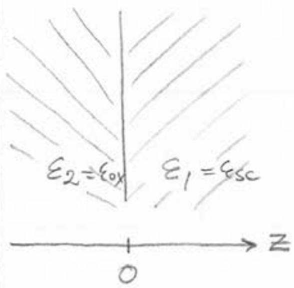
2D - system

The potential due to a charge center located at the position $Q_i(\vec{r}_i, z_i)$ is:

$$\varphi(\vec{r}, z) = \frac{e^2}{4\pi\varepsilon_{sc}}\left[\frac{1}{\sqrt{(\vec{r}-\vec{r}_i)^2+(z-z_i)^2}} + \frac{\varepsilon_{sc}-\varepsilon_{ox}}{\varepsilon_{sc}+\varepsilon_{ox}}\frac{1}{\sqrt{(\vec{r}-\vec{r}_i)^2+(z+z_i)^2}}\right]$$

for $z>0$

$$\varphi(\vec{r}, z) = \frac{e^2}{2\pi(\varepsilon_{sc}+\varepsilon_{ox})}\frac{1}{\sqrt{(\vec{r}-\vec{r}_i)^2+(z-z_i)^2}} \; ; \text{ for } z<0$$



$$\mathcal{F}\left[\frac{1}{[(\vec{r}-\vec{r}_i)^2+(z\pm z_i)^2]^{1/2}}\right] = \frac{2\pi}{q}e^{-q|z-z_i|}e^{-i\vec{k}\cdot\vec{r}_i}$$

If the charge center is located at the interface, then:

$$z_i = 0$$

$$\varphi(q, z, r_i) = \frac{2\pi}{q}e^{-q|z|}e^{-i\vec{k}\cdot\vec{r}_i}$$

Then, if we have a single subband case, the probability for scattering from a state $k_{\parallel}$ in a subband n to a state $k_{\parallel}'$ in a subband m is:

$$S_{nm}(k_{\parallel}, k_{\parallel}') = \frac{2\pi}{\hbar}|H(k_{\parallel}, k_{\parallel}')|^2_{nm}\,\delta(E-E'\pm\hbar\omega)$$

where:

$$\psi_n(k_{\parallel}) = \frac{1}{\sqrt{A}}e^{i\vec{k}_{\parallel}\cdot\vec{r}}\varphi_n(z) \quad ; \; E = E_n + \hbar^2 k_{\parallel}^2/2m^*$$

$$\psi_m(k_{\parallel}') = \frac{1}{\sqrt{A}}e^{i\vec{k}_{\parallel}'\cdot\vec{r}}\varphi_m(z') \quad ; \; E' = E_m + \hbar^2 k_{\parallel}'^2/2m^*$$

and:

$$H(k_{\parallel}, k_{\parallel}')_{nm} = \frac{1}{A}\int d\vec{r}\, e^{i\underbrace{(\vec{k}_{\parallel}-\vec{k}_{\parallel}')}_{q_{\parallel}}\cdot\vec{r}}\int dz\,\varphi_m^*(z)\,H_{qp}(\vec{R})\,\varphi_n(z)$$

$$\downarrow$$

$$\varphi(\vec{r}, z) = \frac{e^2}{2\pi(\varepsilon_{sc}+\varepsilon_{ox})}\frac{1}{\sqrt{r^2+z^2}}$$

Now, using that $\mathcal{F}\left[\frac{1}{\sqrt{r^2+z^2}}\right] = \frac{2\pi}{q_{\parallel}}e^{-q_{\parallel}|z|}$, we get:

$$M(k_{\parallel}, k_{\parallel}')_{nm} = \frac{1}{A} \frac{e^2}{\cancel{2\pi}(\varepsilon_{sc}+\varepsilon_{ox})} \frac{\cancel{2\pi}}{q_{\parallel}} \int dz\, \psi_m^*(z)\, e^{-q_{\parallel}|z|} \psi_n(z)$$

$$= \frac{e^2}{A(\varepsilon_{sc}+\varepsilon_{ox})} \frac{1}{q_{\parallel}} \underbrace{\int dz\, \psi_m^*(z)\, e^{-q_{\parallel}|z|} \psi_n(z)}_{A_{00}(q_{\parallel})}$$

For a single subband, we have:

$$S(k_{\parallel}, k_{\parallel}') = \frac{2\pi}{\hbar} \frac{N_I}{A} \frac{e^4}{(\varepsilon_{sc}+\varepsilon_{ox})^2} \left(\frac{1}{q_{\parallel}+q_D}\right)^2 A_{00}^2(q_{\parallel})$$

$$\frac{1}{\tau_m(k_{\parallel})} = \frac{\cancel{2\pi}}{\hbar} n_I \frac{e^4}{(\varepsilon_{sc}+\varepsilon_{ox})^2} \iint \frac{1}{\cancel{(2\pi)^2}} \frac{\delta(E_{k_{\parallel}'} - E_{k_{\parallel}})\, k_{\parallel}\, dk_{\parallel}\, d\varphi\, (1-\cos\varphi)}{(q_{\parallel}+q_D)^2} A_{00}^2(q_{\parallel})$$

$$E_{\parallel} = \frac{\hbar^2 k_{\parallel}^2}{2m} \Rightarrow k_{\parallel}\, dk_{\parallel} = \frac{m^*}{\hbar^2} dE_{k\parallel}$$

$$\frac{1}{\tau_m(k_{\parallel})} = n_I \frac{e^4 m^*}{2\pi \hbar^3 (\varepsilon_{sc}+\varepsilon_{ox})^2} \int_0^{2\pi} \frac{d\varphi\,(1-\cos\varphi)}{(q_{\parallel}+q_D)^2} \underbrace{A_{00}^2(q_{\parallel})}_{\left(\frac{b}{b+q_{\parallel}}\right)^3}$$

$$q_{\parallel} = 2k_{\parallel} \sin(\varphi/2)$$

$$q_D = \frac{e^2 N_s}{2\varkappa k_B T} \Rightarrow \text{Debye-Hückel screening parameter}$$

$$\psi(z) = \left(\frac{b^3}{2}\right)^{1/2} z\, e^{-bz/2}$$

where $b = \left(\frac{12 m_e^* N^* e^2}{K_s \varepsilon_0 \hbar^2}\right)^{1/3}$

$$N^* = N_{Depl} + \frac{11}{32} N_s$$

$$q_{\parallel}^2 = k_{\parallel}^2 - 2k_{\parallel}k_{\parallel}' + k_{\parallel}'^2 = 2k_{\parallel}^2 - 2k_{\parallel}^2\cos\varphi = 2k_{\parallel}^2(1-\cos\varphi)$$

$$q_{\parallel}^2 = 4k_{\parallel}^2 \sin^2(\varphi/2) \Rightarrow q_{\parallel} = 2k_{\parallel}\sin(\varphi/2)$$

$$\frac{1}{\tau_m(k_{\parallel})} = \frac{n_I e^4 m^* b^3}{\cancel{2}\pi \hbar^3 (\varepsilon_{sc}+\varepsilon_{ox})^2} \int_0^{2\pi} \frac{\cancel{2}\sin^2(\varphi/2)\, d\varphi}{(2k_{\parallel}\sin(\varphi/2)+q_D)^2 [b + 2k_{\parallel}\sin(\varphi/2)]^3}$$

$$b \to \text{large} \longrightarrow \frac{1}{\tau_m(k_{\parallel})} = \frac{n_I e^4 m^*}{\pi \hbar^3 4\varkappa^2} \frac{1}{4k_{\parallel}^2} \cdot 2\pi = \frac{n_I e^4 \cancel{m^*} \cdot \cancel{2\pi}}{\cancel{\pi}\hbar \cdot 4\varkappa^2 4\, \cancel{2m^*} E_K}$$

$$E_K = \frac{\hbar^2 k_{\parallel}^2}{2m} \Rightarrow \hbar^2 k_{\parallel}^2 = 2m^* E_K \qquad \frac{1}{\tau_m(k_{\parallel})} = \frac{n_I e^4}{16\hbar \varkappa^2 E_K}$$