NOTE ON CARRIER-CARRIER SCATTERING

In general, the electron gas displays both collective and individual particle aspects:

- The primary manifestations of the collective behavior are
  - organized oscillations of the system as a whole (so-called plasma oscillations)
  - screening of the field of any individual electron within a Debye length by the remainder of the electron gas

  In the collective oscillation, each individual electron suffers a small periodic perturbation of its velocity and position due to the combined potential of all other electrons in the system. The cumulative potential might be quite large since the long-range of the coulomb potential (interaction) permits a very large number of electrons to contribute to the potential at a given point

  The screening of the electronic fields may be viewed as arising from the coulomb repulsion, which causes the electrons to stay apart, and so it leads to a deficite of a negative charge in the immediate neighborhood of a given electron.

- When do we have binary and when collective behavior?

  The collective behavior of the electron gas is decisive for phenomena that involve distances that are larger than the Debye length

  For smaller distances, the electron gas is best considered as a collection of individual particles that interact weekly by means of screened Coulomb force. Therefore, the individual particles component is associated with the random thermal motion of the electrons and shows no collective behavior; it represents a collection of individual particles (electrons) surrounded by co-moving clouds of charge that acts to screen their fields. The individual particles

component thus includes the effects of the residual short-range screened Coulomb force, which leads only to two body collisions.

- For the collective description to be valid, it is necessary that the mean collision time τ_{coll} for electron collisions, which tend to disrupt the collective motion, be large compared to the period of a collective oscillation. This is because collisions cause damping of the collective oscillation, and the criterion:

$$\tau_{coll} \gg \frac{2\pi}{\omega_p} = 2\pi \left(\frac{m^* \varepsilon_\infty}{N_D e^2}\right)^{1/2}$$

is a criterion that such damping is small. If the above criterion is not satisfied, the damping is large and the whole concept of collective oscillation loses its significance in a description of electron interaction.

- Real values for GaAs samples:

$\underline{N_D = 10^{17}\,cm^{-3}}$, $\omega_p = \left(\frac{N_D e^2}{m^* \varepsilon_\infty}\right)^{1/2} \approx 2\times10^{13}$ $(\hbar\omega_p \approx 0.013\,eV)$

$\tau_{coll} \gg \frac{2\pi}{2\times10^{13}} \approx 3\times10^{-13}\ (s) \Rightarrow \frac{1}{\tau_{coll}} \ll 3\times10^{12}\ (\frac{1}{s})$ not really satisfied

$\underline{N_D = 10^{18}\,cm^{-3}}$, $\omega_p \approx 6.32\times10^{13}$ $(\hbar\omega_p \approx 0.041\,eV)$

$\tau_{coll} \gg \frac{2\pi}{6.32\times10^{13}} \approx 10^{-13} \Rightarrow \underbrace{\frac{1}{\tau_{coll}} \ll 10^{13}}_{\text{more realistic}}$

plasma oscillations couple to LO phonon modes. One must consider plasmon-phonon coupling

$\underline{N_D = 10^{19}\,cm^{-3}}$, $\omega_p \approx 2\times10^{14}$

$\tau_{coll} \gg \frac{2\pi}{2\times10^{14}} \approx 3\times10^{-14}\ (s) \Rightarrow \frac{1}{\tau_{coll}} \ll 3\times10^{13}\ (1/s)$

- To get a better insight into the separation of the force on a short-range and long-range force as well as the treatment of the carrier interactions as simple binary collisions or as a collective motion of all carriers, it is instructive to follow the approach due to Bohm and Pines, in which:

  (1) Rather than following the motion of individual particles, the electron gas is described in terms of the electron density (Fourier components) at each point in space.

  (2) The electrons are assumed to be moving in a medium of positive charges whose average density is that of the electons (to ensure charge neutrality). The positive charge is assumed to be smeared out throughout the entire system.

- Each electron in the system experiences forces arising from all other electrons ⊕ forces arising from ions (smeared out positive charge). The potential energy of the interaction between the i-th and j-th electron is represented using Fourier series expansion in a box of unit volume with periodic boundary conditions, i.e.

$$\frac{e^2}{4\pi\varepsilon_0|\vec{r}_i-\vec{r}_j|} = \sum_{\vec{k}}{}' \frac{e^2}{\varepsilon_0 k^2} e^{+i\vec{k}\cdot(\vec{r}_i-\vec{r}_j)} = \sum_{\vec{k}}{}' \frac{e^2}{\varepsilon_0 k^2} e^{i\vec{k}\cdot(\vec{r}_i-\vec{r}_j)}$$

The equation of motion for the i-th electron is then found from:

$$m\frac{d^2\vec{r}_i}{dt^2} = -\sum_j \frac{\partial V_{ij}}{\partial \vec{r}_i} = -\sum_j \nabla_{\vec{r}_i} V_{ij} = -i\frac{e^2}{\varepsilon_0}\sum_{j,k}{}' \frac{\vec{k}}{k^2} e^{i\vec{k}\cdot(\vec{r}_i-\vec{r}_j)}$$

The prime means that the term $\vec{k}=0$ is excluded from the sum, since it balances the potential from the positive charge to ensure overall charge neutrality.

The equation of motion for the electron at $\vec{r}_i$ is very difficult to solve since one must take into account all other electrons for which similar differential equations hold. The problem is furthermore complicated by the fact that the

Coulomb potential is a long-range potential. Therefore, the electrons are expected to move in an organized fashion.

- Assuming that electrons can be represented as point-like particles, the particle density in a box of unit volume is given by:

$$\rho(\vec{r}) = \sum_i \delta(\vec{r}-\vec{r}_i)$$

The Fourier components of the density are then given by:

$$\rho_k = \int d^3r\, e^{-i\vec{k}\cdot\vec{r}}\, \rho(\vec{r}) = \int d^3r\, e^{-i\vec{k}\cdot\vec{r}} \sum_i \delta(\vec{r}-\vec{r}_i) = \sum_i e^{-i\vec{k}\cdot\vec{r}_i}$$

i.e.

$$\rho(\vec{r}) = \sum_k \rho_k e^{i\vec{k}\cdot\vec{r}} = \sum_{i,k} e^{i\vec{k}\cdot(\vec{r}-\vec{r}_i)}$$

Note: → the term $\rho_{k=0} = n$ represents the mean electron density
→ terms ρ_k, for $\vec{k}\neq 0$, thus give us deviations (fluctuations) about the mean value

- We now ~~integrate~~ differentiate the expression for ρ_k twice with respect to time to get:

$$\frac{d\rho_k}{dt} = \frac{d\rho_k}{dx_i}\frac{dx_i}{dt} + \frac{d\rho_k}{dy_i}\frac{dy_i}{dt} + \frac{d\rho_k}{dz_i}\frac{dz_i}{dt} = \sum_i (-i\vec{k}\cdot\vec{v}_i)\, e^{-i\vec{k}\cdot\vec{r}_i}$$

$$\frac{d^2\rho_k}{dt^2} = -i\sum_i \vec{k}\cdot\underbrace{\frac{d\vec{v}_i}{dt}}_{\frac{d^2\vec{r}_i}{dt^2}} e^{-i\vec{k}\cdot\vec{r}_i} + \sum_i (-i\vec{k}\cdot\vec{v}_i)^2 e^{-i\vec{k}\cdot\vec{r}_i}$$

Expressing now $d^2\vec{r}_i/dt^2$ in terms of the Fourier components of the gradient of the potential gives:

$$\frac{d^2\rho_k}{dt^2} = -i\sum_i \vec{k}\cdot\left(-i\frac{e^2}{\varepsilon_\infty}\right)\sum_{j,k'}\frac{\vec{k}'}{k'^2 m^*} e^{i\vec{k}'\cdot(\vec{r}_i-\vec{r}_j)}\, e^{-i\vec{k}\cdot r_i} - \sum_i (\vec{k}\cdot\vec{v}_i)^2 e^{-i\vec{k}\cdot\vec{r}_i}$$

$$= \sum_{i,j,k'}\left(-\frac{e^2}{\varepsilon_\infty}\right)\underbrace{\frac{\vec{k}\cdot\vec{k}'}{m^*\vec{k}'^2}\, e^{i(\vec{k}'-\vec{k})\cdot\vec{r}_i}\, e^{-i\vec{k}'\cdot\vec{r}_j}}_{\text{term 1}} - \underbrace{\sum_i (\vec{k}\cdot\vec{v}_i)^2 e^{-\vec{k}\cdot\vec{r}_i}}_{\text{term 2}}$$

(a) Consider an ensemble of non-interacting particles when each particle moves with a constant velocity v_{0i}. then:

$$x_i(t) = x_{0i} + v_{0i}\, t$$

and: $$\rho_k = \sum_i e^{-i\vec{k}\cdot(\vec{r}_{0i} + \vec{v}_{0i}\, t)}$$

Now, even though particles have been prepared (to say) at $t=0$ to have same phase, since the velocities are different, they very quickly go out of phase and the contributions to ρ_k when summed over all particles will cancel out. Therefore, if we have a gas of free particles, fluctuations are damped out very quickly. Therefore, a collection of free particles shows no organized behavior, and its characteristic property is that disturbances die out quickly because of the random diffusion of particles.

(b) The inclusion of the Coulomb force gives finite contribution to $\partial^2\rho_k/\partial t^2$, i.e. forces each particle to oscillate with the same angular frequency as that of every other particle.

$\Rightarrow$ If random thermal motions were not present, Coulomb forces would produce perfectly organized behavior of ρ_k.

$\Rightarrow$ Since both Coulomb forces and random thermal motion are present simultaneously, the net behavior of the electron gas will show some collective aspects and some aspects of an ensemble of randomly moving individual particles.

$\Rightarrow$ Rough criterion for the applicability of the collective description is that:

$$\frac{ne^2}{m^*\varepsilon_\infty} \gg \langle(\vec{k}\cdot\vec{v}_i)^2\rangle_{av}$$

$\rightarrow$ high particle density favors collective behavior and high random velocity opposes it.

For electron gas with isotropic Maxwellian distribution this criterion gives: $k^2 \ll 1/L_D^2$, where L_D is the Debye length.

CARRIER-CARRIER SCATTERING

When carrier densities are high, collisions between carriers become important scattering mechanism. Two types of processes can be distinguished:

→ Binary Carrier-Carrier scattering
→ Collective carrier-carrier scattering

The binary carrier-carrier scattering can, in turn, be separated into:
- electron-hole interaction (narrow band-gap materials, InSb)
- electron-electron interaction
- hole-hole interaction

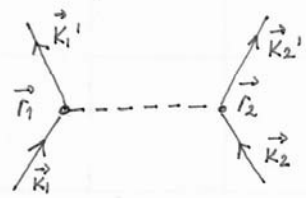
The focus of this section will be on electron-electron scattering, and for this purpose we will consider:
(A) Binary Electron-electron scattering
(B) Collective electron-electron scattering

(A) BINARY ELECTRON-ELECTRON INTERACTION

- This scattering mechanism is closely related to charged impurity scattering and the interaction between two electrons can be approximated by a screened coulomb interaction between point-like particles, i.e.

$$H_{e-e}(\vec{r}_{12}) = \frac{e^2}{4\pi\varepsilon_0 r_{12}} e^{-r_{12}/L_D} \quad , \quad r_{12} = |\vec{r}_1 - \vec{r}_2|$$

- Then, one can obtain the scattering rate in the Born approximation as one usually does in Brooks-Herring approach. To write the collision term, one needs to define a pair transition rate $S(\vec{k}_1, \vec{k}_2, \vec{k}_1', \vec{k}_2')$, which represents the probability per unit time, that electrons in states $\vec{k}_1$ and $\vec{k}_2$ collide and scatter to states $\vec{k}_1'$ and $\vec{k}_2'$, as shown diagramatically in the figure below:



This pair transition rate is defined as:

$$S(\vec{k}_1, \vec{k}_2, \vec{k}_1', \vec{k}_2') = \frac{2\pi}{\hbar} |M_{12}|^2 \delta(E_{k_1}' + E_{k_2}' - E_{k_1} - E_{k_2})$$

where:

$$M_{12} = \langle \vec{k}_1', \vec{k}_2' | H_{ee}(r_{12}) | \vec{k}_1, \vec{k}_2 \rangle .$$

- Using plane-waves basis for the electronic states description, we get:

$$M_{12} = \frac{1}{V^2} \int d\vec{r}_1 \int d\vec{r}_2 \, e^{-i(\vec{k}_1{}'\cdot\vec{r}_1 + \vec{k}_2{}'\cdot\vec{r}_2)} \frac{e^2}{4\pi\varepsilon_\infty} \frac{e^{-r_{12}/L_D}}{r_{12}} e^{i(\vec{k}_1\cdot\vec{r}_1 + \vec{k}_2\cdot\vec{r}_2)}$$

Since the interaction potential depends only upon the distance between the two particles, it is easier to work in a center-of-mass coordinate system defined as:

$$\left.\begin{aligned} \vec{r}_{cm} &= \frac{m_1^*\vec{r}_1 + m_2^*\vec{r}_2}{m_1^* + m_2^*} = \frac{\vec{r}_1 + \vec{r}_2}{2} \\ \vec{r}_{12} &= \vec{r}_1 - \vec{r}_2 \end{aligned}\right\} \Rightarrow \begin{aligned} \vec{r}_1 &= \vec{r}_{cm} + \frac{\vec{r}_{12}}{2} \\ \vec{r}_2 &= \vec{r}_{cm} - \frac{\vec{r}_{12}}{2} \end{aligned}$$

Similar definitions hold for the momentum: $\left.\begin{aligned} k_{cm} &= \frac{\vec{k}_1 + \vec{k}_2}{2} \\ K_{12} &= \frac{\vec{k}_1 - \vec{k}_2}{2} \end{aligned}\right\} \Rightarrow \begin{aligned} \vec{k}_1 &= \vec{k}_{cm} + \vec{K}_{12} \\ \vec{k}_2 &= \vec{k}_{cm} - \vec{K}_{12} \end{aligned}$

- With the use of the center-of-mass and relative coordinates, the exponential factor in M_{12} becomes:

$$\begin{aligned} &-(\vec{k}_1{}'\cdot\vec{r}_1 + \vec{k}_2{}'\cdot\vec{r}_2) + \vec{k}_1\cdot\vec{r}_1 + \vec{k}_2\cdot\vec{r}_2 = \\ &= \vec{k}_1\cdot(\vec{r}_{cm} + \tfrac{1}{2}\vec{r}_{12}) + \vec{k}_2\cdot(\vec{r}_{cm} - \tfrac{1}{2}\vec{r}_{12}) - \vec{k}_1{}'\cdot(\vec{r}_{cm} + \tfrac{1}{2}\vec{r}_{12}) \\ &- \vec{k}_2{}'\cdot(\vec{r}_{cm} - \tfrac{1}{2}\vec{r}_{12}) = (\vec{k}_1 + \vec{k}_2 - \vec{k}_1{}' - \vec{k}_2{}')\cdot\vec{r}_{cm} + \\ &\qquad + (\vec{k}_1 - \vec{k}_2 - \vec{k}_1{}' + \vec{k}_2{}')\cdot\frac{\vec{r}_{12}}{2} \\ &\qquad = (\vec{k}_1 + \vec{k}_2 - \vec{k}_1{}' - \vec{k}_2{}')\cdot\vec{r}_{cm} + (\vec{K}_{12} - \vec{K}_{12}{}')\cdot\vec{r}_{12} \end{aligned}$$

- Now, rather than integrating with respect to $\vec{r}_1$ and $\vec{r}_2$, we need to perform integration with respect to $\vec{r}_{cm}$ and $\vec{r}_{12}$, to get:

$$M_{12} = \underbrace{\frac{1}{V^2}\int d\vec{r}_{cm}\, e^{i(\vec{k}_1 + \vec{k}_2 - \vec{k}_1{}' - \vec{k}_2{}')\cdot\vec{r}_{cm}}}_{\frac{1}{V}\delta(\vec{k}_1 + \vec{k}_2 - \vec{k}_1{}' - \vec{k}_2{}') \text{ since the integral itself gives } V\delta(\ )} \int d\vec{r}_{12} \frac{e^2}{4\pi\varepsilon_\infty} \frac{e^{-r_{12}/L_D}}{r_{12}} e^{i(\vec{K}_{12} - \vec{K}_{12}{}')\cdot\vec{r}_{12}}$$

$$= \frac{1}{V}\frac{e^2}{4\pi\varepsilon_\infty} \underbrace{\int d\vec{r}_{12} \frac{e^{-r_{12}/L_D}}{r_{12}} e^{i\vec{q}\cdot\vec{r}_{12}}}_{4\pi \frac{1}{q^2 + 1/L_D^2}} \quad , \text{ where } \vec{q} = \vec{K}_{12} - \vec{K}_{12}{}'$$

$$M_{12} = \frac{e^2}{V\varepsilon_\infty} \frac{1}{q^2 + 1/L_D^2} \quad \text{where } q^2 = |\vec{K}_{12} - \vec{K}_{12}{}'|^2.$$

- From the momentum and energy conservation we have that:

$$E_{K_1'} + E_{K_2'} = E_{K1} + E_{K2} \Rightarrow \frac{\hbar^2}{2m^*}(k_1'^2 + k_2'^2) = \frac{\hbar^2}{2m^*}(k_1^2 + k_2^2)$$

$$\vec{k}_1' + \vec{k}_2' = \vec{k}_1 + \vec{k}_2 \Rightarrow K_1'^2 + k_2'^2 + 2\vec{k}_1'\cdot\vec{k}_2' = k_1^2 + k_2^2 + 2\vec{k}_1\cdot\vec{k}_2$$

which gives: $\vec{k}_1'\cdot\vec{k}_2' = \vec{k}_1\cdot\vec{k}_2$. We now use this result to show that $|K_{12}'| = |K_{12}|$:

$$\left.\begin{aligned} |k_{12}'|^2 &= \vec{K}_{12}'\cdot\vec{K}_{12}' = \frac{1}{4}(k_1'^2 + k_2'^2 - 2\vec{k}_1'\cdot\vec{k}_2') \\ |K_{12}|^2 &= \vec{K}_{12}\cdot\vec{K}_{12} = \frac{1}{4}(k_1^2 + k_2^2 - 2\vec{k}_1\cdot\vec{k}_2) \end{aligned}\right\} \Rightarrow |K_{12}'| = |K_{12}|$$

Denoting by θ the angle between $\vec{K}_{12}'$ and $\vec{K}_{12}$ we have:

$$|\vec{K}_{12} - \vec{K}_{12}'|^2 = K_{12}^2 + K_{12}'^2 - 2K_{12}K_{12}'\cos\theta = 2K_{12}^2(1-\cos\theta)$$

i.e. $|\vec{K}_{12} - \vec{K}_{12}'|^2 = 4K_{12}^2\sin^2(\theta/2) = 2K_{12}^2(1-\cos\theta)$. Therefore, the matrix element M_{12} becomes:

$$M_{12} = \frac{e^2}{V\varepsilon_\infty}\frac{1}{2K_{12}^2(1-\cos\theta) + \frac{1}{L_D^2}} = \frac{e^2}{V\varepsilon_\infty 2K_{12}^2}\frac{1}{1-\cos\theta + \frac{1}{2K_{12}^2L_D^2}}$$

To summarize:

$$S(\vec{k}_1, \vec{k}_2, \vec{k}_{12}', \vec{k}_2') = \frac{2\pi}{\hbar}\frac{e^4}{4K_{12}^4V^2\varepsilon_\infty^2}\frac{1}{\left[(1-\cos\theta) + \frac{1}{2K_{12}^2L_D^2}\right]^2}\delta(E_{K_1'} + E_{K_2'} - E_{K_1} - E_{K_2})$$

- The energy conservation function gives:

$$\begin{aligned} E_{K_1'} + E_{K_2'} - E_{K_1} - E_{K_2} &= \frac{\hbar^2}{2m^*}[K_1'^2 + K_2'^2 - K_1^2 - K_2^2] \\ &= \frac{\hbar^2}{2m^*}(k_1'^2 + k_2'^2 - 2\vec{k}_1'\cdot\vec{k}_2') \\ &\quad - \frac{\hbar^2}{2m^*}(k_1^2 + k_2^2 - 2\vec{k}_1\cdot\vec{k}_2) = \\ &= \frac{\hbar^2}{2m^*}|\vec{k}_1' - \vec{k}_2'|^2 - \frac{\hbar^2}{2m^*}|\vec{k}_1 - \vec{k}_2|^2 \\ &= \frac{2\hbar^2}{m^*}[K_{12}'^2 - K_{12}^2] = \\ &= 4\left[\frac{\hbar^2K_{12}'^2}{2m^*} - \frac{\hbar^2K_{12}^2}{2m^*}\right] = 4(E_{12}' - E_{12}) \end{aligned}$$

i.e. $\delta(E_{K_1'} + E_{K_2'} - E_{K1} - E_{K_2}) = \delta[4(E_{12}' - E_{12})] = \frac{1}{4}\delta(E_{12}' - E_{12})$

This result further simplifies the expression for the pair transition rate:

$$S(\vec{k}_1, \vec{k}_2, \vec{k}_{12}', \vec{k}_2') = \frac{2\pi}{\hbar}\frac{e^4}{4K_{12}^4V^2\varepsilon_\infty^2}\frac{1}{4}\cdot\frac{\delta(E_{12}' - E_{12})}{\left[(1-\cos\theta) + \frac{1}{2K_{12}^2L_D^2}\right]^2}$$

- To evaluate the scattering rate due to binary carrier-carrier scattering, we weight the pair transition rate that a target carrier is present and by the probability that the final states at $\vec{k}_1'$ and $\vec{k}_2'$ are empty, i.e.

$$\frac{1}{\tau(k_1)} = \sum_{\vec{k}_2}\sum_{\vec{k}_2'} S(\vec{k}_1, \vec{k}_2, \vec{k}_1', \vec{k}_2')\, f(k_2)\, \underbrace{[1-f(k_1')][1-f(k_2')]}_{\approx 1 \text{ for non-degenerate electron gas.}}$$

Note that a separate sum over $\vec{k}_1'$ is not needed because of the momentum conservation δ-function. For non-degenerate semiconductors, it reduces to:

$$\frac{1}{\tau(\vec{k}_1)} = \sum_{\vec{k}_2} f(\vec{k}_2) \underbrace{\sum_{\vec{k}_{12}'} S(\vec{k}_1, \vec{k}_2, \vec{k}_1', \vec{k}_2')}_{I_{12}}$$

where the integral I_{12} is given by:

this factor of 2 comes from the fact that carrier k_{12}' can have spin-up or spin down.

$$I_{12} = \frac{\bar{u} e^4}{8\hbar K_{12}^4 V^2 \varepsilon_\infty^2} \frac{2V}{(2\bar{u})^3} \cdot 2\bar{u} \underbrace{\int_{-1}^{1} \frac{d(\cos\theta)}{\left[(1-\cos\theta) + \frac{1}{2K_{12}^2 L_D^2}\right]^2}}_{I_1} \underbrace{\int_0^\infty d\vec{k}_{12}'\, \delta(E_{12}' - E_{12})\, k_{12}'^2}_{I_2}$$

$$E_{12}' = \frac{\hbar^2 K_{12}'^2}{2m^*} \Rightarrow K_{12}' dK_{12}' = \frac{m^*}{\hbar^2} dE_{12}'$$

$$I_1 = + \frac{1}{1-\cos\theta + \frac{1}{2K_{12}^2 L_D^2}}\Bigg|_{-1}^{1} = \frac{1}{\frac{1}{2K_{12}^2 L_D^2}} - \frac{1}{2 + \frac{1}{2K_{12}^2 L_D^2}} =$$

$$= \frac{2 + \cancel{\frac{1}{2K_{12}^2 L_D^2}} - \cancel{\frac{1}{2K_{12}^2 L_D^2}}}{\frac{1}{2K_{12}^2 L_D^2} \cdot \left(2 + \frac{1}{2K_{12}^2 L_D^2}\right)} = \frac{4K_{12}^2 L_D^2}{2\left[1 + \frac{1}{4K_{12}^2 L_D^2}\right]}$$

$$I_2 = \frac{m^*}{\hbar^2} \int_0^\infty \sqrt{\frac{2m^* E_{12}'}{\hbar^2}}\, \delta(E_{12}' - E_{12})\, dE_{12}' = \frac{m^* \sqrt{2m^* E_{12}}}{\hbar^3} = \frac{m^*}{\hbar^2} |K_{12}|$$

Using the results obtained for the integrals I_1 and I_2 gives:

$$I_{12} = \frac{\cancel{2}\bar{u} e^4}{\cancel{32}_{2}\bar{u}^2 \hbar \cancel{K_{12}^4} V \varepsilon_\infty^2} \cdot \frac{\cancel{4K_{12}^2} L_D^2}{2\left[4K_{12}^2 + \frac{1}{L_D^2}\right]} \cdot \cancel{4K_{12}^2} \cdot \frac{m^*}{\hbar^2} |K_{12}|$$

$$= \frac{2m^* e^4 L_D^2}{4\bar{u}\hbar^3 V \varepsilon_\infty^2} \frac{|K_{12}|}{4K_{12}^2 + \frac{1}{L_D^2}} = \frac{m^* e^4 L_D^2}{4\bar{u}\hbar^3 V \varepsilon_\infty^2} \frac{\cancel{2}|\vec{k}_1 - \vec{k}_2|/\cancel{2}}{|\vec{k}_1 - \vec{k}_2|^2 + \frac{1}{L_D^2}}$$

If there are N-electrons in the system, we will have:

$I_{12} \rightarrow N I_{12}$ and $N/V = n$ gives the electron density.

To summarize:

$$\frac{1}{\tau(\vec{k}_1)} = \sum_{\vec{k}_2} f(\vec{k}_2) \frac{m^* n e^4 L_D^2}{4\pi\hbar^3 \varepsilon_\infty^2} \frac{|\vec{k}_1 - \vec{k}_2|}{|\vec{k}_1 - \vec{k}_2|^2 + 1/L_D^2}$$

For applications in Ensemble Monte Carlo programs, the above form is particularly useful for studying the variations in the scattering rate as the distribution changes shape. The sum over $\vec{k}_2$ is converted into a sum over the particles and the result given in (1) becomes:

$$\frac{1}{\tau(\vec{k})} = \frac{n m^* e^4 L_D^2}{4\pi\hbar^3 \varepsilon_\infty^2} \frac{1}{N_0} \sum_{i=1}^{N_0} \frac{|\vec{k} - \vec{k}_i|}{|\vec{k} - \vec{k}_i|^2 + \frac{1}{L_D^2}}$$

where N_0 is the number of electrons in the ensemble. Note that, as the distribution changes, the scattering rate changes as well, which makes simulations involving electron-electron interactions rather consuming because of the need of frequent renormalization of the scattering rates.

<u>Important notes:</u>

- Although the total momentum and energy of the carrier ensemble cannot change by carrier-carrier scattering, the distribution of momenta can be affected. By altering the distribution, carrier-carrier scattering affects the average relaxation times and, therefore, the values of the average carrier velocity and energy.
- The requirement that we know the distribution function makes carrier-carrier scattering extremly difficult to treat. The techniques used to solve the BTE for the distribution function will be discussed later.
- The binary collisions describe short-range interactions between the electrons, i.e. momentum transfers in the scattering process $q > q_D = 1/\lambda_D$ (λ_D being the debye length). Therefore, in the occupation representation it is represented by the Hamiltonian:

$$H_{ee}^{(SC)} = \frac{e^2}{2\varepsilon_\infty} \sum_{\vec{q} > \vec{q}_D} \frac{1}{q^2} \sum_{\lambda\mu} C^+_{\vec{k}_\lambda + \vec{q}} C^+_{\vec{k}_\mu - \vec{q}} C_{\vec{k}_\mu} C_{\vec{k}_\lambda}$$

Terms involving $\vec{q} < \vec{q}_D$ represent long-range terms, and those are treated via collective carrier-carrier scattering.

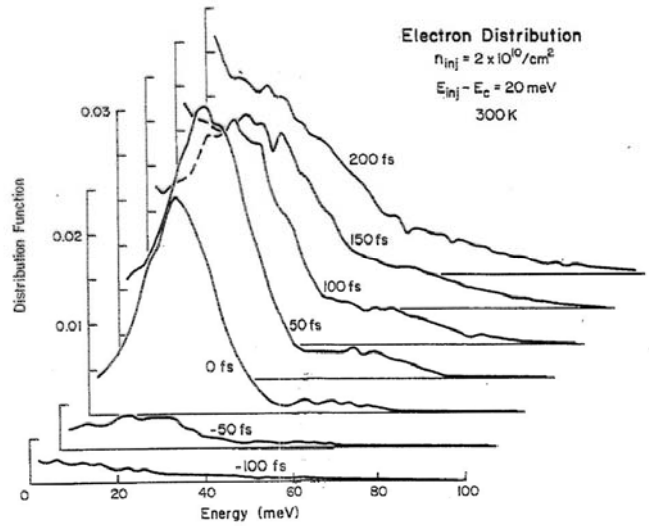


FIG. 2. Electron distribution function as a function of energy at various times before and after a pump pulse at time $t=0$.

those experiments. Time-resolved observation of this feature in absorption experiments could provide a direct measure of the hole–optical-phonon coupling and possible nonequilibrium phonon effects at lower temperatures. As shown in Fig. 3, the athermal phonon replica is observed to smear out with time due to hole-hole scattering and eventually form a Maxwellian tail after 200 fs.

Finally, in Fig. 4 we plot the differential transmission spectra as a function of photon energy at various times using Eq. (2) and the results of Figs. 2 and 3. The excess energies of f_e and f_h coupled by a photon of energy $h\nu$ are chosen to satisfy the momentum conservation rules for band-to-band optical transitions. A quantitative comparison between Fig. 4 and experimental spectra[4,5] is difficult at this time due to the overlap of the excitonic absorption spectrum at low energies which changes dynamically dur-

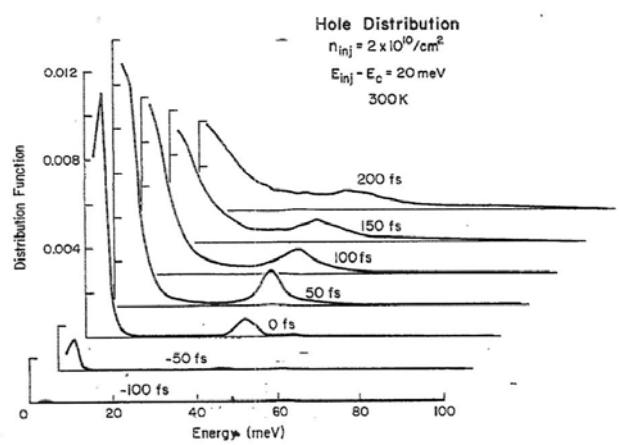
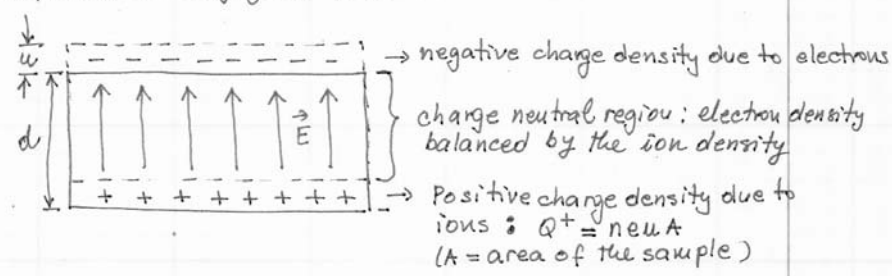


FIG. 3. Hole distribution function as a function of energy at various times during a pump pulse at $t=0$.

(B) COLLECTIVE CARRIER-CARRIER SCATTERING

- Electrons in the bottom of the conduction band move through a relatively immobile background of positively charged atoms. On average, charge neutrality prevails, but if fluctuations in the electron density occur, strong electric fields appear which attempt to restore neutrality, but succeed only in causing oscillations. This effect can be best described by assuming that electrons form a continuum with density n, and writing down the equations of motion for a test electron or the whole plasma.

- Consider a situation that corresponds to the mode $q=0$, when all electrons in the system have been displaced by a same amount u, as depicted in the figure below:



- Because of the positive (negative) surface charge density (neu) at the bottom (top) slab, an electric field is produced inside the slab. The electric field can be calculated using a simple parallel plate capacitor model for which:

$$C = \frac{\varepsilon_\infty A}{d} = \frac{Q}{V_{voltage}} = \frac{neuA}{V_{voltage}} = \frac{neuA}{E \cdot d} \Rightarrow \varepsilon_\infty \frac{\cancel{A}}{\cancel{d}} = \frac{neu\cancel{A}}{E\cancel{d}}$$

$$E = \frac{neu}{\varepsilon_\infty}$$

- The equation of motion of a unit volume of the electron gas of concentration n is:

$$nm^* \frac{d^2u}{dt^2} = -neE = -\frac{n^2e^2u}{\varepsilon_\infty} \Rightarrow \frac{d^2u}{dt^2} + \frac{n^{\cancel{2}}e^2}{\cancel{n}m^*\varepsilon_\infty} u = 0$$

or: $$\boxed{\frac{d^2u}{dt^2} + \omega_p^2 u = 0} \quad \text{where} \quad \omega_p = \left(\frac{ne^2}{m^*\varepsilon_\infty}\right)^{1/2}$$

$\Uparrow$

This represents the equation of motion of a simple harmonic oscillator used to describe the fluctuations in the charge density in the system.

- An alternative derivation is obtained by solving the equation of motion of an electron in the presence of electric field:

$$m^* \frac{d\vec{v}}{dt} = -e\vec{E} \ ,$$

using Gauss law:

$$\nabla \cdot \vec{E} = -\frac{e(n-n_0)}{\varepsilon_\infty}$$

and the continuity equation for electron conservation, i.e.

$$\frac{\partial n}{\partial t} = -\nabla \cdot (n\vec{v}) = -n\nabla \cdot \vec{v} - \vec{v} \cdot \nabla n \approx -n\nabla \cdot \vec{v} \ .$$

Differentiating last equation gives:

$$\frac{d^2 n}{dt^2} \cong -\underbrace{\cancel{\frac{\partial n}{\partial t}\nabla \cdot \vec{v}}}_{\approx 0} - n\nabla \cdot \frac{\partial \vec{v}}{\partial t} \approx -n\nabla \cdot \left(-\frac{e}{m^*}\vec{E}\right)$$

$$\frac{d^2 n}{dt^2} = \frac{ne}{m^*}\nabla \cdot E = \frac{ne}{m^*}(-e)\frac{n-n_0}{\varepsilon_\infty} = -\frac{ne^2}{m^*\varepsilon_\infty}(n-n_0)$$

Defining $\delta\rho = -e(n-n_0)$, we obtain differential equation for the charge density fluctuation, i.e.

$$\frac{d^2}{dt^2}\left[-e(n-n_0)\right] + \frac{ne^2}{m^*\varepsilon_\infty}\left[-e(n-n_0)\right] = 0$$

$$\boxed{\frac{d^2\delta\rho}{dt^2} + \omega_p^2\,\delta\rho = 0}$$

- Important notes:

(1) Plasma oscillation is a collective longitudinal excitation of the conduction electron gas.

(2) A <u>plasmon</u> is a quantum of plasma oscillations. PLASMONS obey Bose-Einstein statistics.

(3) An electron couples with the electrostatic field fluctuations due to plasma oscillations, in a similar manner as the charge of the electron couples to the electrostatic field fluctuations due to longitudinal POP.

(4) For plasma oscillation to be established, the condition

$$\omega_p \tau \geqslant 2\pi \quad \Rightarrow \quad \tau_{coll} \gg 2\pi\left(\frac{m^*\varepsilon_\infty}{ne^2}\right)^{1/2}$$

must be fullfilled, where τ is the collision time. Since τ is about $10^{-13} \div 10^{-14}$ s, the condition is satisfied in semiconductors for carrier densities $> 10^{18}\ cm^{-3}$. Hence, this mechanism is important in narrow-gap SC or wide-gap SC illuminated with intense laser.

- The process is identical to the Fröhlich interaction if plasmon damping is neglected except that $\omega_L \to \omega_p$ and $\varepsilon(0) \to \infty$. Therefore:

$$\frac{1}{\tau(k)} = \frac{m^* e^2 \omega_p}{4\hbar^2 k \varepsilon_\infty} \left\{ N_0 \ln\left|\frac{q_{max}^{ab}}{q_{min}^{ab}}\right| + (N_0+1) \ln\left|\frac{q_{max}^{em}}{q_{min}^{em}}\right| \right\}$$

where q_{max} and q_{min} were discussed and expressions were given in connection with POP scattering.

- Note on q_{max}:

  → large q_{max} refers to short-wavelength oscillations, but one Debye length is needed to screen the interaction. Therefore, when q_{max} exceeds $1/L_D$, the scattering should be treated as a binary collision.

  → To conclude q_c, the upper limit, equals to:

$$q_c = \min\{q_{max}, 1/L_D\}$$

- Importance of plasmon scattering:

  → As noted earlier plasma oscillations and plasmon scattering are important for high carrier densities.

  → When the electron density exceeds about $10^{18}\,cm^{-3}$, the plasma oscillations couple to the LO phonons and one must consider scattering from these coupled modes.