

NCN Summer School: July 2011

Near-equilibrium Transport: Fundamentals and Applications

Lecture 1b: Introduction

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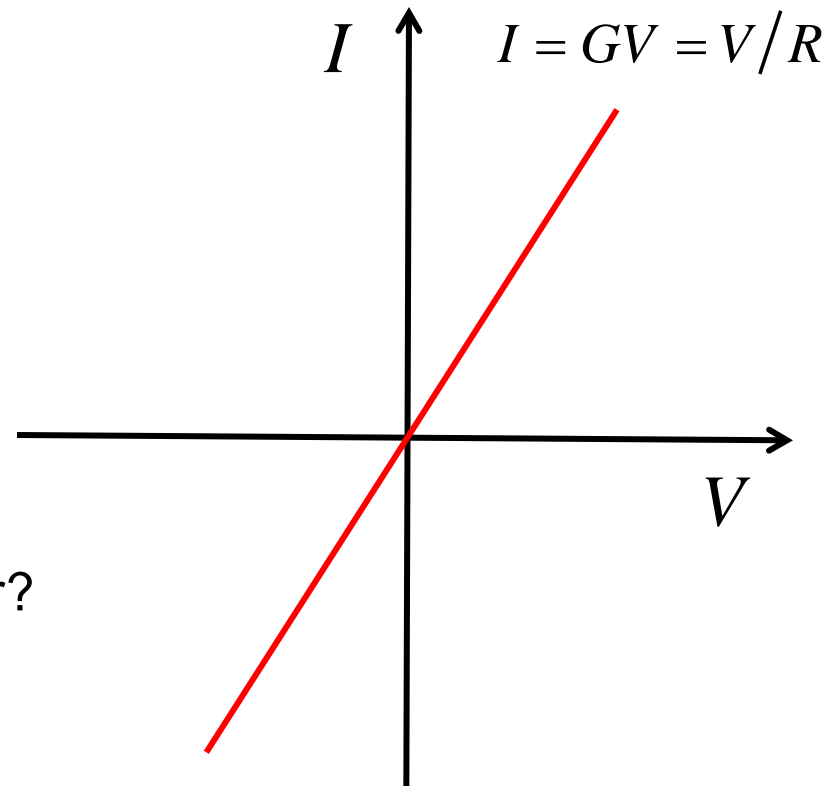
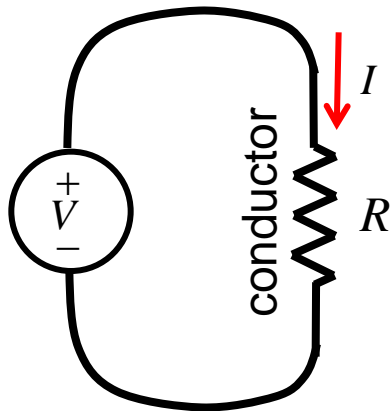
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about these lectures

This is a course about how electrons flow in semiconductors and metals.

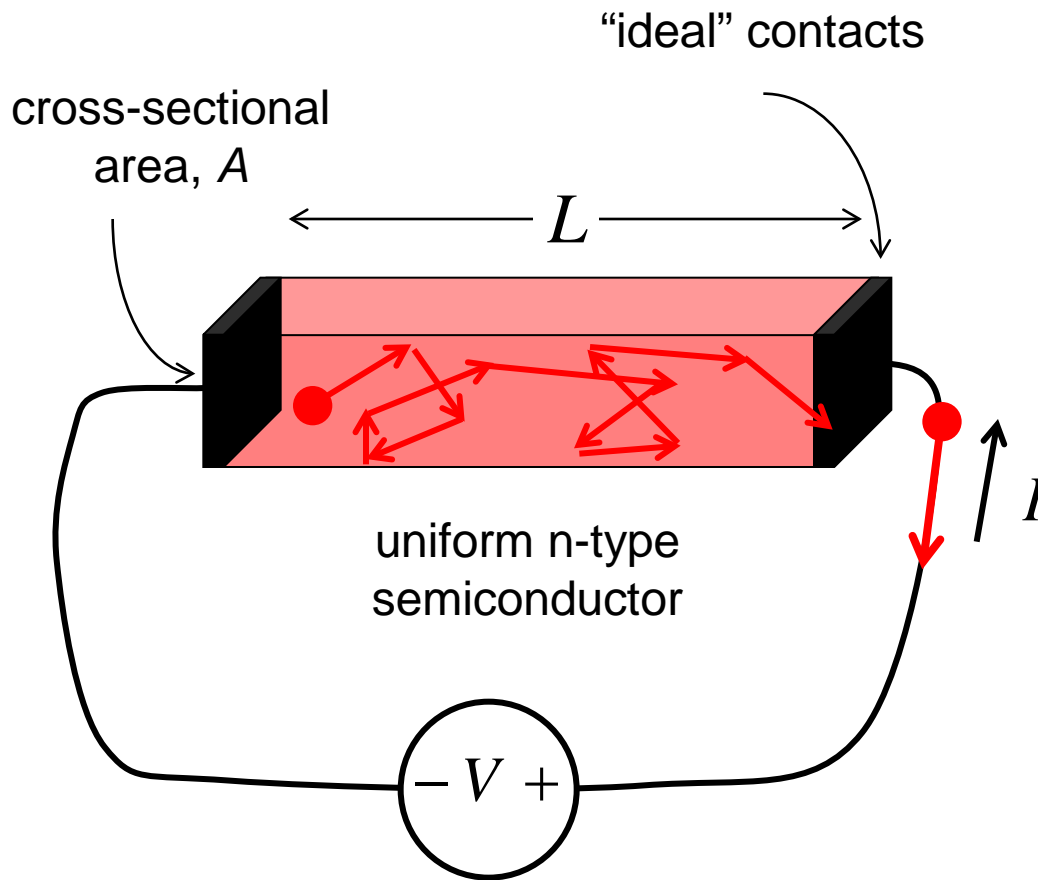
Ohm's Law



- 1) Why is the characteristic linear?
How large can V be?
- 2) What determines G (or R)?

Georg Ohm, 1927

“diffusive” transport



1) random walk with a small bias from left to right

2) electric field

$$\mathcal{E}_x = -\frac{dV}{dx} = -\frac{V}{L}$$

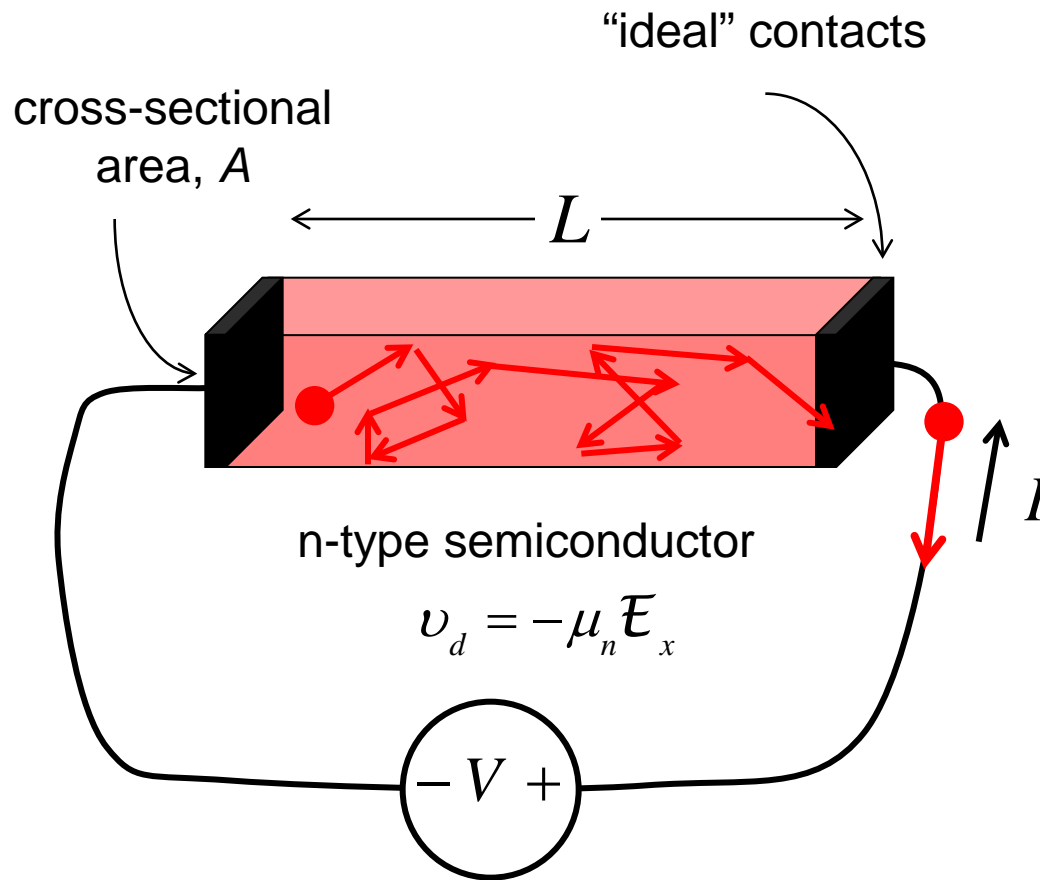
3) force on an electron

$$F_e = -q\mathcal{E}_x$$

4) average velocity:

$$v_d = -\mu_n \mathcal{E}_x$$

conductance (resistance)



$$I = Q/t_t$$

$$Q = nqAL$$

$$t_t = L/v_d$$

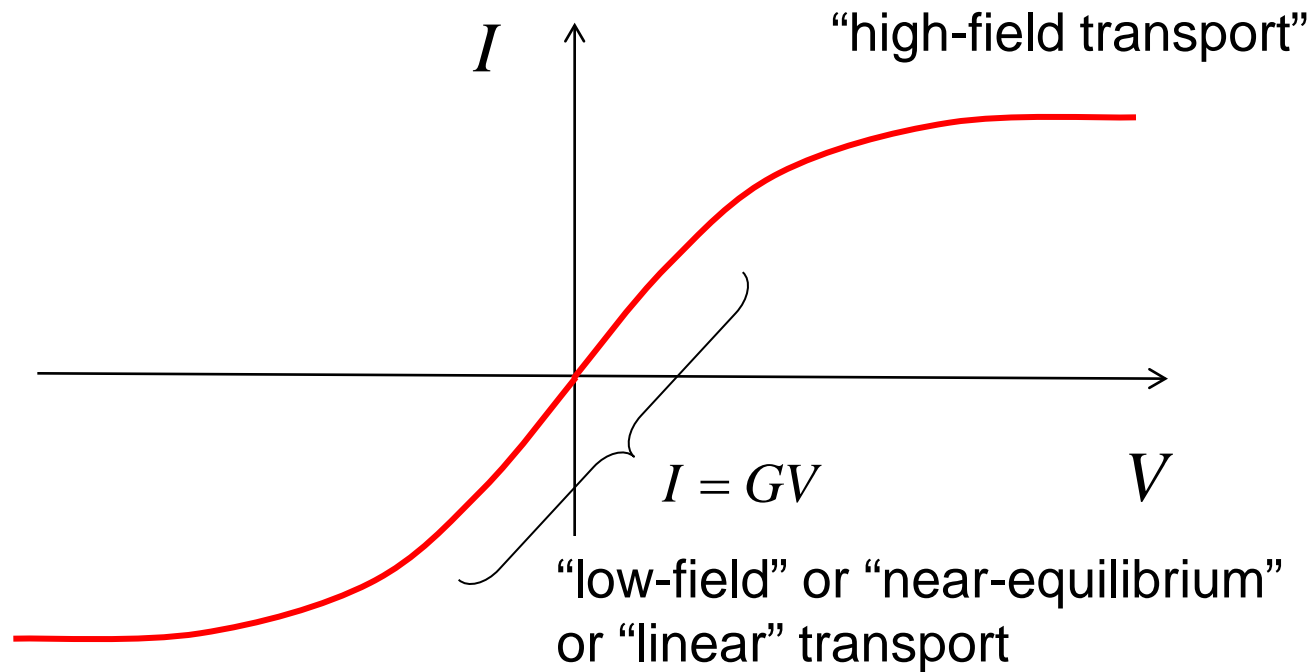
$$I = nqv_d A = nq\mu_n \mathcal{E}_x A$$

$$I = nq\mu_n \frac{A}{L} V$$

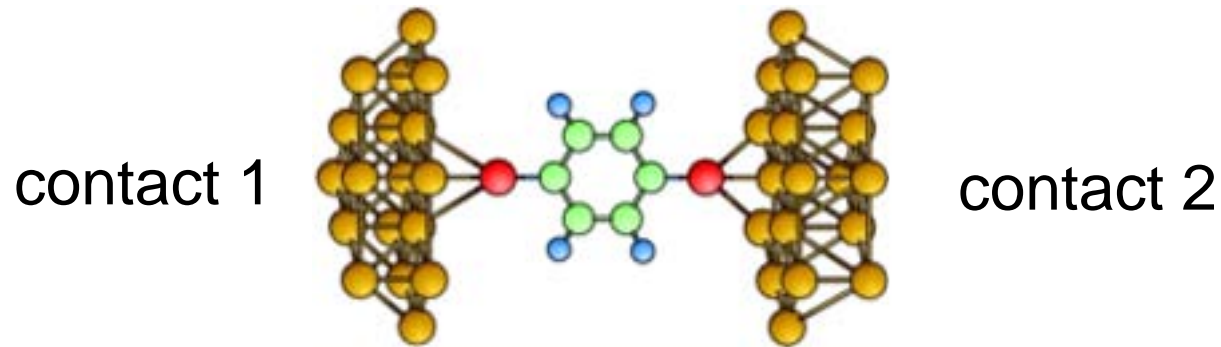
$$G = nq\mu_n \frac{A}{L} = \sigma_n \frac{A}{L}$$

$$G = \sigma_n \frac{A}{L} \quad \sigma_n = nq\mu_n$$

carrier transport in semiconductors



transport at the molecular scale



$$I = GV$$

$$G = nq\mu_n \frac{A}{L}?$$

We will find that resistance is quantized in small conductors.

carrier transport

- 1) *near-equilibrium transport*
- 2) high-field (hot carrier) transport
- 3) non-local transport In small devices
- 4) quantum transport
- 5) transport in random / disordered / nanostructured media

why study near-equilibrium transport?

- 1) It is the foundation for understanding transport in general.
- 2) It is useful in characterizing and understanding materials.
- 3) It plays an important (sometime dominant) role in most electronic devices.

why take a new approach?

- 1) More physically transparent
- 2) Mathematically simpler
- 3) More broadly applicable

the lectures

Lecture 2: General Model

Lecture 3: Resistance: Ballistic to diffusive

Lecture 4: Thermoelectric effects: Physical Approach

Lecture 5: Thermoelectric effects: Mathematics

Lecture 6: Carrier scattering

Lecture 7: The Boltzmann Transport Equation

Lecture 8: Measurements

Lecture 9: Introduction to phonon transport

Lecture 10: Graphene: A Case Study

Lecture 2: General Model for Transport

Datta's model of a nanodevice (a version of the Landauer approach) is introduced as a general way of describing nanodevices – molecular, semiconductor nanowires, carbon nanotubes, graphene, etc. – as well as bulk metals and semiconductors.

Supriyo Datta, *Lessons from Nanoelectronics: A new approach to transport theory*, World Scientific Publishing Company, Singapore, 2011.

Lecture 3: Resistance: Ballistic to Diffusive

The resistance of a ballistic conductor and concepts such as the quantum contact resistance are introduced and discussed. The results are then generalized to treat transport all the way from the ballistic to diffusive regimes.

We will discuss how to apply these concepts to 2D conductors (electrons free to move in a plane).

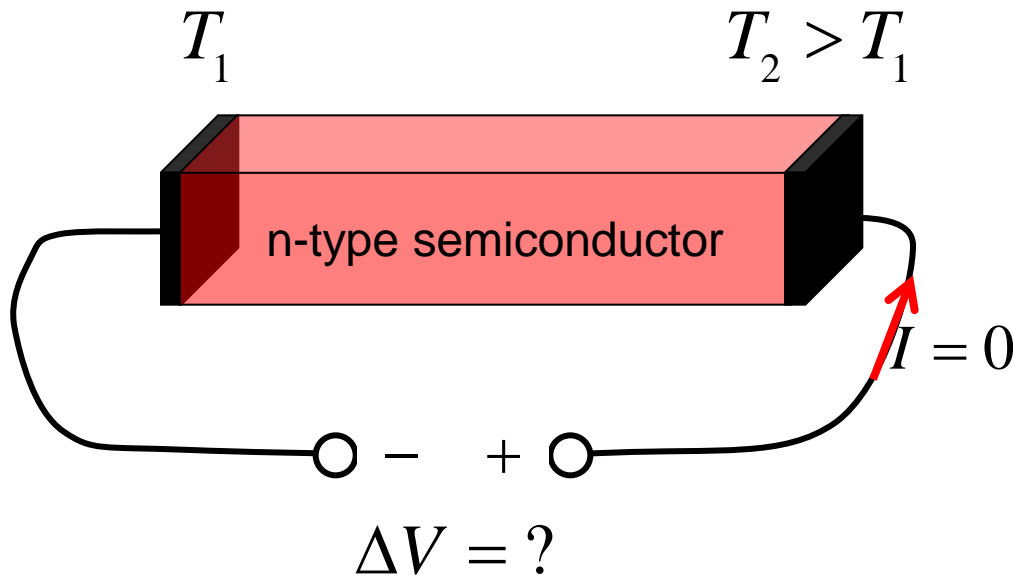
$$R_{2D} = \rho_S \frac{L}{W} ? \quad \sigma_S = \frac{1}{\rho_S} = n_S q \mu_n ?$$

We will also discuss 3D conductors (electrons free to move in 3D) and to 1D conductors (electrons free to move along a wire).

Lecture 4: Thermoelectric Effects: Physical Approach

We will discuss how temperature gradients affect current flow and how electrical currents produce heat currents. Coupled equations for the electric and heat currents will be presented and applications to electronic cooling and the generation of electrical power from thermal gradients will be briefly discussed.

also near-equilibrium transport



$$\Delta V = -S\Delta T$$

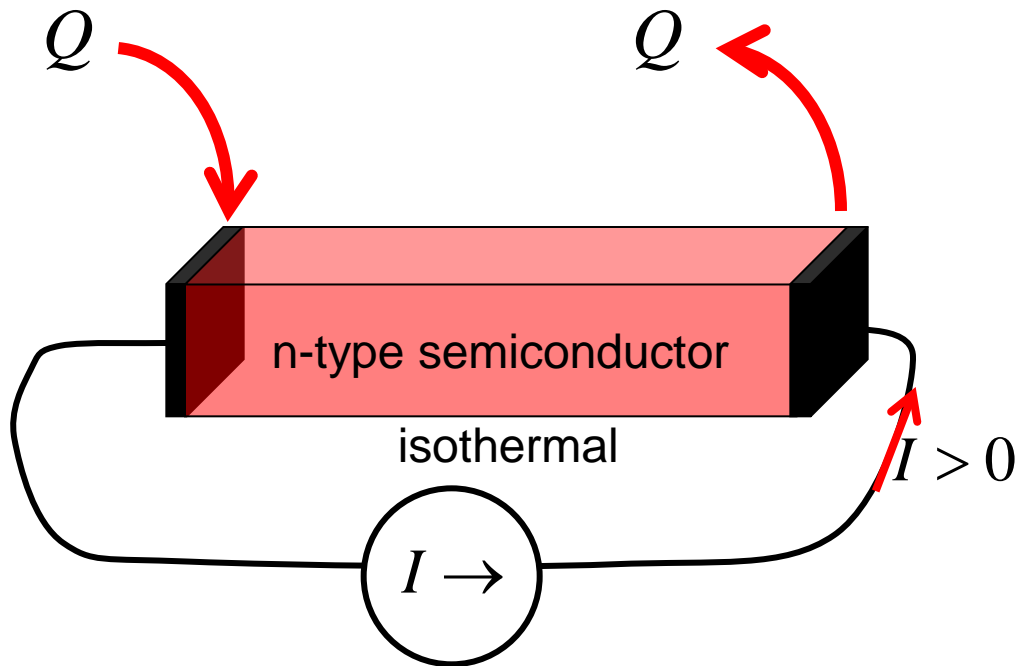
S is the “Seebeck coefficient” in V/K

$S < 0$ for n -type conduction

S is also called the “thermopower”

The Seebeck effect was discovered in 1821 by Thomas Seebeck. It also occurs between the junction of two dissimilar metals at different temperatures. It is the basis for temperature measurement with thermocouples and for thermoelectric power generation.

still more on near-equilibrium transport



$$I_Q = \pi I$$

π is the "Peltier coefficient"
in W/A

*There is a close connection
between the Peltier
coefficient and the Seebeck
coefficient.*

$$\pi = TS \quad \text{Kelvin relation}$$

The Peltier effect was discovered in 1834 by Jean-Charles Peltier and explained in 1838 by Lenz. It finds use in thermoelectric cooling.

Lecture 5: Thermoelectric Effects: Mathematics

Beginning with the general model for transport, we mathematically derive expressions for the four thermoelectric transport coefficients:

- i) electrical conductivity
- ii) Seebeck coefficient (thermopower)
- iii) Peltier coefficient
- iv) Electronic heat conductivity

We also discuss the relationship of the coefficients (e.g. the Kelvin relation and the Weidemann-Franz Law).

Lecture 6: Carrier Scattering

In Lectures 1-5, we describe scattering by a mean-free-path for backscattering. In this lecture, we show how the mfp is related to the time between scattering events and briefly discuss how the scattering time is related to underlying physical processes.

Lecture 7: The Boltzmann Transport Equation

Semi-classical carrier transport is traditionally described by the Boltzmann Transport Equation (BTE). In this lecture, we present the BTE, show how it is solved, and relate it to the Landauer Approach used in these lectures. As an example of the use of the BTE, we derive the conductivity in the presence of an applied B-field.

Lecture 8: Measurement Considerations

Measurements of near-equilibrium transport are commonly used to characterize electronic materials. This lecture is a brief introduction to commonly-used techniques such as 4-probe measurements, transmission line method, van der Pauw techniques, and Hall effect measurements.

Lecture 9: Introduction to Phonon Transport

Most of the heat flow in semiconductors is carried by phonons. In the presence of a small temperature gradient, phonon transport is also a problem in near-equilibrium transport, and the techniques developed for electron transport can be readily applied to phonons. This lecture is an introduction to phonon transport and to the key similarities and differences between electron and phonon transport.

Lecture 10: Case Study: Transport in Graphene

In previous lectures we largely consider applications of near-equilibrium transport to traditional materials, such as semiconductors with a parabolic energy band, but the theory is much more general. As an example of how to apply the concepts in these lectures, we discuss near-equilibrium transport in graphene, a material that has recently attracted a lot of attention and was the subject of the 2010 Nobel Prize in Physics.

course objectives

- 1) To introduce students to carrier transport using a “bottom up” approach that works at the nanoscale as well as at the (traditional) macroscale.
- 2) To acquaint students with some key results (e.g. the quantum of conductance, common measurement techniques).
- 3) To provide a starting point, a basic foundation upon which you can build.

for more information

“Electronics from the Bottom Up”

<http://nanohub.org/topics/ElectronicsFromTheBottomUp>

“ECE 656: Electronic Transport in Semiconductors”

<http://nanohub.org/resources/7281>

Questions?

