\partial^2 \Delta p}{\partial x^2} = \frac{\Delta p}{D_p \tau_p^{g-r}} = \frac{\Delta p}{L_p^2}$

WHERE $L_p = \sqrt{D_p \tau_p^{g-r}}$ (HOLE DIFFUSION LENGTH)

ASSUME THAT HOLES ARE INJECTED INTO A SEMICONDUCTOR SUCH THAT $\Delta p(x=0) = p_i$ (CONSTANT CARRIER INJECTION)

WHAT IS THE DISTRIBUTION OF EXCESS HOLES (Δp) AS A FUNCTION OF x?

GENERAL SOLUTION OF THE DIFFUSION EQUATION:

$$\Delta p(x) = C_1 e^{x/L_p} + C_2 e^{-x/L_p}$$

APPLY BOUNDARY CONDITIONS: $\Delta p(x=0) = p_i$, $\Delta p(x=\infty) = 0$

THE SECOND BOUNDARY CONDITION $\Rightarrow$ $C_1 = 0$.

THE FIRST BOUNDARY CONDITION $\Rightarrow$ $C_2 = p_i$

$\therefore \; \Delta p(x) = p_i \, e^{-x/L_p}$

$\Rightarrow$ L_p IS THE AVERAGE DISTANCE A HOLE DIFFUSES BEFORE RECOMBINING

$\Rightarrow$ THE SPATIALLY VARYING HOLE CONCENTRATION IMPLIES THAT A CURRENT WILL FLOW:

$$J_p(x) = -eD_p \frac{dp}{dx} = -eD_p \frac{d\,\Delta p}{dx} = e\frac{D_p}{L_p} p_i \, e^{-x/L_p}$$

$$\therefore \; \boxed{J_p(x) = e\frac{D_p}{L_p}\Delta p(x)}$$

THIS EXAMPLE IS PARTICULARLY IMPORTANT FOR THE INJECTION OF MINORITY CARRIERS ACROSS SEMICONDUCTOR JUNCTIONS (e.g., p-n JUNCTIONS)

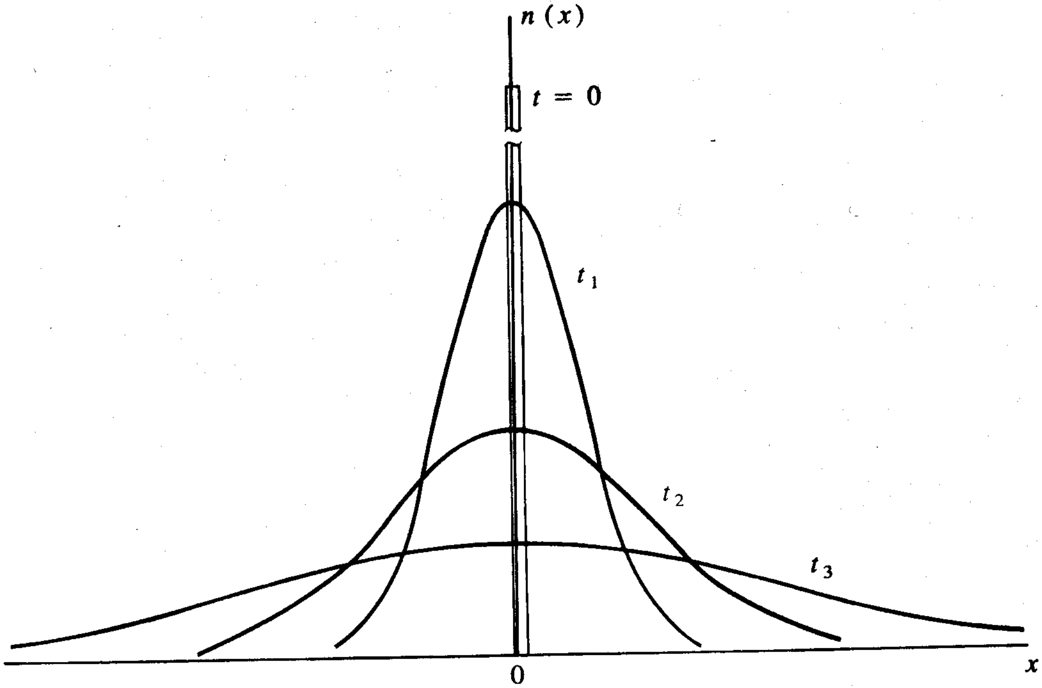
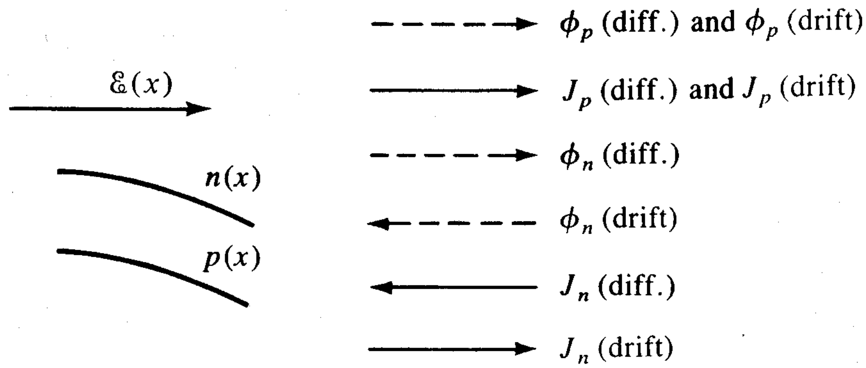


Figure 4-12
Spreading of a pulse of electrons by diffusion.



Drift and diffusion directions for electrons and holes in a carrier gradient and an electric field. Particle flow directions are indicated by dashed arrows, and the resulting currents are indicated by solid arrows.

Figure 4-14