NCN Summer School: July 2011

Near-equilibrium Transport:
Fundamentals and Applications

Lecture 8: Measurements

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measurement of conductivity / resistivity

1) Commonly-used to characterize electronic materials.

2) Results can be clouded by several effects – e.g. contacts, thermoelectric effects, etc.

3) Measurements in the absence of a magnetic field are often combined with those in the presence of a B-field.

This lecture is a brief introduction to the measurement and characterization of near-equilibrium transport.

outline

8.1 Introduction
8.2 Resistivity / conductivity measurements
8.3 Hall effect measurements
8.4 The van der Pauw method
8.5 Temperature-dependent measurements
8.6 Discussion
8.7 Summary

resistivity / conductivity measurements

$$J_{nx} = \sigma_n \frac{d\left(F_n/q\right)}{dx}$$

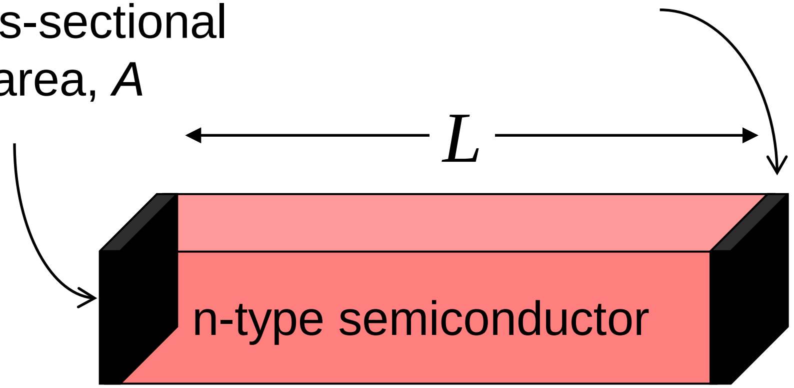
$$\left(\text{diffusive transport assumed}\right)$$

For uniform carrier concentrations:

$$J_{nx} = \sigma_n \mathcal{E}_x \quad \mathcal{E}_x = \rho_n J_{nx}$$

We generally measure **resisitivity** (or **conductivity**) because for diffusive samples, these parameters depend on material properties and not on the length of the resistor or its width or cross-sectional area.

Landauer conductance and conductivity



"ideal" contacts

cross-sectional
area, *A*

L

n-type semiconductor

$$G = \sigma_n \frac{A}{L}$$

For ballistic or quasi-ballistic transport, replace the mfp with the "apparent" mfp:

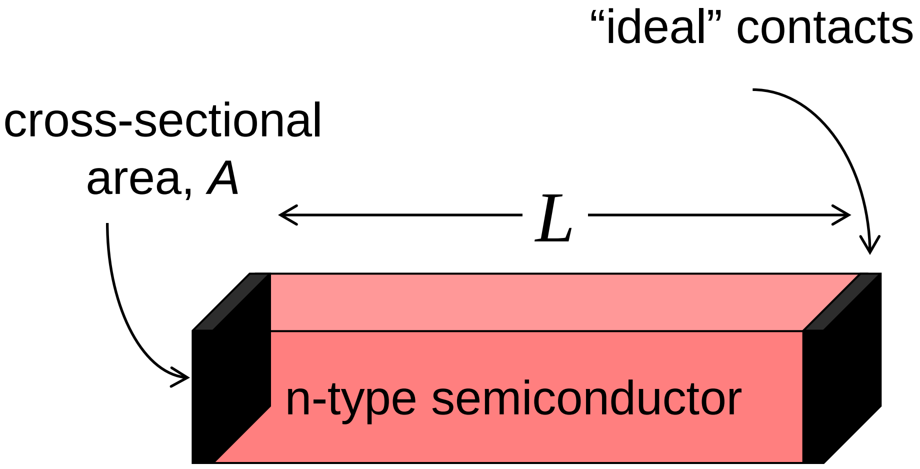
$$1/\lambda_{app}(E) = 1/\lambda(E) + 1/L$$

$$G = \frac{2q^2}{h}\int M(E)T(E)\left(-\frac{\partial f_0}{\partial E}\right)dE \qquad T(E) = \frac{\lambda(E)}{L}$$

$$\sigma = \frac{G}{A/L} = \frac{2q^2}{h}\int M_{3D}(E)\lambda(E)\left(-\frac{\partial f_0}{\partial E}\right)dE \qquad M_{3D}(E) = \frac{M(E)}{A}$$

(diffusive)

conductivity and mobility



$$\sigma = \frac{G}{A/L} = \frac{2q^2}{h}\int M_{3D}(E)\lambda(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

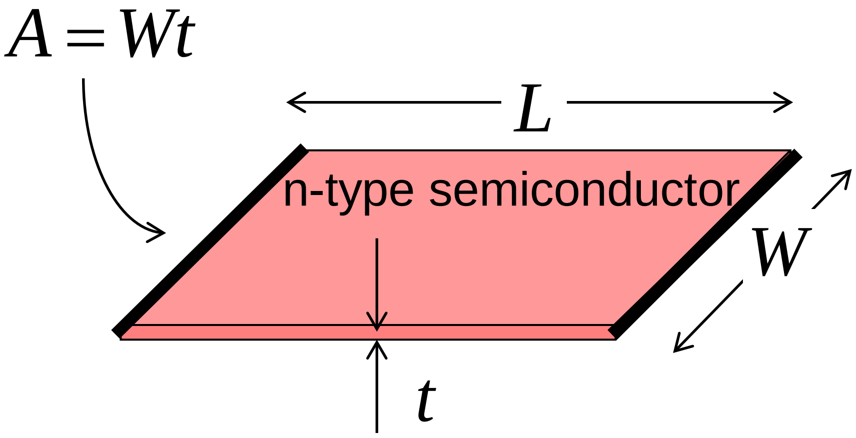
1) Conductivity depends on E_F.
2) E_F *depends on* carrier density.
3) So it is common to characterize the conductivity at a given carrier density.
4) Mobility is often the quantity that is quoted.

So we need techniques to measure two quantities:

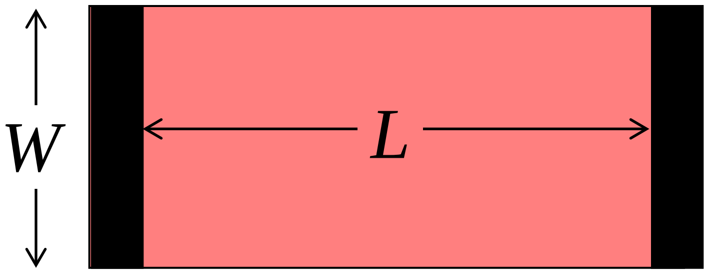
1) conductivity 2) carrier density

$$\sigma_n = nq\mu_n$$

2D: conductivity and sheet conductance



Top view



$$I = GV$$

2D electrons

$$G = \sigma_n \frac{A}{L}$$

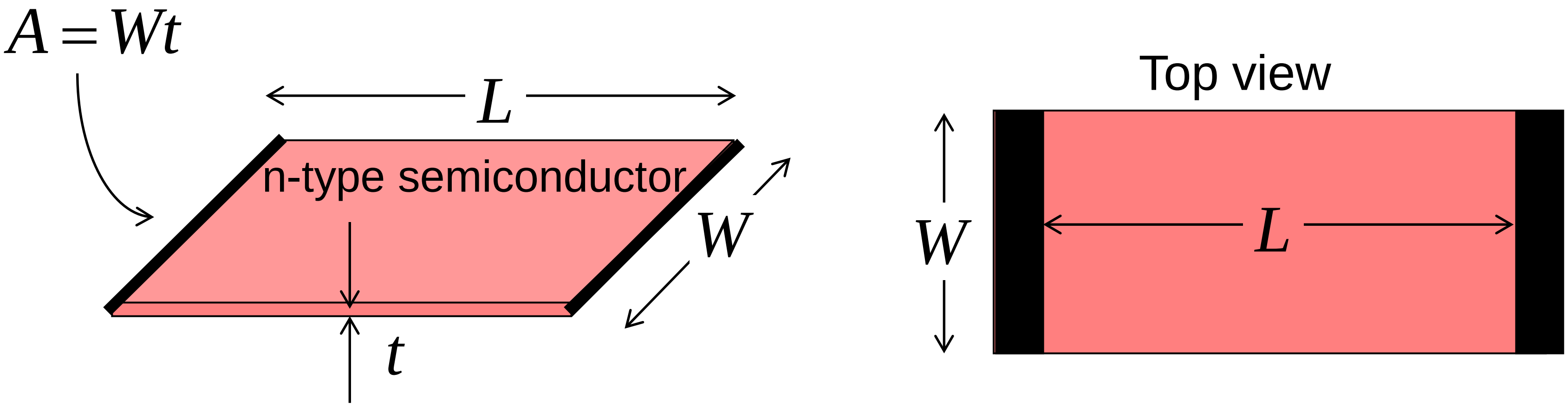
$$G = \sigma_n \left(\frac{Wt}{L} \right) = ntq\mu_n \left(\frac{W}{L} \right)$$

$$G = \sigma_S \left(\frac{W}{L} \right)$$

$$\sigma_S = n_S \, q \, \mu_n \quad (1/\Omega)$$

"sheet conductance"

2D electrons vs. 3D electrons



3D electrons:

$$G = \sigma \frac{A}{L} = \sigma \frac{Wt}{L} \rightarrow \sigma_S = \frac{G}{W/L} = \frac{2q^2}{h}\int t M_{3D}(E)\lambda(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

2D electrons:

$$G = \sigma_S \frac{W}{L} \rightarrow \sigma_S = \frac{G}{W/L} = \frac{2q^2}{h}\int \boxed{M_{2D}(E)}\lambda(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

mobility

1) Measure the conductivity: σ_S

2) Measure the sheet carrier density: n_S

3) Deduce the mobility from: $\sigma_S \equiv n_S q \mu_n$

4) Relate the mobility to material parameters:

$$\sigma_S = \frac{2q^2}{h} \int M_{2D}(E)\lambda(E)\left(-\frac{\partial f_0}{\partial E}\right) \equiv n_S q \mu_n$$

recap

There are three near-equilibrium transport coefficients: conductivity, Seebeck (and Peltier) coefficient, and the electronic thermal conductivity. We can measure all three, but in this brief lecture, we will just discuss the conductivity.

Conductivity depends on the location of the Fermi level, which can be set by controlling the carrier density.

So we need to discuss how to measure the conductivity (or resistivity) and the carrier density. Let's discuss the resistivity first.

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8.2 Resistivity / conductivity measurements
8.3 Hall effect measurements
8.4 The van der Pauw method
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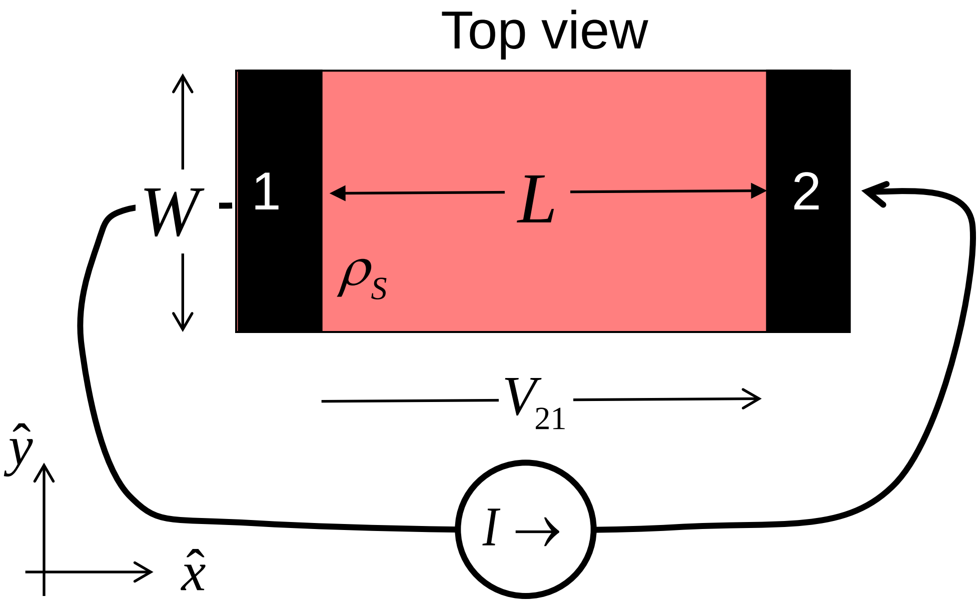
2-probe measurements



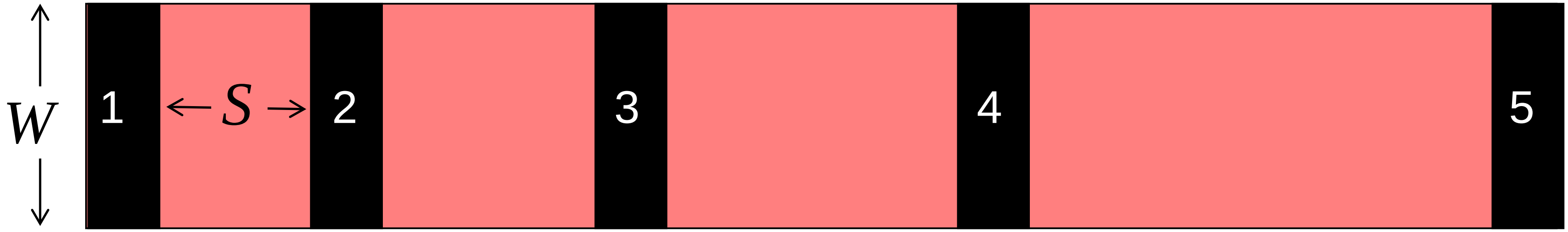
$$R_{CH} = \rho_S \frac{L}{W}$$

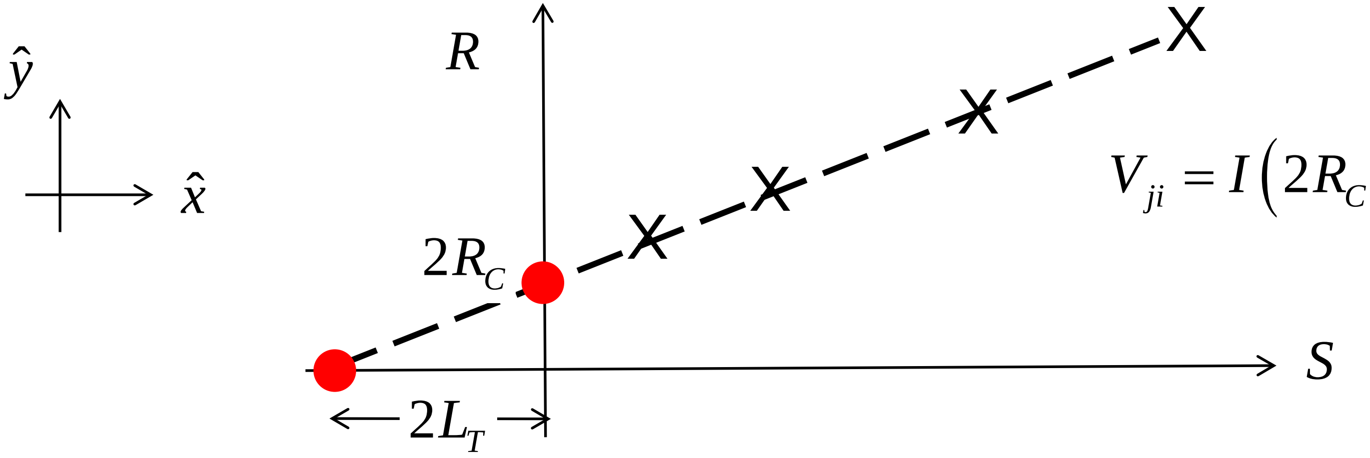
$$V_{21} = I\left(2R_C + R_{CH}\right)$$

$$R_{CH} \neq \frac{V_{21}}{I}$$

transmission line measurements

Top view

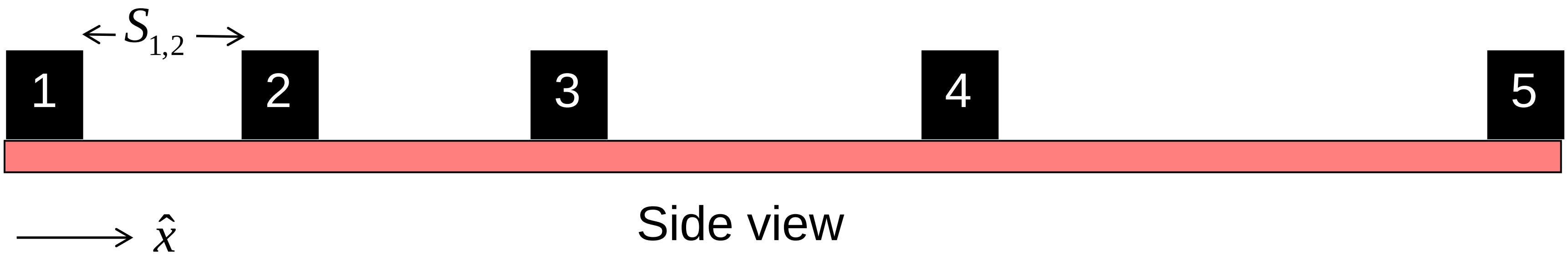




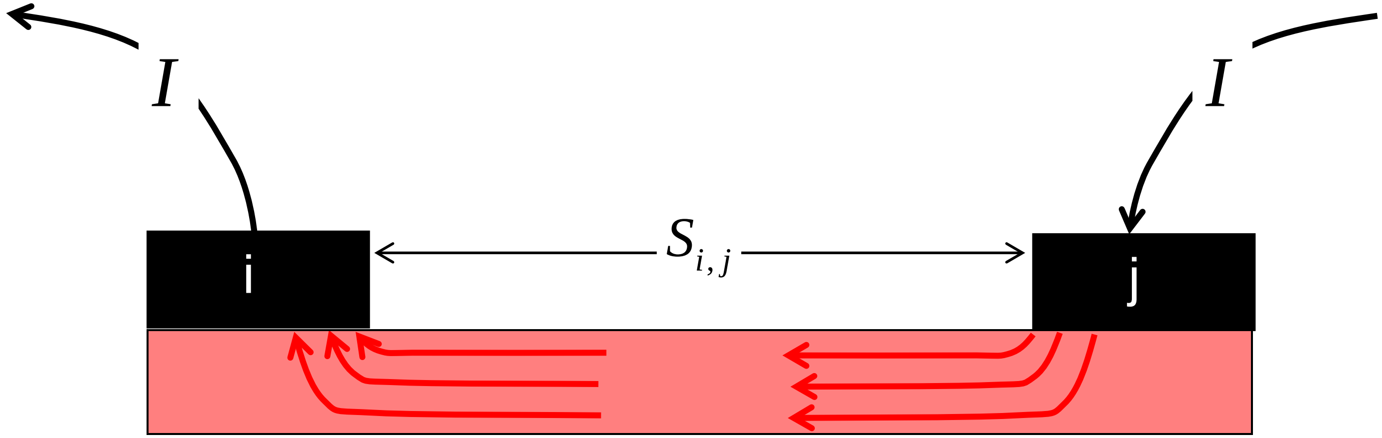
$$V_{ji} = I\left(2R_C + \rho_S S_{ij}/W\right)$$

H.H. Berger, "Models for Contacts to Planar Devices," *Solid-State Electron*., **15**, 145-158, 1972.

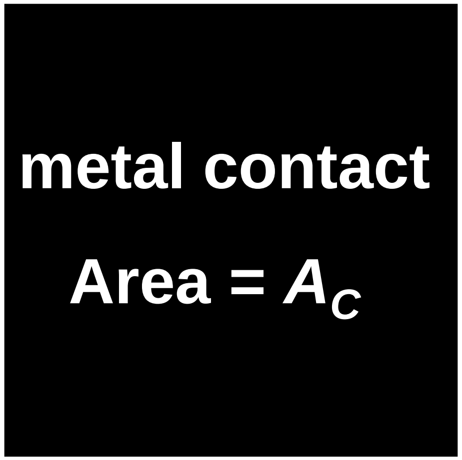
transmission line measurements (TLM)



Side view



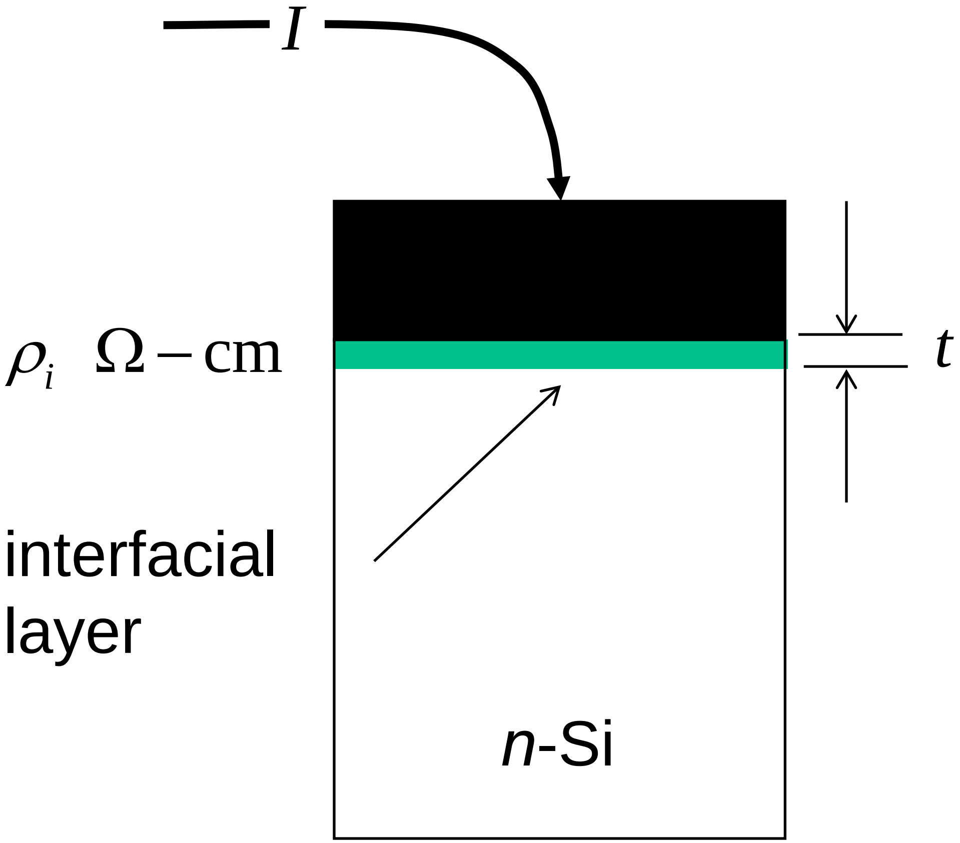
contact resistance (vertical flow)



metal contact

Area = A_C

Top view



I

$\rho_i \;\; \Omega - \text{cm}$

t

interfacial layer

n-Si

Side view

contact resistance (vertical flow)

$$R_C = \frac{\rho_i t}{A_C} = \frac{\rho_C}{A_C}\ \Omega$$

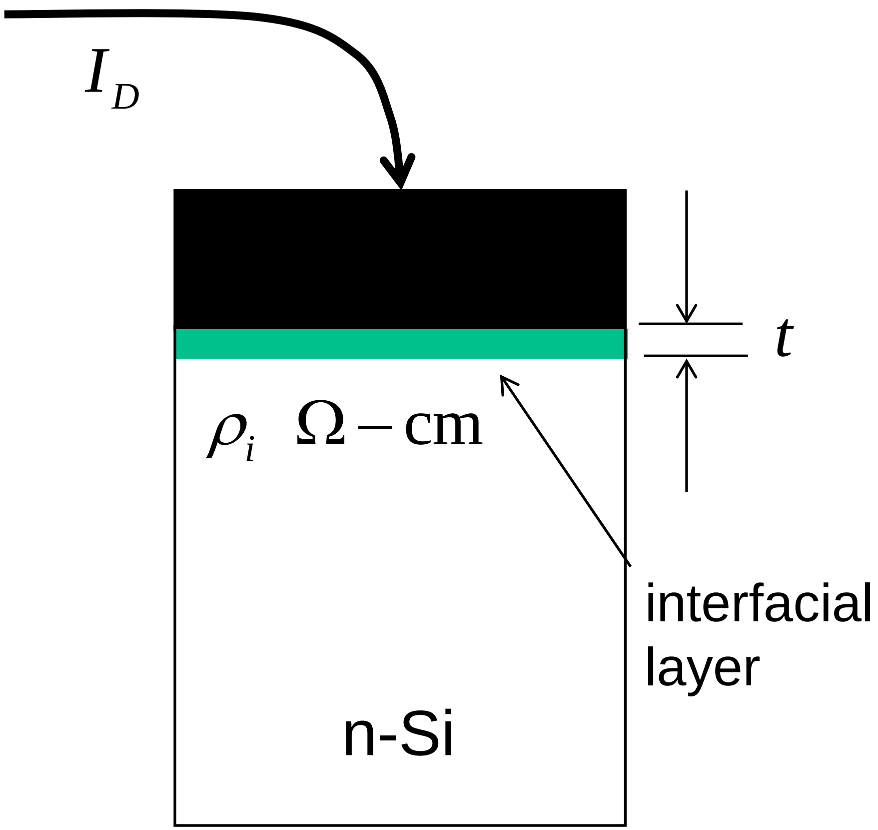
$$10^{-8} \le \rho_C \le 10^{-6}\ \Omega\text{-cm}^2$$

"interfacial contact resistivity"

$$A_C = 0.10\,\mu m \times 1.0\,\mu m$$

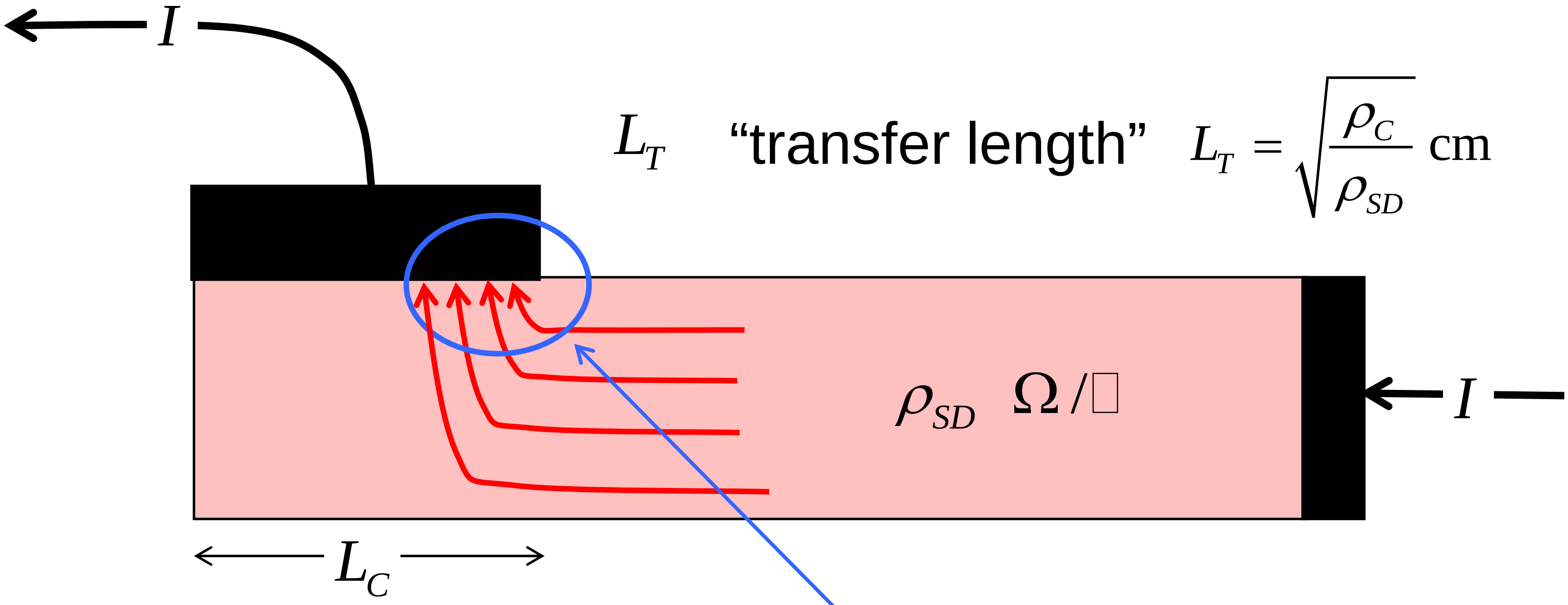
$$\rho_C = 10^{-7}\ \Omega\text{-cm}^2$$

$$R_C = 100\ \Omega\text{-}\mu\text{m}$$



Side view

contact resistance (vertical + lateral flow)



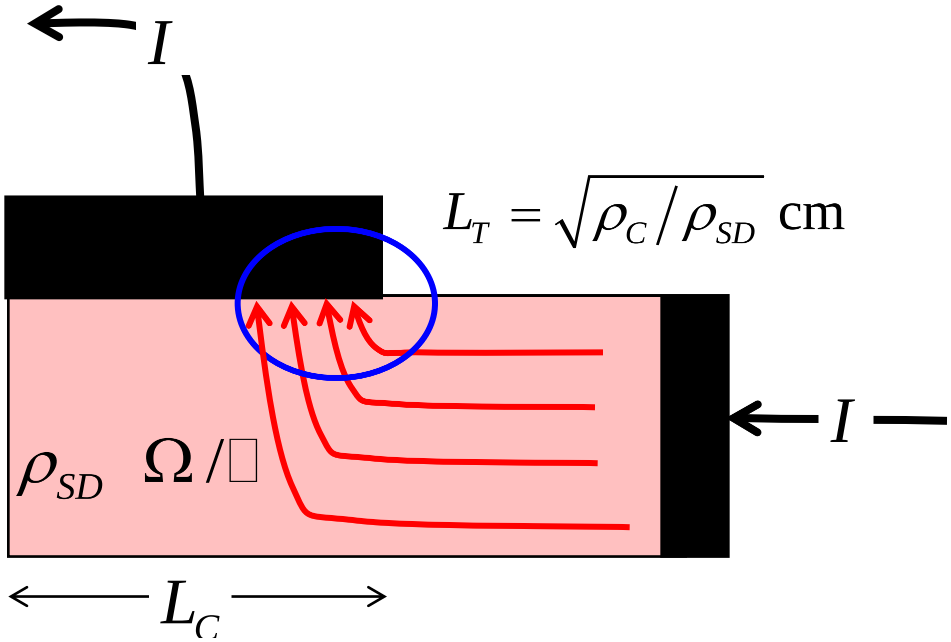
L_T “transfer length” $L_T = \sqrt{\dfrac{\rho_C}{\rho_{SD}}}$ cm

$A_C = WL_C$

(*W* into page)

$A_C(\text{eff}) < WL_C$

contact resistance



$$L_T = \sqrt{\rho_C / \rho_{SD}}\ \text{cm}$$

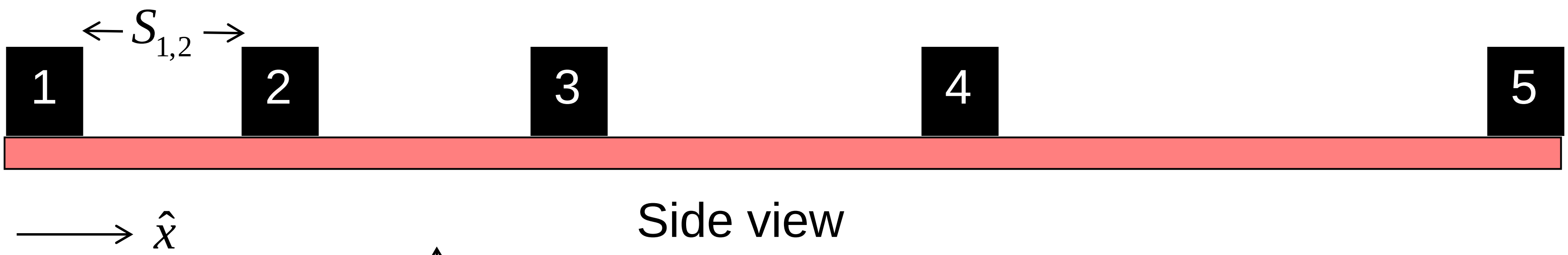
$$A_C = W L_C$$

$$R_C = \frac{\sqrt{\rho_C \rho_{SD}}}{W} \coth\left(L_C / L_T\right)$$

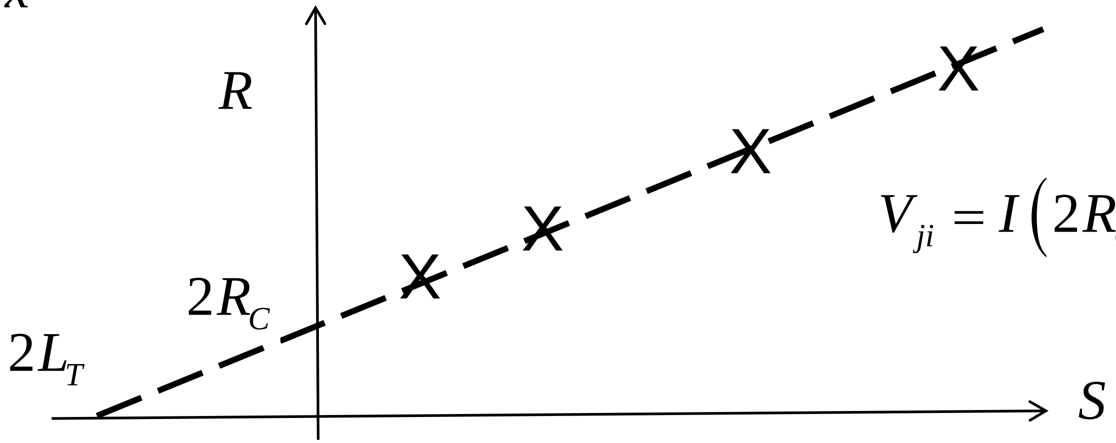
i) $L_C \ll L_T$: $\quad R_C = \dfrac{\rho_C}{L_C W}$

ii) $L_C \gg L_T$: $\quad R_C = \dfrac{\rho_C}{L_T W}$

transfer length measurments (TLM)



Side view



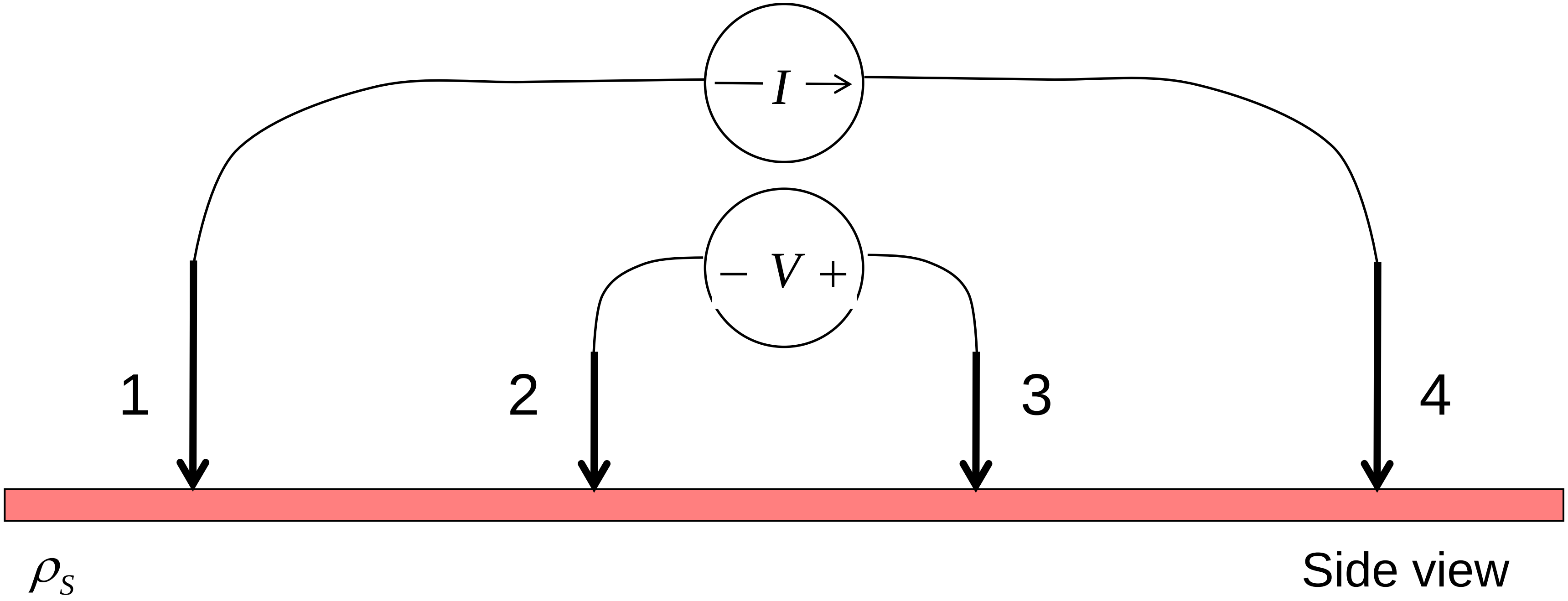
$$V_{ji} = I\left(2R_C + \rho_S S_{ji}/W\right)$$

1) Slope gives sheet resistance, intercept gives contact resistance

2) Determine specific contact resistivity and transfer length:

$$R_C = \frac{\sqrt{\rho_C \rho_{SD}}}{W} \coth\left(L_C / L_T\right) \qquad L_T = \sqrt{\rho_C / \rho_{SD}} \text{ cm}$$

four probe measurements



1) force a current through probes 1 and 4
2) with a high impedance voltmeter, measure the voltage between probes 2 and 3

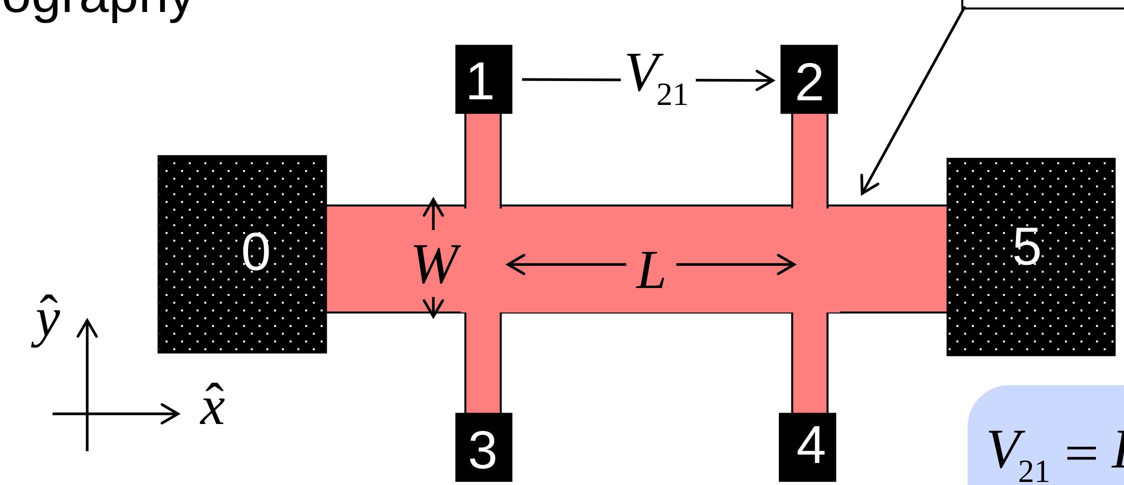
$$R = \frac{V}{I} = f(\rho_S) \quad \text{(no series resistance)}$$

Hall bar geometry

pattern created with
photolithography

Top view

thin film
isolated from substrate



$$V_{21} = I \times \rho_S \frac{L}{W}$$

(high impedance voltmeter)

no contact resistance

Contacts 0 and 5: “current probes”
Contacts 1 and 2 (3 and 4): “voltage probes”

outline

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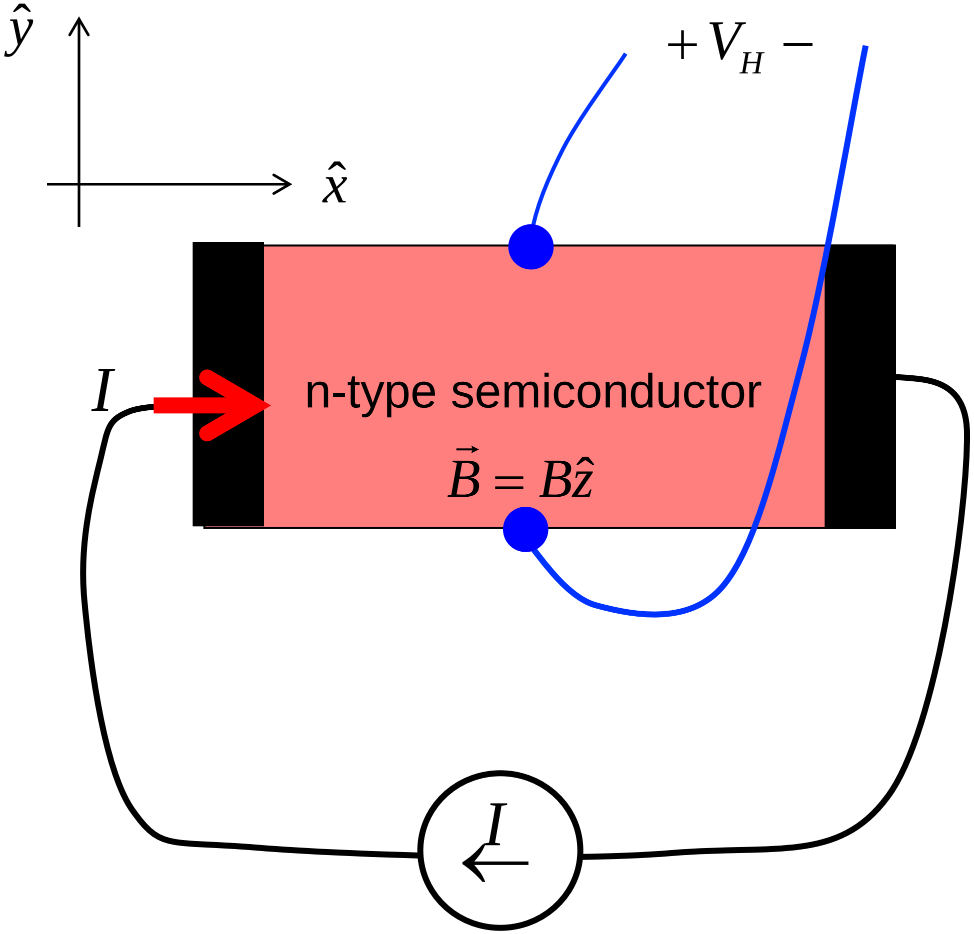
Hall effect



current in *x*-direction:

$$I_x$$

B-field in *z*-direction:

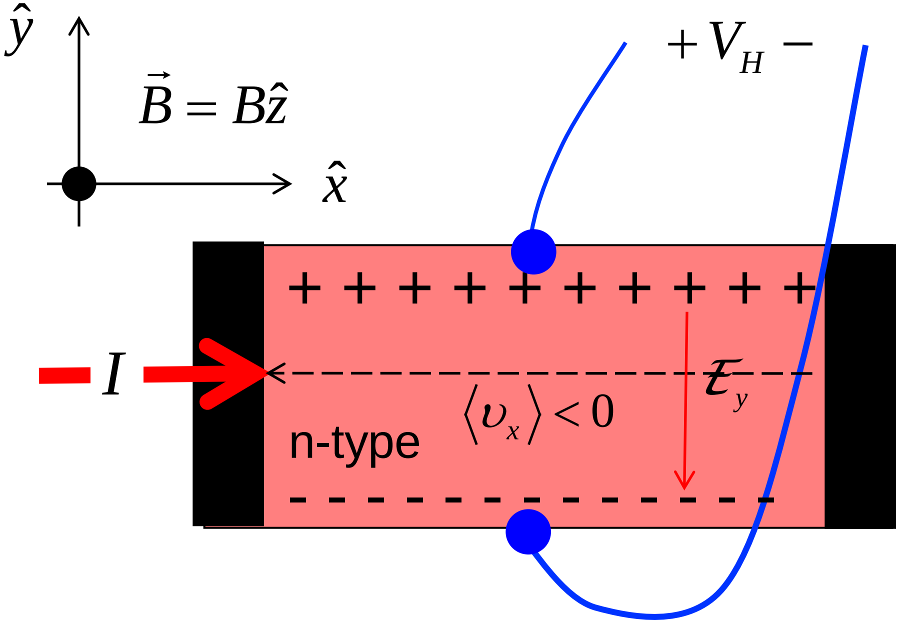
$$\vec{B}=B\hat{z}$$

Hall voltage measured in the *y*-direction:

$$V_H>0\quad(\text{n-type})$$

The Hall effect was discovered by Edwin Hall in 1879 and is widely used to characterize electronic materials. It also finds use magnetic field sensors.

Hall effect: physics



$$I_x = nq\langle \upsilon_x \rangle$$

$$\langle \upsilon_x \rangle < 0$$

$$\vec{F}_e = -q\vec{\upsilon}\times\vec{B}$$

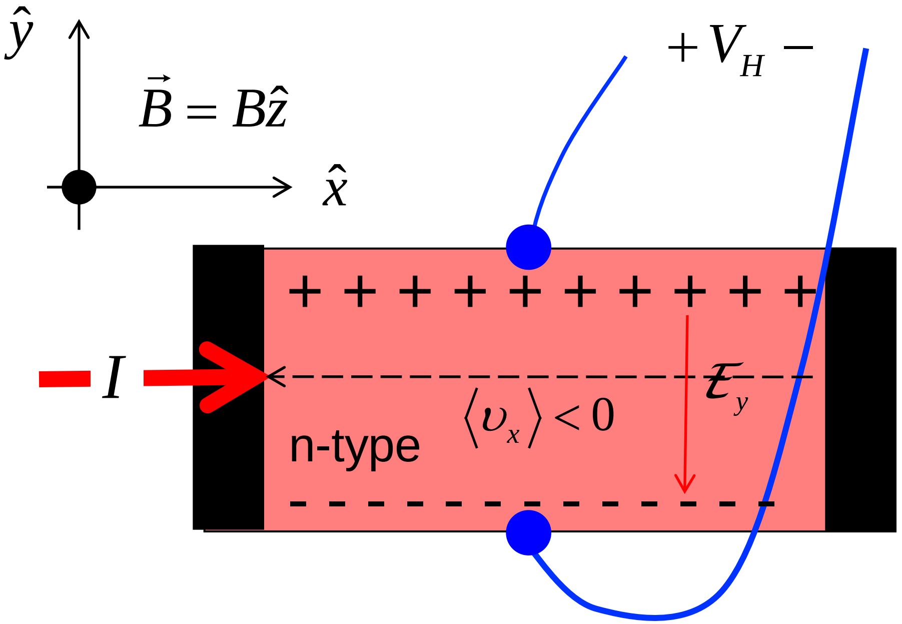
$$\langle F_{ey} \rangle < 0$$

$$\mathcal{E}_y < 0$$

$$V_H > 0 \quad (\text{n-type})$$

Hall effect: analysis

Top view of a 2D film



$$\vec{J}_n = \sigma_n \vec{\mathcal{E}} - (\sigma_n \mu_n r_H)\vec{\mathcal{E}} \times \vec{B}$$

$$J_x = \sigma_n \mathcal{E}_x - (\sigma_n \mu_n r_H)\mathcal{E}_y B_z \approx \sigma_n \mathcal{E}_x$$

$$J_y = 0 = n_S q \mu_n \mathcal{E}_y - (\sigma_n \mu_n r_H)\mathcal{E}_x B_z$$

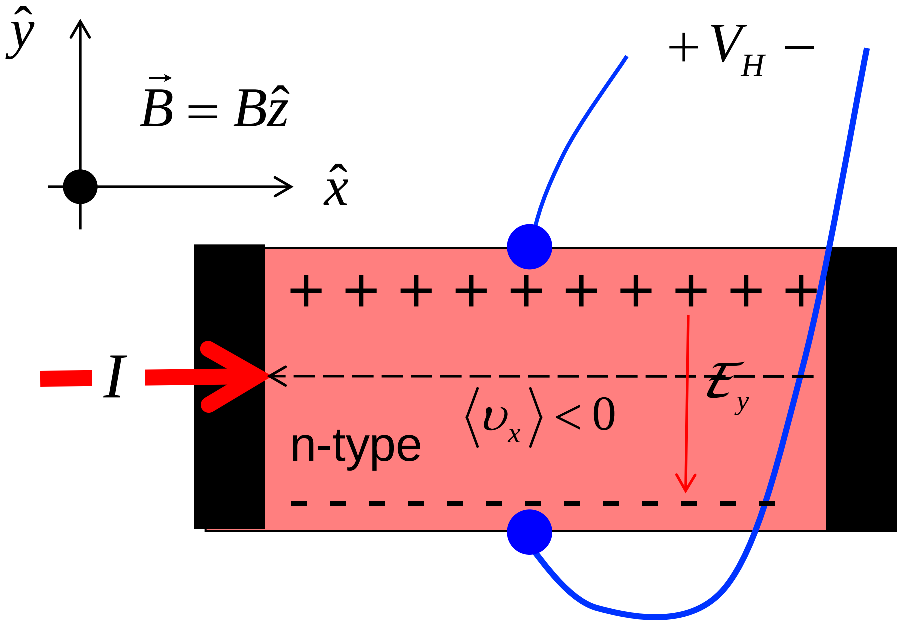
$$\mathcal{E}_y = -\mu_n r_H B_z \mathcal{E}_x = -\frac{r_H B_z J_x}{n_S q}$$

$$\frac{\mathcal{E}_y}{J_x B_z} \equiv R_H = \frac{r_H}{(-q)n_S}$$

R_H is the “Hall coefficient”
$R_H < 0$ for n-type
$R_H > 0$ for p-type

Hall effect: analysis

Top view of a 2D film



$$\vec{J}_n = nq\mu_n\vec{\mathcal{E}} - \left(\sigma_n\mu_n r_H\right)\vec{E}\times\vec{B}$$

$$R_H \equiv \frac{\mathcal{E}_y}{J_x B_z} = \frac{-V_H}{I_x B_z}$$

$$R_H = \frac{r_H}{(-q)n_S}$$

$$r_H \equiv \frac{\langle\langle \tau_m^2 \rangle\rangle}{\langle\langle \tau_m \rangle\rangle^2}$$

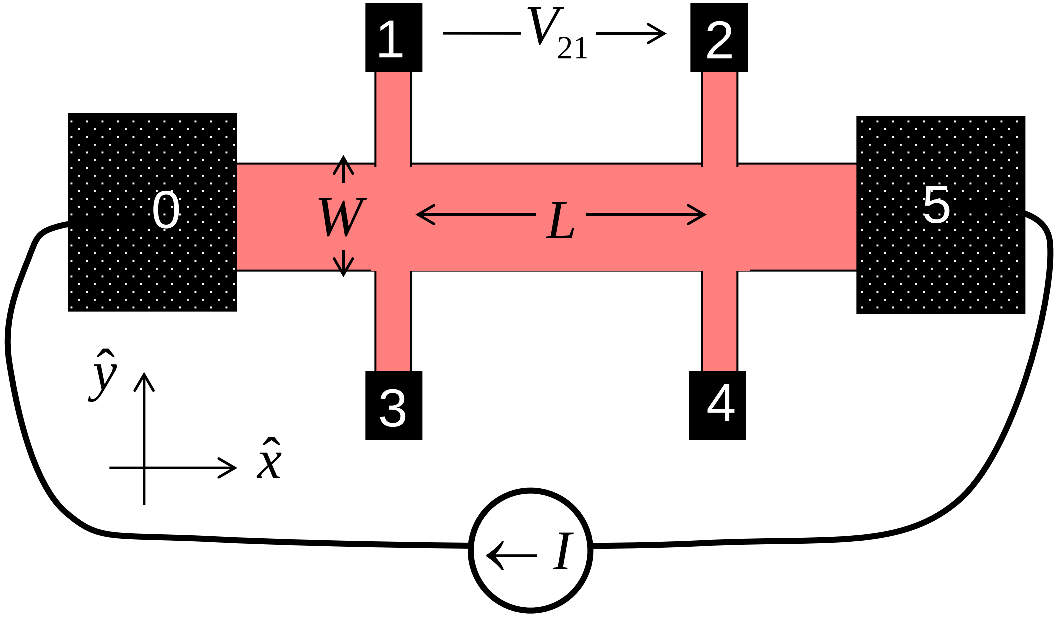
“Hall factor”

$$n_H \equiv \frac{n_S}{r_H}$$

“Hall concentration”

example

Top view



What are the:

1) resistivity?
2) sheet carrier density?
3) mobility?

$$I = I_x = 1\ \mu\text{A}$$

$$B_z = 2{,}000\ \text{Gauss}$$

$$\left(1\ \text{Tesla} = 10^4\ \text{Gauss}\right)$$

$$L = 100\ \mu\text{m}$$

$$W = 50\ \mu\text{m}$$

$B = 0$:

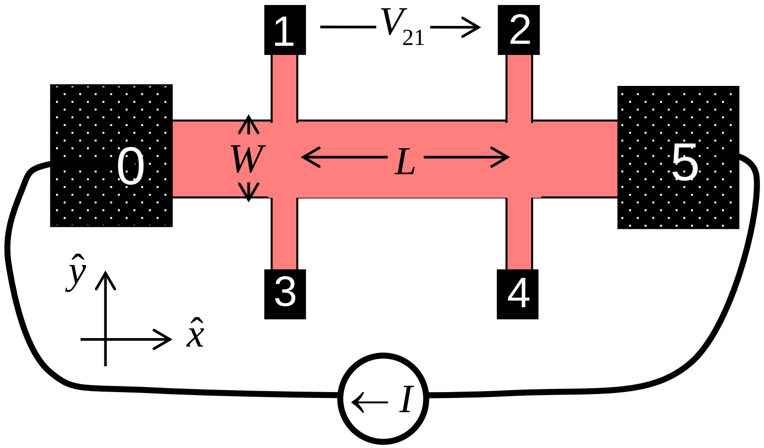
$$V_{21} = 0.4\ \text{mV}$$

$B \neq 0$:

$$V_{24} = 13\ \mu\text{V}$$

example: resistivity

Top view



$$I = I_x = 1\ \mu\text{A}$$

resistivity:

$$R_{xx} = \frac{V_{21}}{I} = 400\ \Omega$$

$$R_{xx} = \rho_S \frac{L}{W} \rightarrow \rho_S = 200\ \Omega/\square$$

$$B_z = 2{,}000 \text{ Gauss}$$

$$\left(1 \text{ Tesla} = 10^4 \text{ Gauss}\right)$$

$$L = 100\ \mu\text{m}$$

$$W = 50\ \mu\text{m}$$

B = 0:

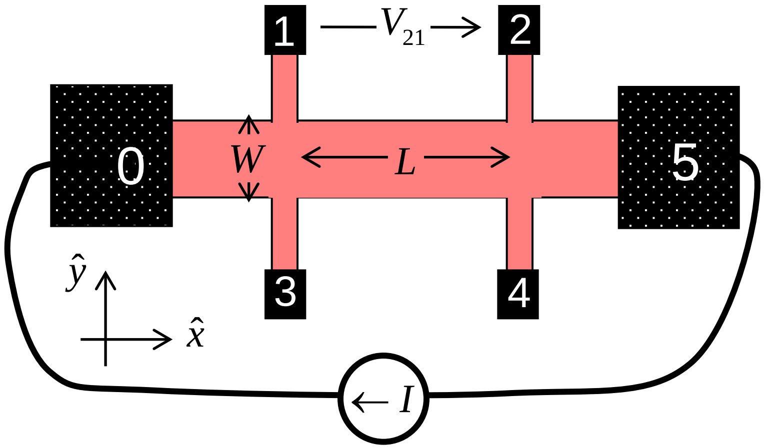
$$V_{21} = 0.4 \text{ mV}$$

B = 0.2T:

$$V_{24} = 13\ \mu$$

example: sheet carrier density

Top view



$$I = I_x = 1\ \mu\text{A}$$

sheet carrier density:

$$n_H \equiv \frac{n_S}{r_H} = \frac{I_x B_z}{qV_H} = \frac{I_x B_z}{qV_{24}}$$

$$n_H = 9.6 \times 10^{12}\ \text{cm}^{-2}$$

$$B_z = 2{,}000\ \text{Gauss}$$

$$\left(1\ \text{Tesla} = 10^4\ \text{Gauss}\right)$$

$$L = 100\ \mu\text{m}$$

$$W = 50\ \mu\text{m}$$

$B = 0$:

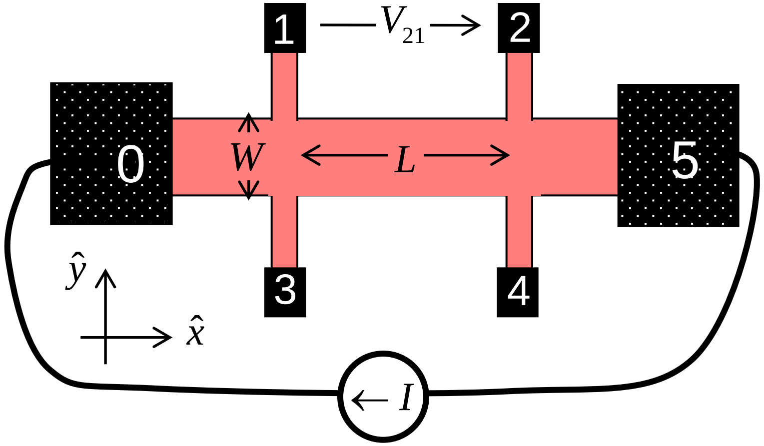
$$V_{21} = 0.4\ \text{mV}$$

$B = 0.2$T:

$$V_{24} = 13\ \mu\text{V}$$

example: mobility

Top view



$$I = I_x = 1\ \mu\text{A}$$

mobility:

$$\sigma_S = \frac{1}{\rho_S} = n_S q \mu_n = \left(\frac{n_S}{r_H}\right) q \left(r_H \mu_n\right)$$

$$\mu_H \equiv r_H \mu_n = 3125\ \text{cm}^2/\text{V-s}$$

$$B_z = 2{,}000\ \text{Gauss}$$

$$\left(1\ \text{Tesla} = 10^4\ \text{Gauss}\right)$$

$$L = 100\ \mu\text{m}$$

$$W = 50\ \mu\text{m}$$

B = 0:

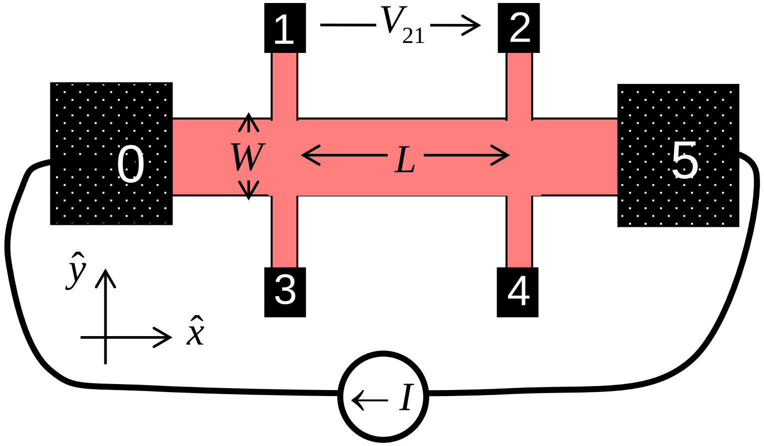
$$V_{21} = 0.4\ \text{mV}$$

B = 0.2T:

$$V_{24} = 13\ \mu\text{V}$$

re-cap

Top view



$$I = I_x = 1\,\mu\text{A}$$

1) Hall coefficient:

$$R_H \equiv \frac{-V_H}{I_x B_z} = \frac{r_H}{(-q)\,n_S}$$

2) Hall factor:

$$r_H \equiv \left\langle\left\langle \tau_m^2 \right\rangle\right\rangle \Big/ \left\langle\left\langle \tau_m \right\rangle\right\rangle$$

3) Hall concentration:

$$n_H \equiv n_S / r_H$$

4) Hall mobility:

$$\mu_H \equiv r_H \mu_n$$

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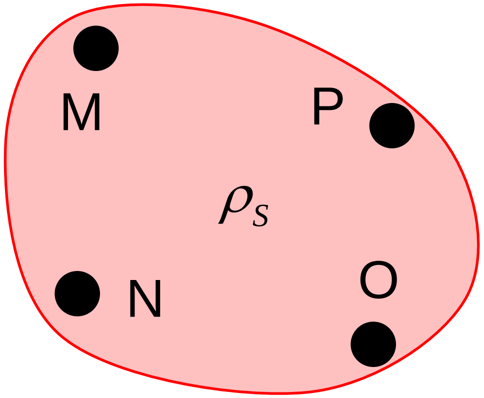
van der Pauw sample

2D film
arbitrarily shaped
homogeneous, isotropic
(no holes)

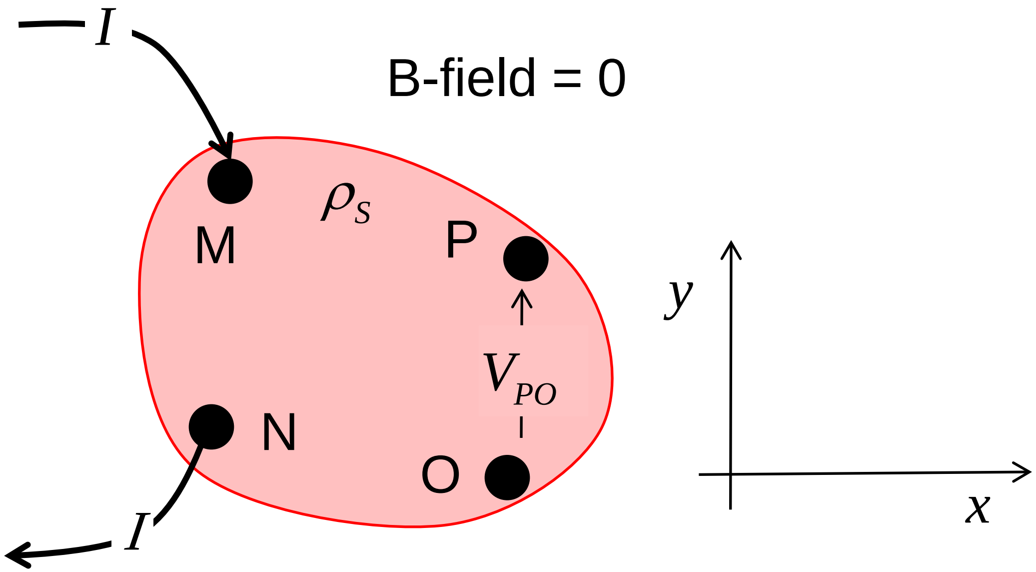
Top view



Four small contacts
along the perimeter

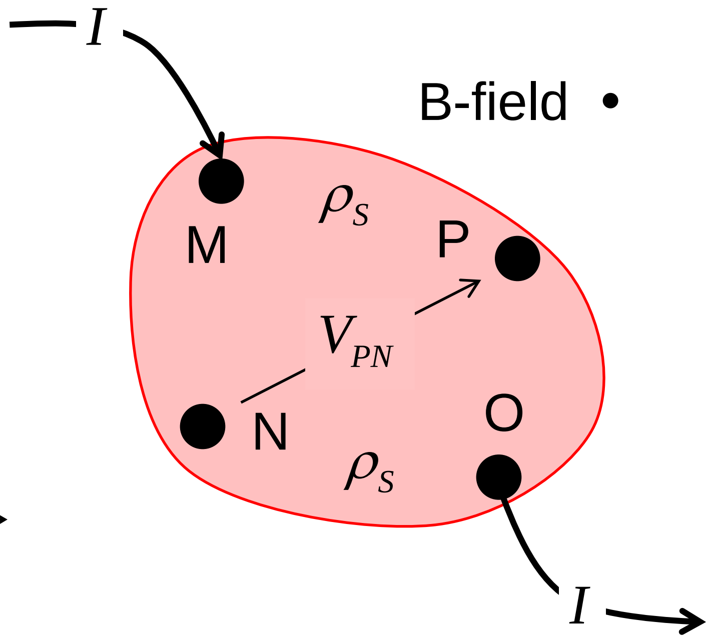
van der Pauw approach

Resistivity



1) force a current in *M* and out *N*
2) measure V_{PO}
3) $R_{MN,\,OP} = V_{PO} / I$ related to ρ_S

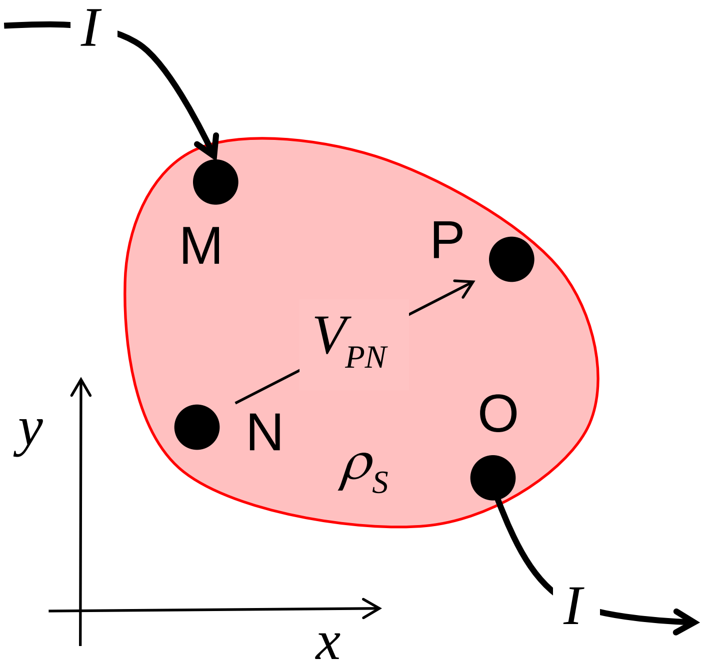
Hall effect



1) force a current in *M* and out *O*
2) measure V_{PN}
3) $R_{MO,\,NP} = V_{PN} / I$ related to V_H

van der Pauw approach: Hall effect

Hall effect



$$\vec{J}_n = \sigma_n \vec{\mathcal{E}} - \left(\sigma_n \mu_n r_H\right) \vec{E} \times \vec{B}$$

$$J_x = \sigma_n \mathcal{E}_x - \left(\sigma_n \mu_n r_H\right) E_y B_z$$

$$J_y = \sigma_n \mathcal{E}_y + \left(\sigma_n \mu_n r_H\right) E_x B_z$$

$$\mathcal{E}_x = \rho_{nn} J_x + \left(\rho_{nn} \mu_H B_z\right) J_y$$

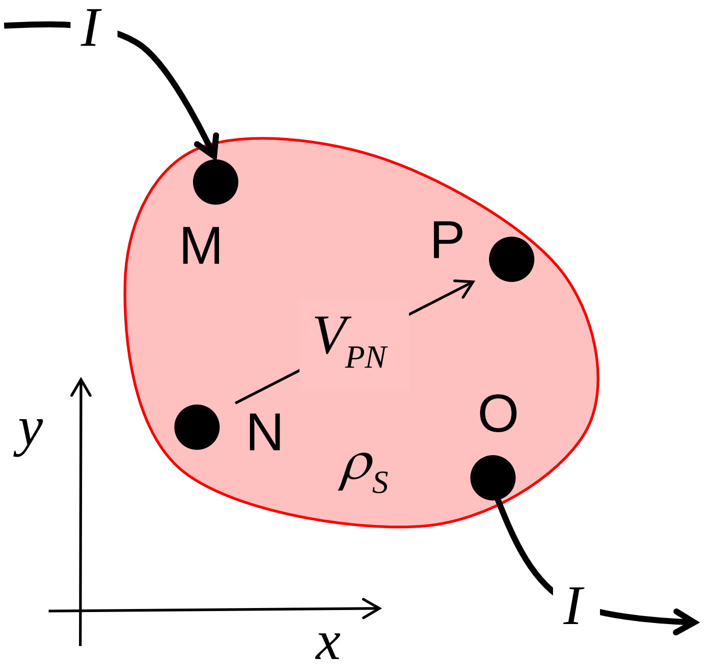
$$\mathcal{E}_y = -\left(\rho_{nn} \mu_H B_z\right) J_x + \rho_{nn} J_y$$

$$V_{PN}\left(B_z\right) = -\int_N^P \vec{\mathcal{E}} \bullet d\vec{l} = -\int_N^P \mathcal{E}_x dx + \mathcal{E}_y dy$$

$$V_H \equiv \frac{1}{2}\left[V_{PN}\left(+B_z\right) - V_{PN}\left(-B_z\right)\right]$$

van der Pauw approach: Hall effect

Hall effect



$$\vec{J}_n = nq\mu_n\vec{\mathcal{E}} - \left(\sigma_n\mu_n r_H\right)\vec{E}\times\vec{B}$$

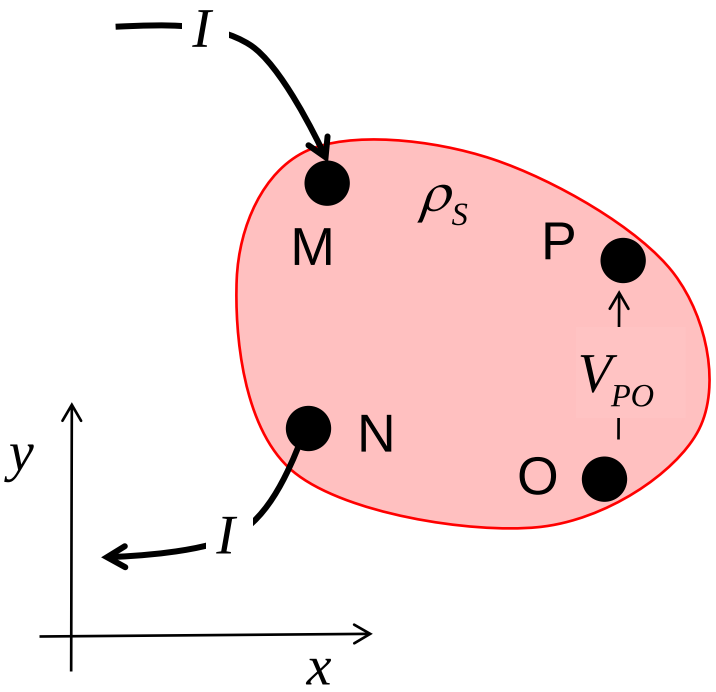
$$V_H = \rho_n\mu_H B_z \underbrace{\left[\int_{y_N}^{y_P} J_x dy - \int_{x_N}^{x_P} J_y dx\right]}_{I = \int_N^P \vec{J}\bullet\hat{n}dl}$$

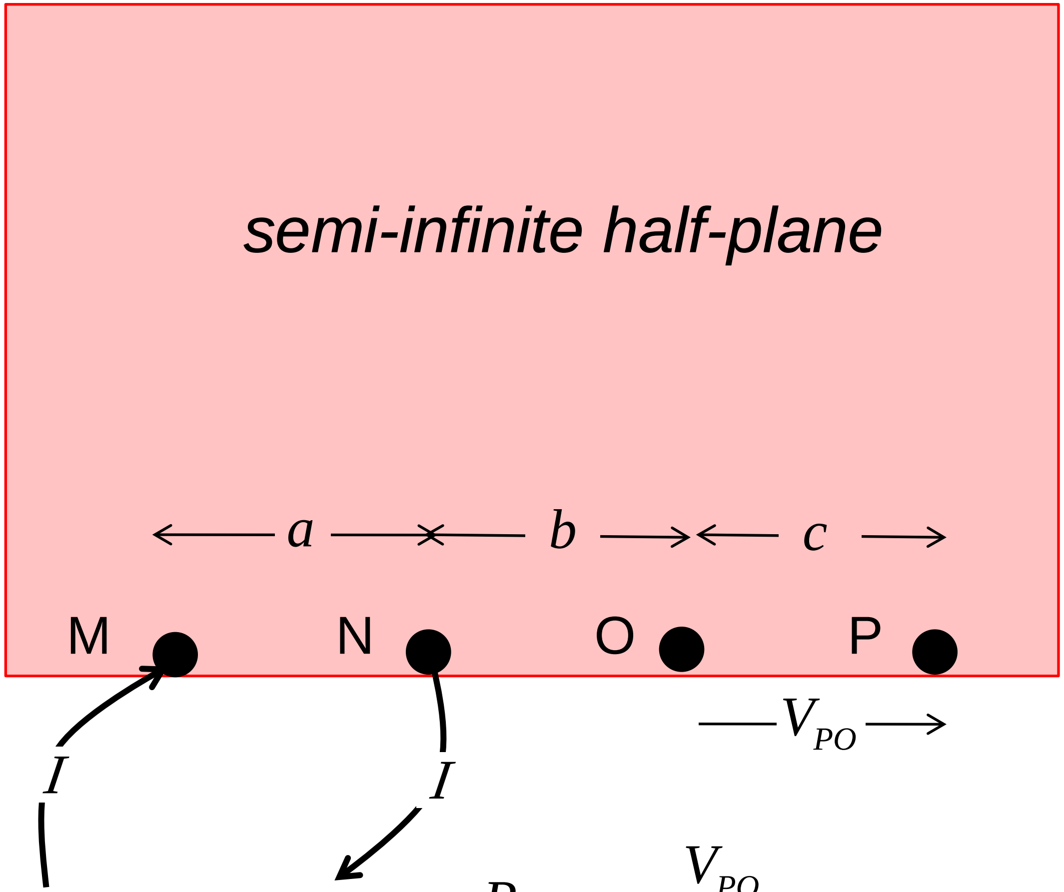
$$V_H = \rho_n\mu_H B_z I$$

So we can do Hall effect measurements on such samples.

van der Pauw approach: resistivity

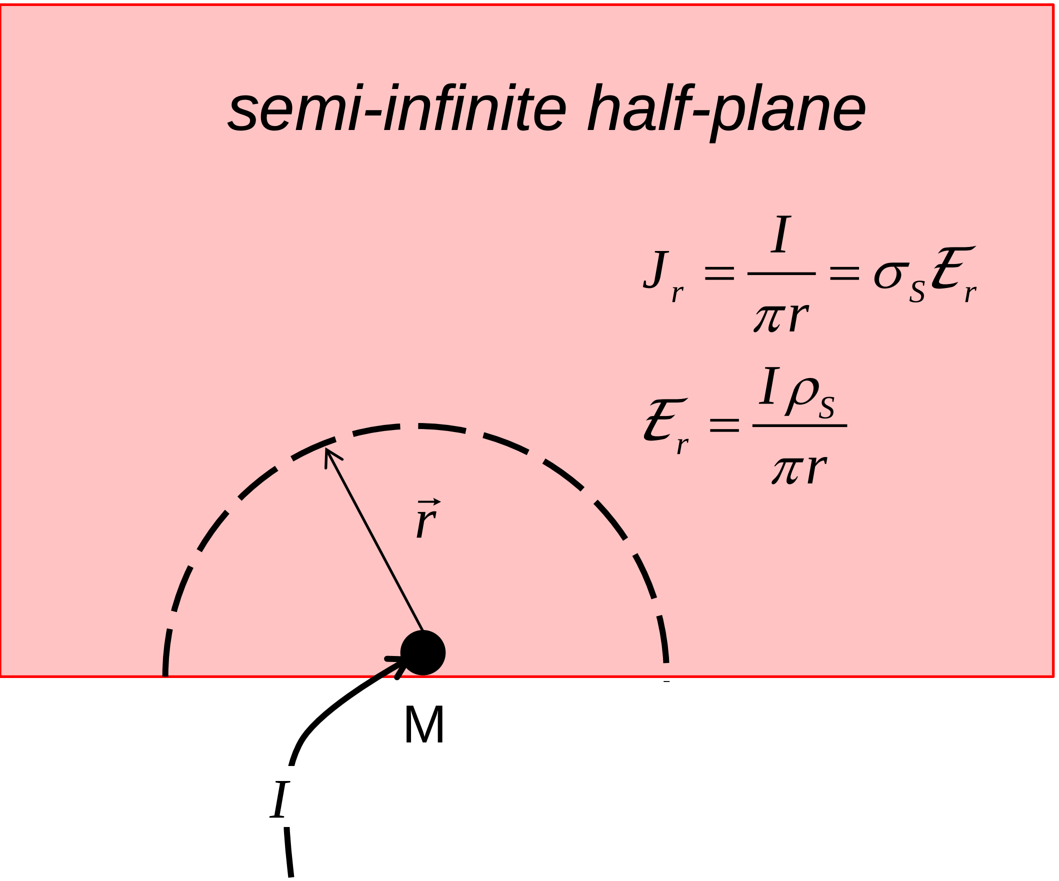
Resistivity





$$R_{MN,OP} = \frac{V_{PO}}{I}$$

van der Pauw approach: resistivity



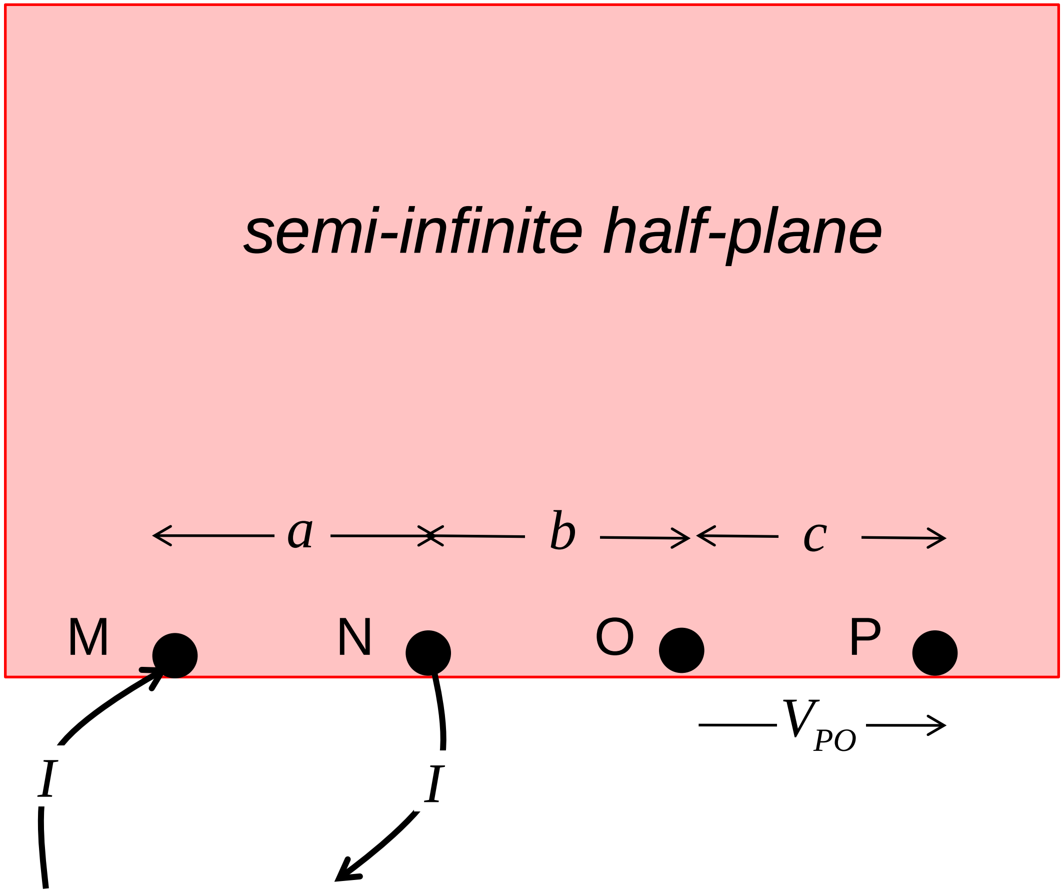
semi-infinite half-plane

$$J_r = \frac{I}{\pi r} = \sigma_S \mathcal{E}_r$$

$$\mathcal{E}_r = \frac{I \rho_S}{\pi r}$$

$$V(r) - V(r_0) = -\frac{I \rho_S}{\pi} \ln\left(\frac{r}{r_0}\right)$$

van der Pauw approach: resistivity



$$V(r)-V(r_0)=-\frac{I\rho_S}{\pi}\ln\left(\frac{r}{r_0}\right)$$

$$V(P)=-\frac{I\rho_S}{\pi}\ln\left(\frac{a+b+c}{r_0}\right)$$

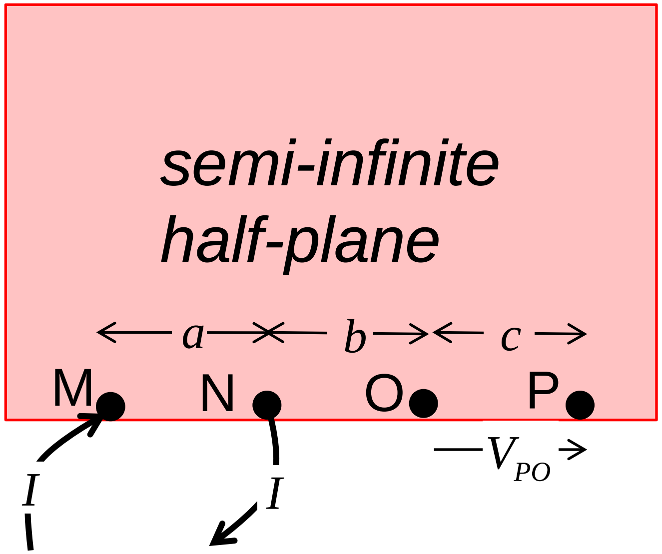
$$V(O)=-\frac{I\rho_S}{\pi}\ln\left(\frac{a+b}{r_0}\right)$$

$$V_{PO}=-\frac{I\rho_S}{\pi}\ln\left(\frac{a+b+c}{a+b}\right)$$

but there is also a contribution from contact *N*

$$V'_{PO}=+\frac{I\rho_S}{\pi}\ln\left(\frac{b+c}{b}\right)$$

van der Pauw approach: resistivity



$$R_{MN,OP} = \frac{V_{PO} + V'_{PO}}{I} = \frac{\rho_S}{\pi} \ln\left(\frac{(a+b)(b+c)}{b(a+b+c)}\right)$$

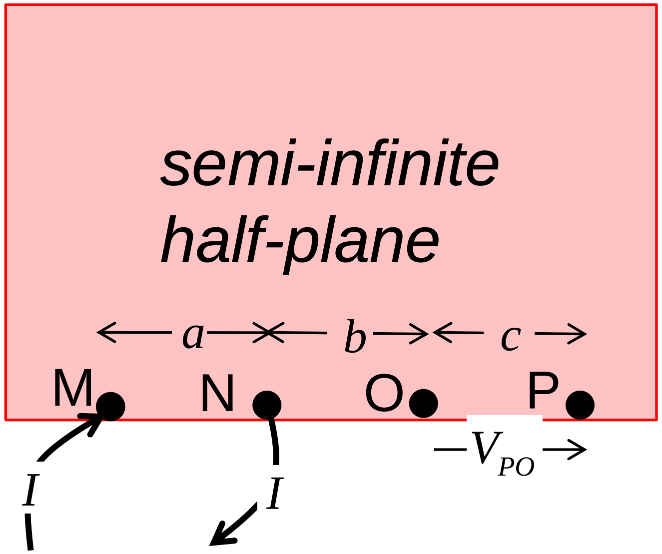
$$R_{NO,PM} = \frac{\rho_S}{\pi} \ln\left(\frac{(a+b)(b+c)}{ac}\right)$$

it can be shown that:

$$e^{-\frac{\pi}{\rho_S} R_{MN,OP}} + e^{-\frac{\pi}{\rho_S} R_{NO,PM}} = 1$$

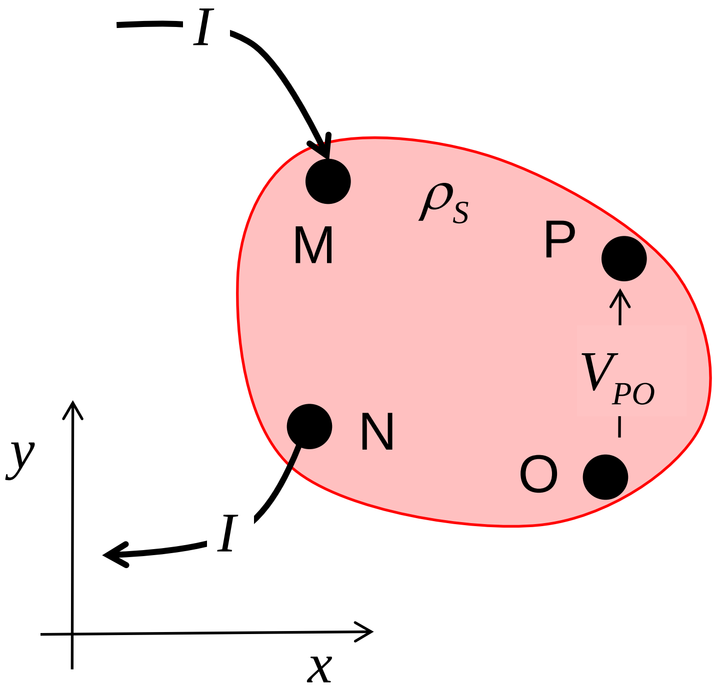
Given two measurements of resistance, this equation can be solved for the sheet resistance.

van der Pauw approach: resistivity



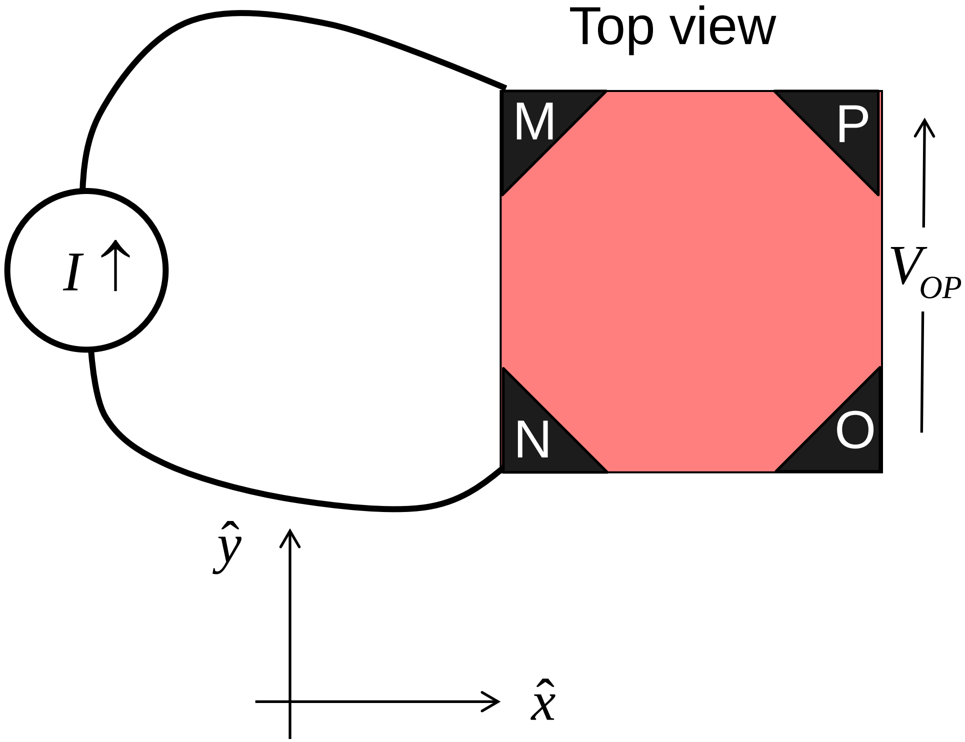
semi-infinite half-plane

$$e^{-\frac{\pi}{\rho_S}R_{MN,OP}} + e^{-\frac{\pi}{\rho_S}R_{NO,PM}} = 1$$



The same equation applies for an arbitrarily shaped sample!

van der Pauw technique: regular sample



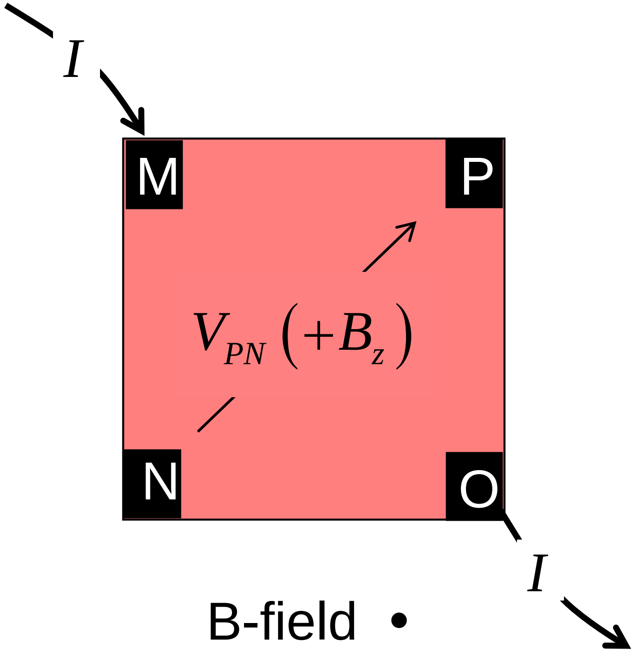
$$e^{-\frac{\pi}{\rho_S} R_{MN,OP}} + e^{-\frac{\pi}{\rho_S} R_{NO,PM}} = 1$$

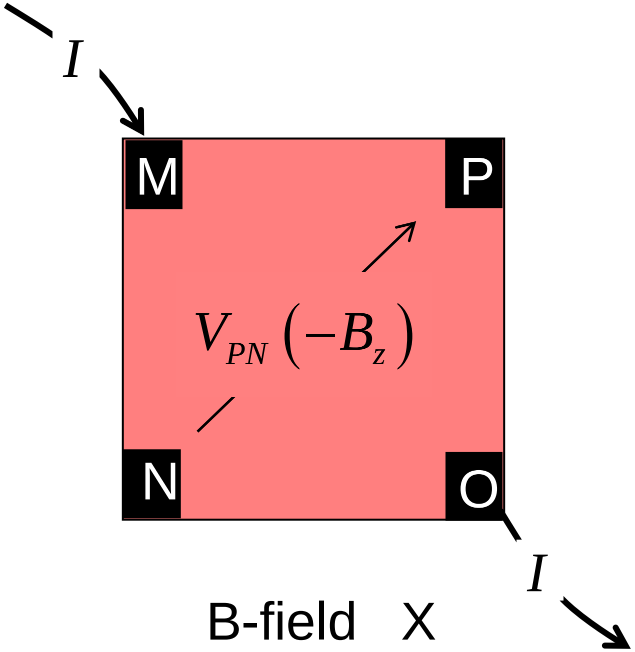
$$R_{MN,OP} = R_{NO,PM} = \frac{V}{I}$$

$$\rho_S = \frac{\pi}{\ln 2} \frac{V}{I}$$

Force *I* through two contacts, measure *V* between the other two contacts.

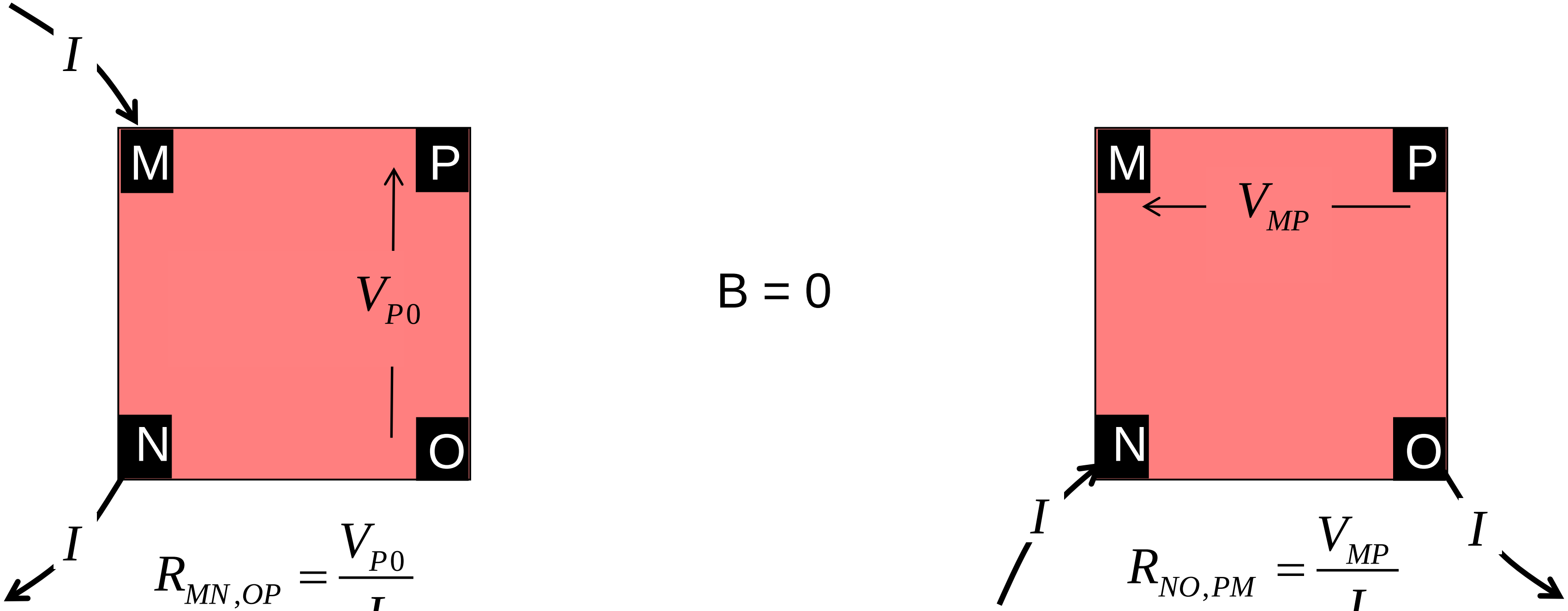
van der Pauw technique: summary





$$V_H = \frac{1}{2}\left[V_{PN}(+B_Z) - V_{PN}(-B_Z)\right] = \frac{r_H}{qn_S} B_z I = \frac{B_z I}{qn_H}$$

van der Pauw technique: summary



B = 0

$$R_{MN,OP} = \frac{V_{P0}}{I}$$

$$R_{NO,PM} = \frac{V_{MP}}{I}$$

$$e^{-\frac{\pi}{\rho_S} R_{MN,OP}} + e^{-\frac{\pi}{\rho_S} R_{NO,PM}} = 1$$

$$\sigma_S = n_S q \mu_n = \frac{n_S}{r_H} q \, r_H \mu_n = n_H \, q \, \mu_H$$

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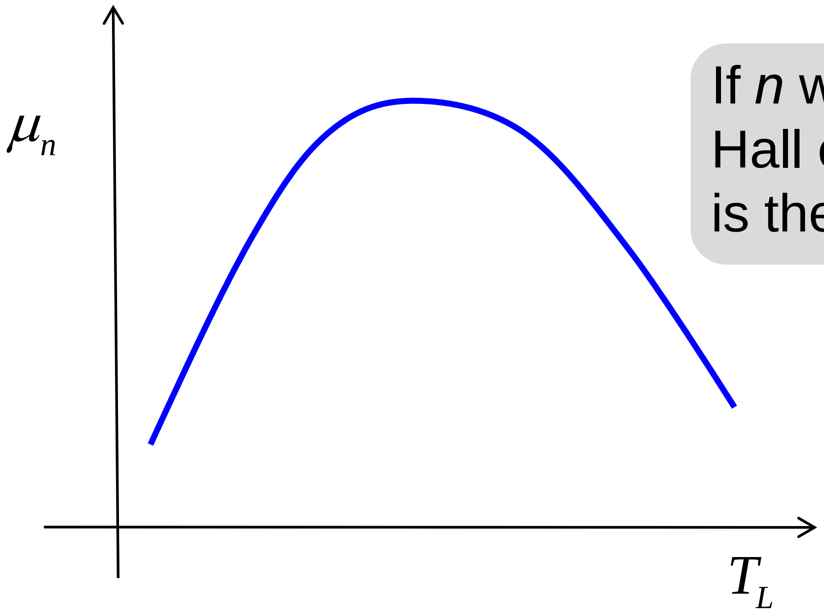
temperature-dependent measurements

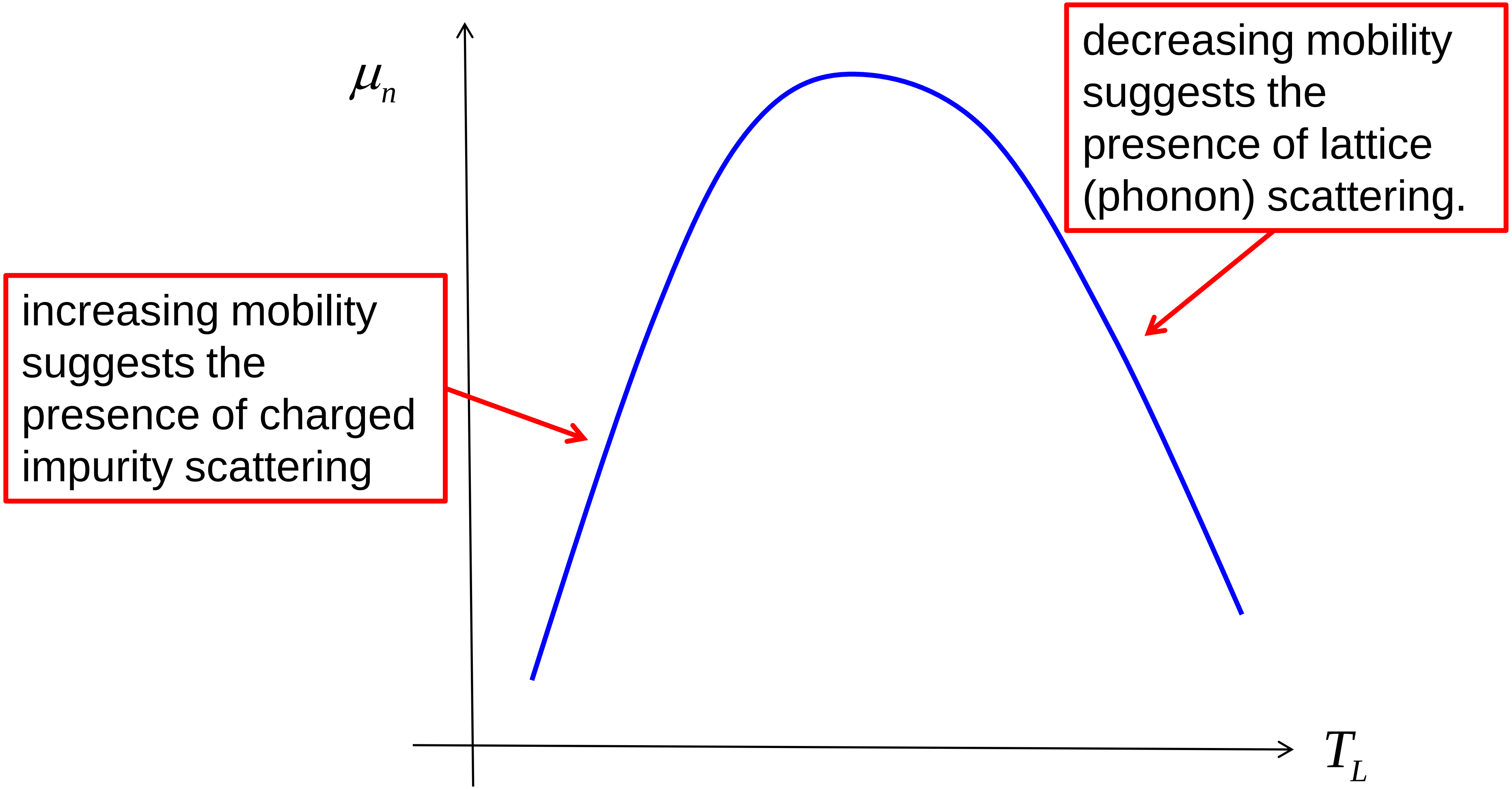
It is common practice to measure the temperature-dependent conductivity.

Assuming that the carrier density is known (or can be measured), a mobility is then extracted from: $\sigma = n q \mu_n$



If *n* was measured by Hall effect, then mobility is the Hall mobility.

interpretation

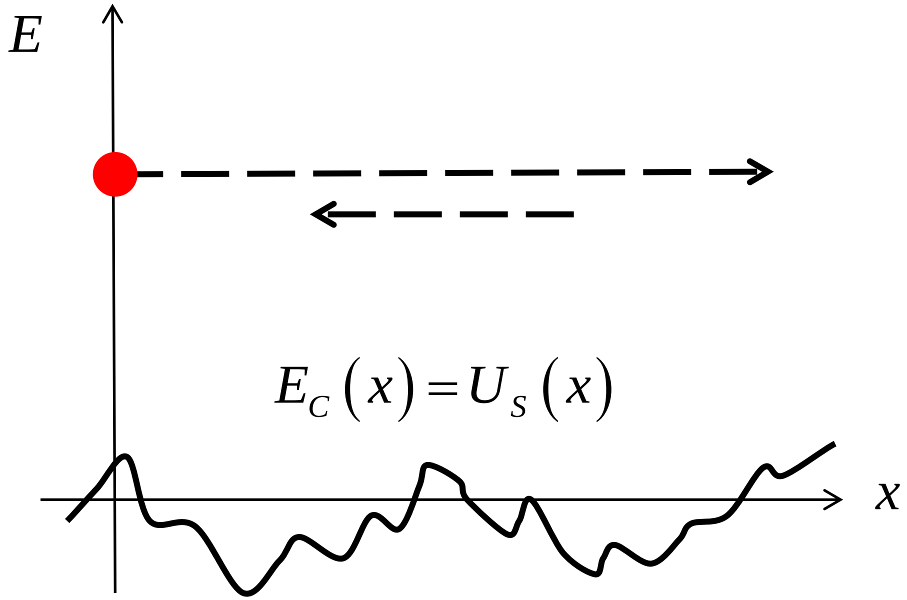


increasing mobility suggests the presence of charged impurity scattering

decreasing mobility suggests the presence of lattice (phonon) scattering.

charged impurity scattering

$\tau(E)\uparrow$ as $E\uparrow$



Random charges introduce random fluctuations in E_C, which act a scattering centers.

High energy electrons don't "see" these fluctuations and are not scattered as strongly.

Average carrier energy ~ $k_B T_L$.

lattice (phonon) scattering

$$\frac{1}{\tau(E)} \propto n_{ph}$$

$$n_{ph} = \frac{1}{e^{\hbar\omega/k_B T_L} - 1}$$

$$n_{ph} \uparrow \quad \text{as} \quad T_L \uparrow$$

Carrier scattering rate is proportional to the number of phonons.

Phonon occupation number given by the Bose-Einstein distribution.

Number of phonons increases as temperature increase. Scattering time decreases, and mobility decreases.

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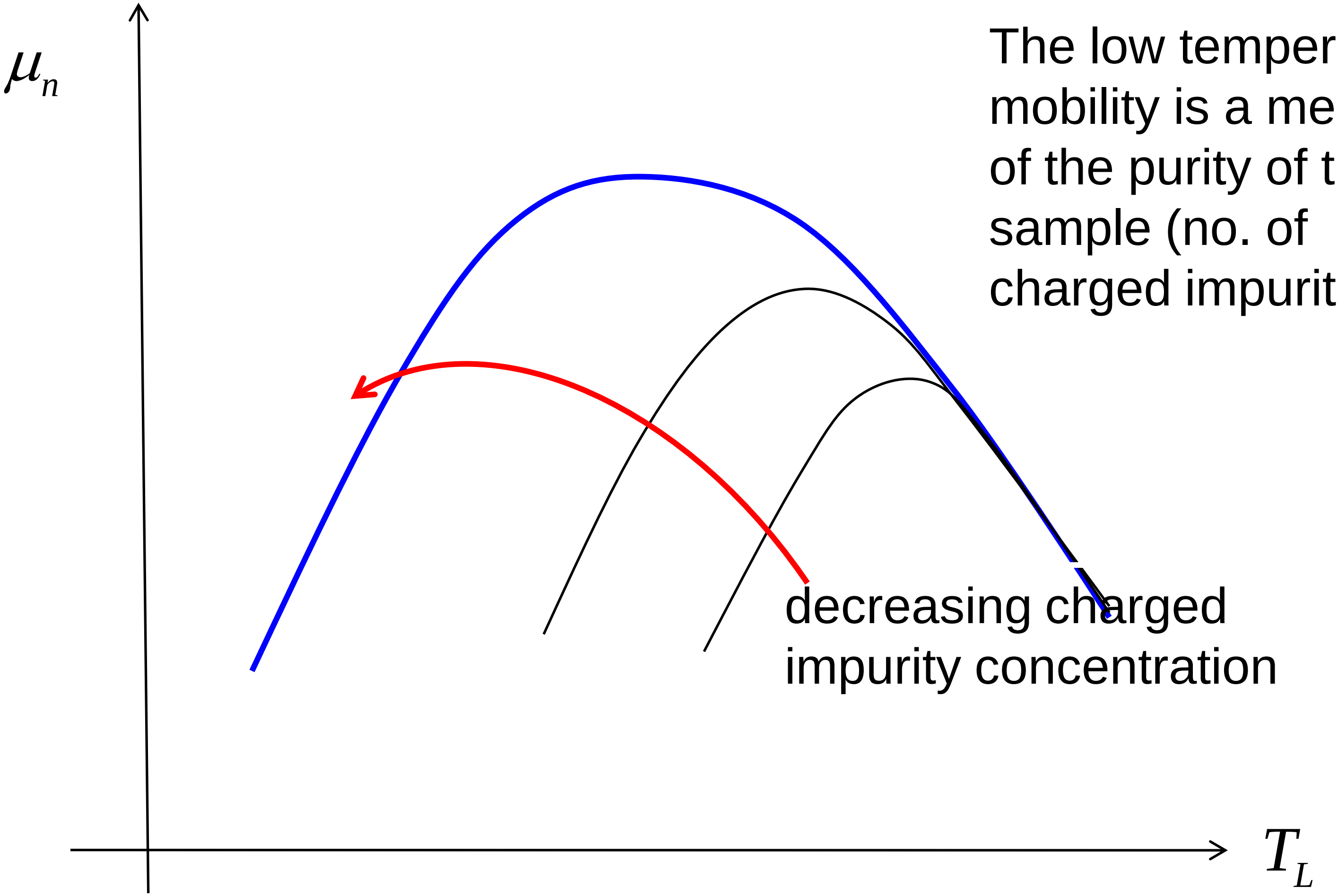
mobility vs. temperature



decreasing charged impurity concentration

The low temperature mobility is a measure of the purity of the sample (no. of charged impurities).

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    i) errors in Hall effect measurements
    ii) low B-field criterion
    iii) high B-fields
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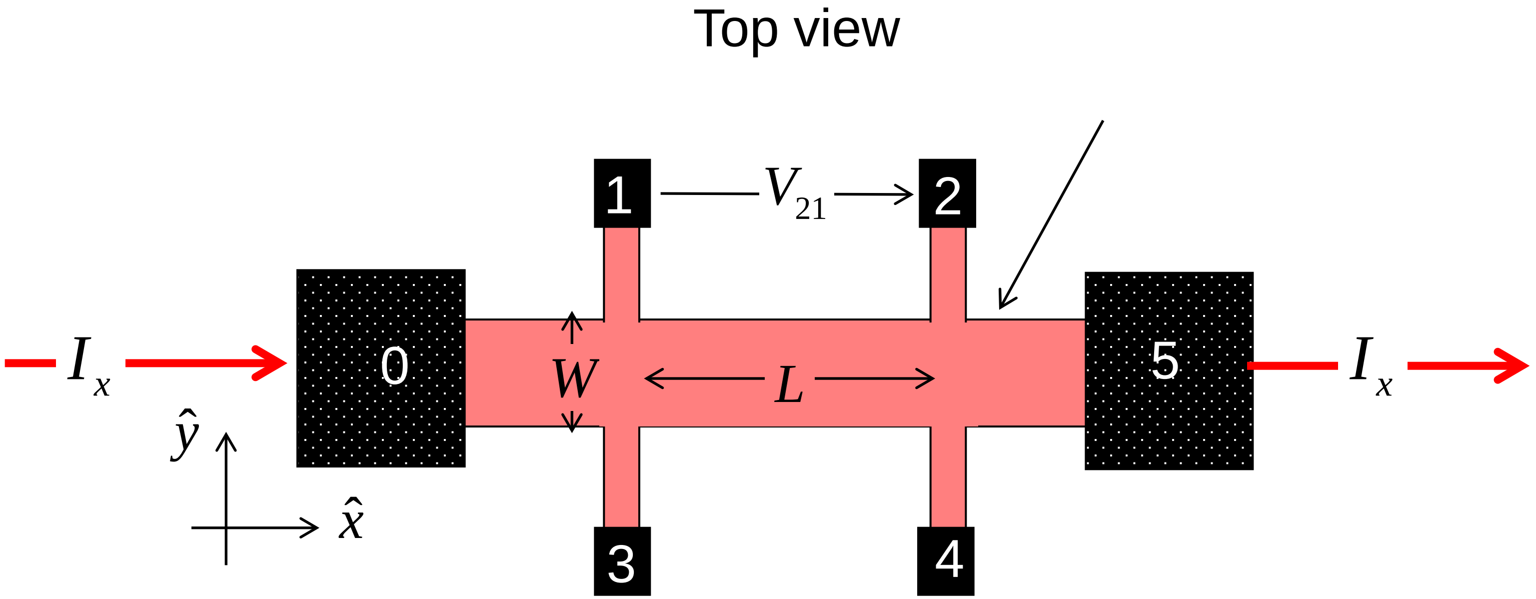
i) Hall effect measurements (errors)



We have assumed isothermal conditions to compute the Hall voltage, but we expect Peltier cooling at contact 0 and Peltier heating at contact 1. If the sample is not isothermal, how does the Hall voltage change?

magnetoconductivity tensor

$$\begin{pmatrix} J_{nx} \\ J_{ny} \end{pmatrix} = \begin{pmatrix} \sigma_S & -\sigma_S \mu_H B_z \\ +\sigma_S \mu_H B_z & \sigma_S \end{pmatrix} \begin{pmatrix} \mathcal{E}_x \\ \mathcal{E}_y \end{pmatrix}$$

$$J_{ni} = \sum_j \sigma_{ij}\left(B_z\right)\mathcal{E}_j \qquad \begin{pmatrix} \sigma_{11} & \sigma_{12z} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} = \begin{pmatrix} \sigma_S & -\sigma_S \mu_H B_z \\ +\sigma_S \mu_H B_z & \sigma_S \end{pmatrix}$$

$J_{ni} = \sigma_{ij}\left(B_z\right)\mathcal{E}_j$ (summation convention)

$$J_i = \sigma_S \mathcal{E}_i - \sigma_S \mu_H \varepsilon_{ijk} B_k \mathcal{E}_j$$

$$\begin{aligned} \varepsilon_{ijk} &= +1\left(i, j, k \ \text{ cyclic}\right) \\ &= -1\left(i, j, k \ \text{ anti-cyclic}\right) \\ &= 0\left(\text{otherwise}\right) \end{aligned}$$

from Lecture 7

$$\mathcal{E}_i = \rho_{ij}\left(\vec{B}\right) J_j + S_{ij}\left(\vec{B}\right)\partial_j T_L$$

$$J_i^Q = \pi_{ij}\left(\vec{B}\right) J_j - \kappa_{ij}^e\left(\vec{B}\right)\partial_j T_L$$

$$\rho_{ij}\left(\vec{B}\right) = \rho_0 + \rho_0 \mu_H \varepsilon_{ijk} B_k + \ldots$$

$$S_{ij}\left(\vec{B}\right) = S_0 + S_1 \varepsilon_{ijk} B_k + \ldots$$

$$\pi_{ij}\left(\vec{B}\right) = \pi_0 + \pi_1 \varepsilon_{ijk} B_k + \ldots$$

$$\kappa_{ij}^e\left(\vec{B}\right) = \kappa_0^e + \kappa_1 \varepsilon_{ijk} B_k + \ldots$$

For parabolic energy bands

Nernst effect

Assume that there is a temperature gradient in the *x*-direction. How is the electric field (Hall voltage) affected?)

$$\mathcal{E}_i = \rho_{ij}\left(\vec{B}\right) J_j + S_{ij}\left(\vec{B}\right)\partial_j T_L$$

$$\rho_{ij}\left(\vec{B}\right) = \rho_0 + \rho_0 \mu_H \varepsilon_{ijk} B_k + \ldots$$

$$S_{ij}\left(\vec{B}\right) = S_0 + S_1 \varepsilon_{ijk} B_k + \ldots$$

$$\mathcal{E}_y = \rho_0 J_y + \rho_0 \mu_H \varepsilon_{yjz} B_z J_j + S_0 \partial_y T_L + S_1 \varepsilon_{yjz} B_z \partial_j T_L$$

$$\mathcal{E}_y = +\rho_0 \mu_H \varepsilon_{yxz} B_z J_x + S_1 \varepsilon_{yxz} B_z \partial_x T_L$$

$$\mathcal{E}_y = -\rho_0 \mu_H B_z J_x - S_1 B_z \partial_x T_L$$

Nernst voltage
Reverse direction of B_z and J_x and average results to eliminate.

other effects

Other "thermomagnetic effects" such as the Ettingshausen and Righi-Leduc effects also occur and affect the measured Hall voltage. See Lundstrom, Chapter 4, Sec. 4.6.2 for a discussion.

ii) small B-field criterion

$\omega_c \tau_m << 1$

$\mu_n B_z << 1$

$$\omega_c = \frac{qB}{m^*}$$

$$\omega_c \tau_m = \frac{q\tau_m B}{m^*} = \mu_n B$$

What does this mean physically?

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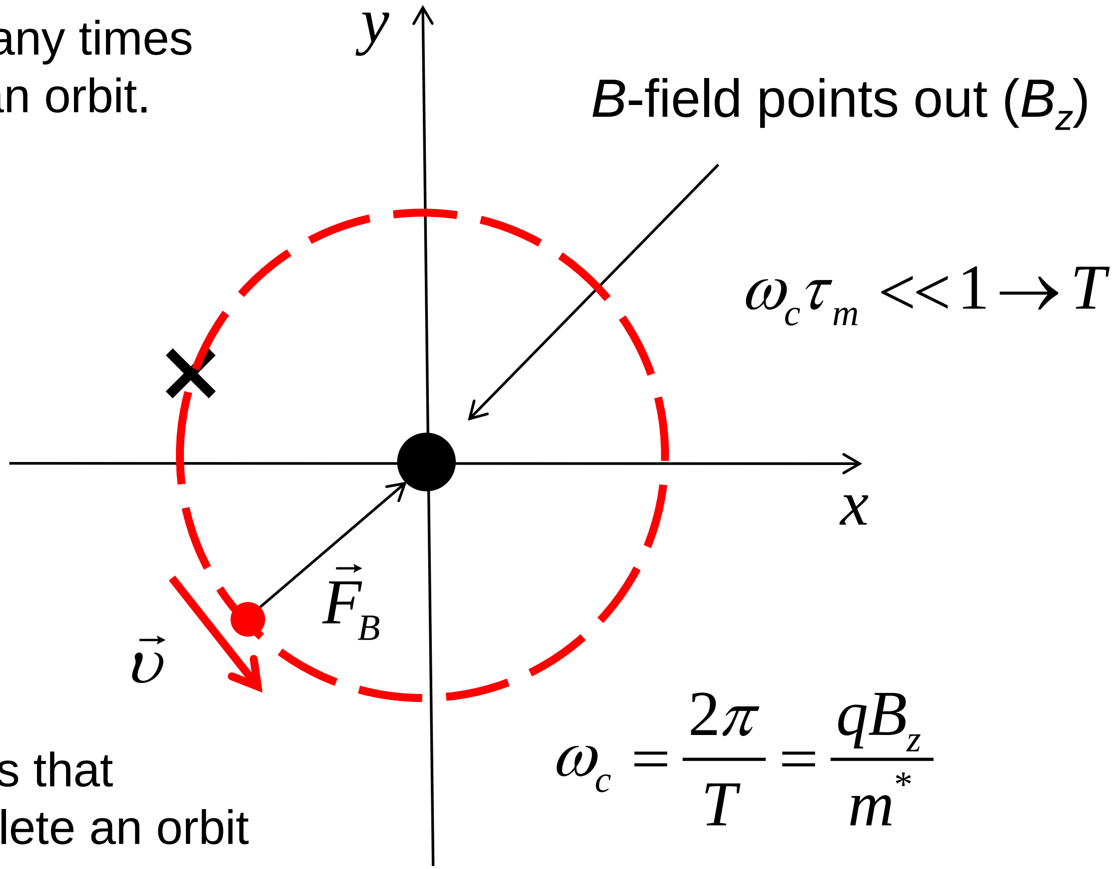
small B-field: physical meaning

"Low B-field" means that electrons scatter many times before completing an orbit.



$$\omega_c \tau_m << 1 \rightarrow T >> \tau_m$$

$$\omega_c = \frac{2\pi}{T} = \frac{qB_z}{m^*}$$

"High B-field" means that electrons can complete an orbit without scattering.

some numbers

silicon

$$\mu_n = 1000 \quad \text{cm}^2/\text{V-s}$$

$$r_H = 1$$

$$B_z = 2,000 \quad \text{Gauss}$$

$$B_z = 0.2 \quad \text{Tesla}$$

$$\mu_H B_z \approx 0.02 << 1$$

Hall effect measurements with typical laboratory-sized magnets are in the low B-field regime. Except – for very high mobility sample such as modulation doped films.)

Birck Nanotechnology Center:	1-8 T
National High Magnetic Field Lab (Florida State Univ.):	45 T

some numbers (III-V modulation-doped)

InAlAs/InGaAs

T_L = 300K

$$\mu_n \approx 10{,}000 \quad \mathrm{cm}^2/\mathrm{V\text{-}s}$$

$$r_H = 1$$

$$B_z = 2{,}000 \quad \mathrm{Gauss}$$

$$B_z = 0.2 \quad \mathrm{Tesla}$$

$$\mu_H B_z \approx 0.2 << 1$$

InAlAs/InGaAs

T_L = 77K

$$\mu_n \approx 100{,}000 \quad \mathrm{cm}^2/\mathrm{V\text{-}s}$$

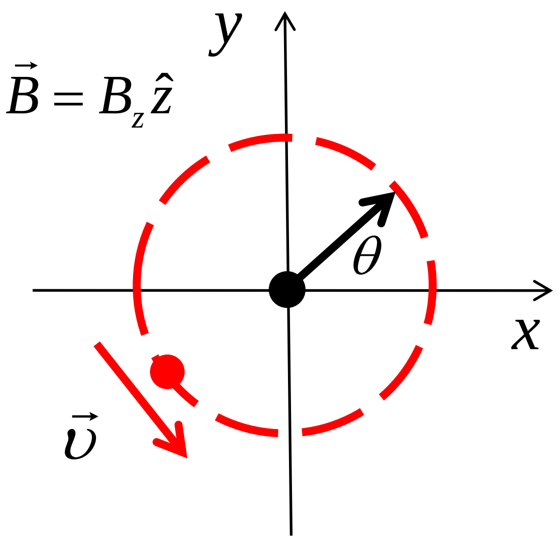
$$r_H = 1$$

$$B_z = 2{,}000 \quad \mathrm{Gauss}$$

$$B_z = 0.2 \quad \mathrm{Tesla}$$

$$\mu_H B_z \approx 2 > 1$$

iii) high B-fields



$$\cos\theta(t) = \cos\theta(0)\, e^{i\omega_c t}$$

harmonic oscillator:

Quantum mechanically:

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c$$

"Landau levels"

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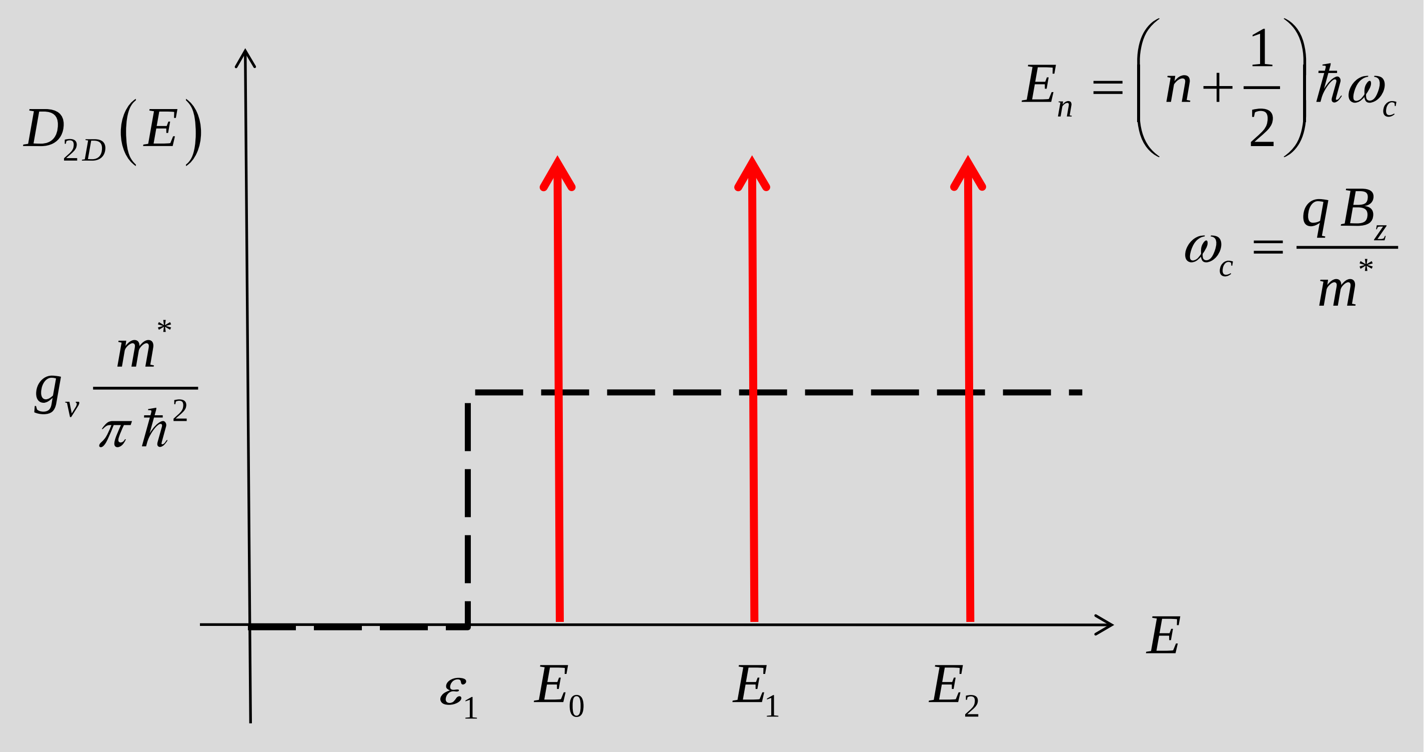
effect on DOS

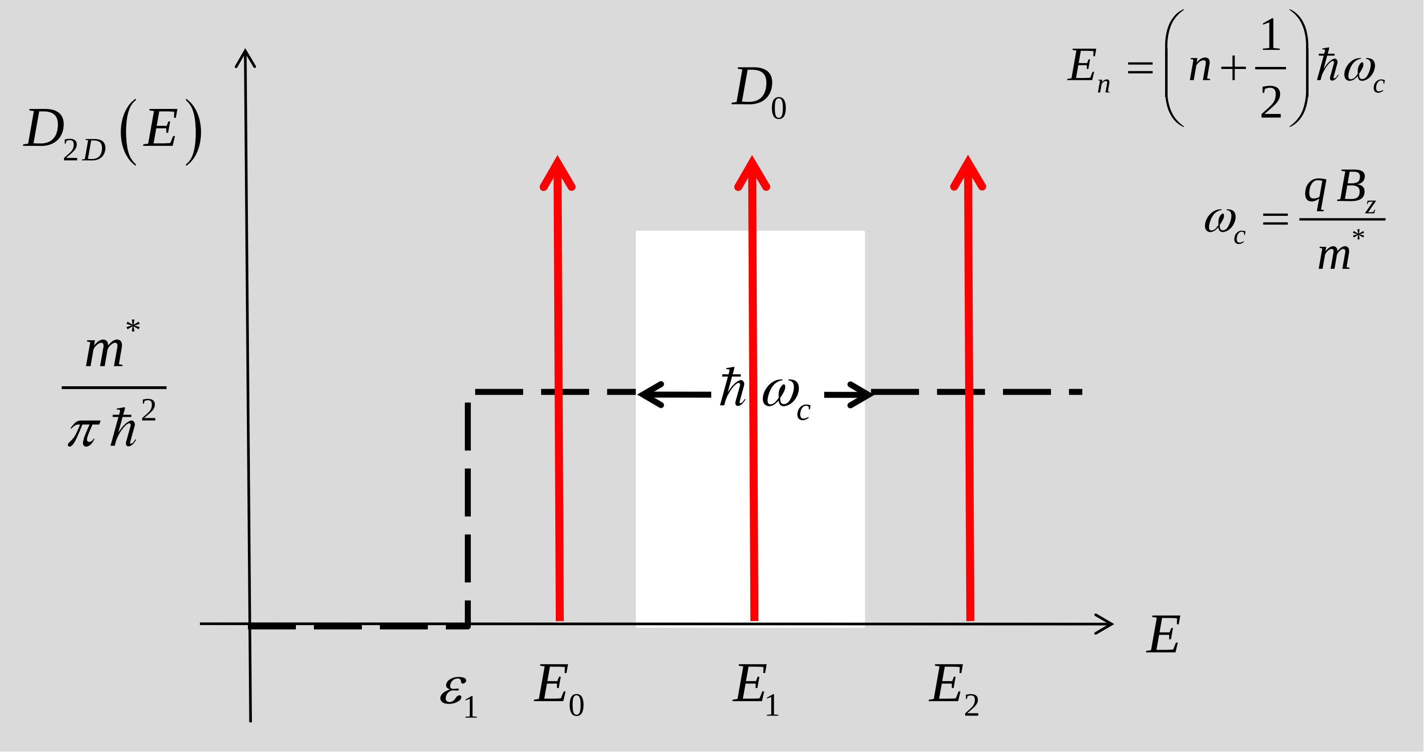


$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c$$

$$\omega_c = \frac{qB_z}{m^*}$$

$$D_{2D}(E, B_z) = D_0 \sum_{n=0}^{\infty} \delta\left[E - \varepsilon_1 - \left(n + \frac{1}{2}\right)\hbar\omega_c\right]$$

degeneracy of Landau levels

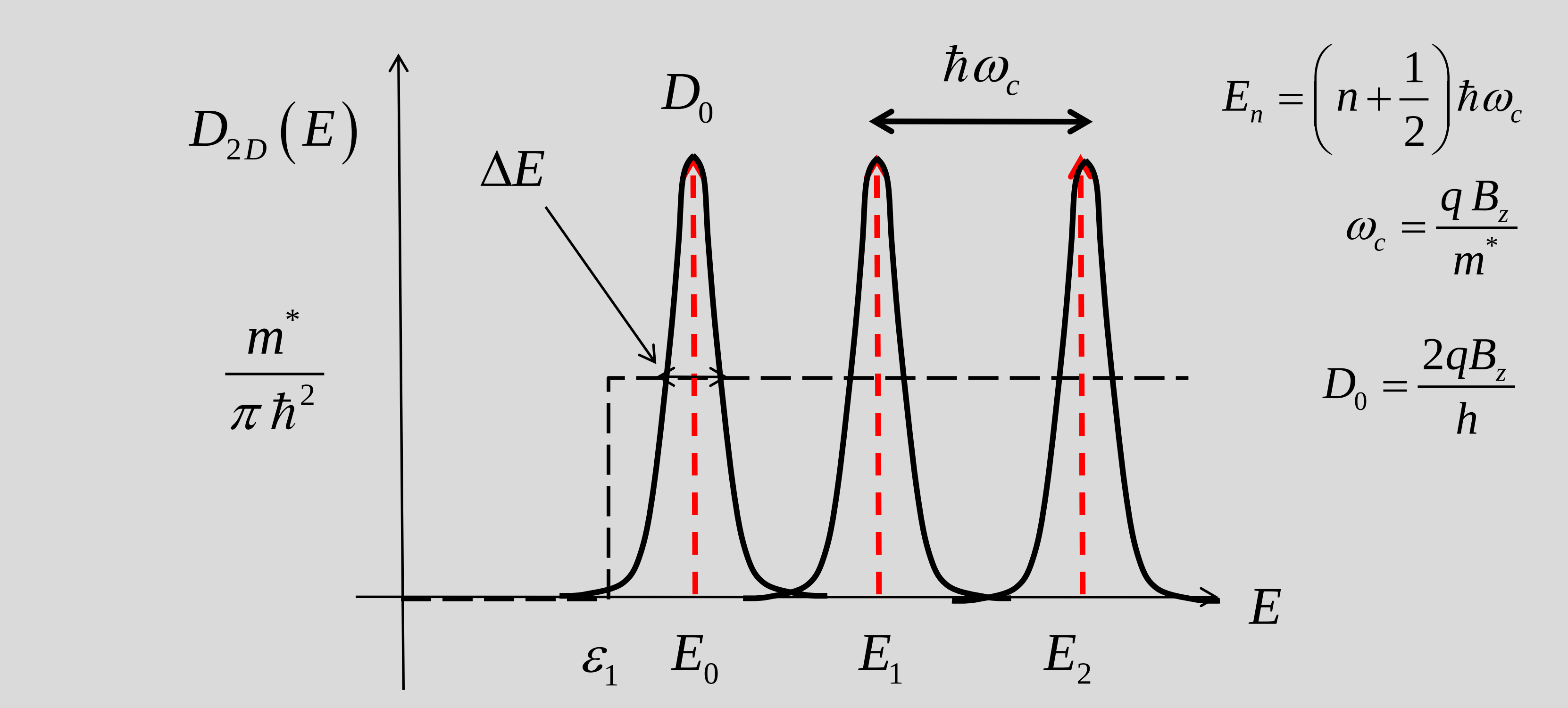


$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c$$

$$\omega_c = \frac{qB_z}{m^*}$$

$$D_0 = \hbar\omega_c \times \frac{m^*}{\pi\hbar^2} = \frac{2qB_z}{h}$$

broadening of Landau levels



$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c$$

$$\omega_c = \frac{q B_z}{m^*}$$

$$D_0 = \frac{2qB_z}{h}$$

$$\Delta E \Delta t = \hbar$$

$$\Delta E = \frac{\hbar}{\tau}$$

to observe Landau levels: $\hbar\omega_c >> \Delta E \rightarrow \omega_c\tau >> 1$

example

If B = 1T, how many states are there in each LL?

$$D_0 = \frac{2qB_z}{h} = 4.8\times10^{10}\ \text{cm}^{-2}$$

If $n_S = 5 \times 10^{11}\ \text{cm}^{-2}$, then 10.4 LL's are occupied.

How high would the mobility need to be to observe these LL's?

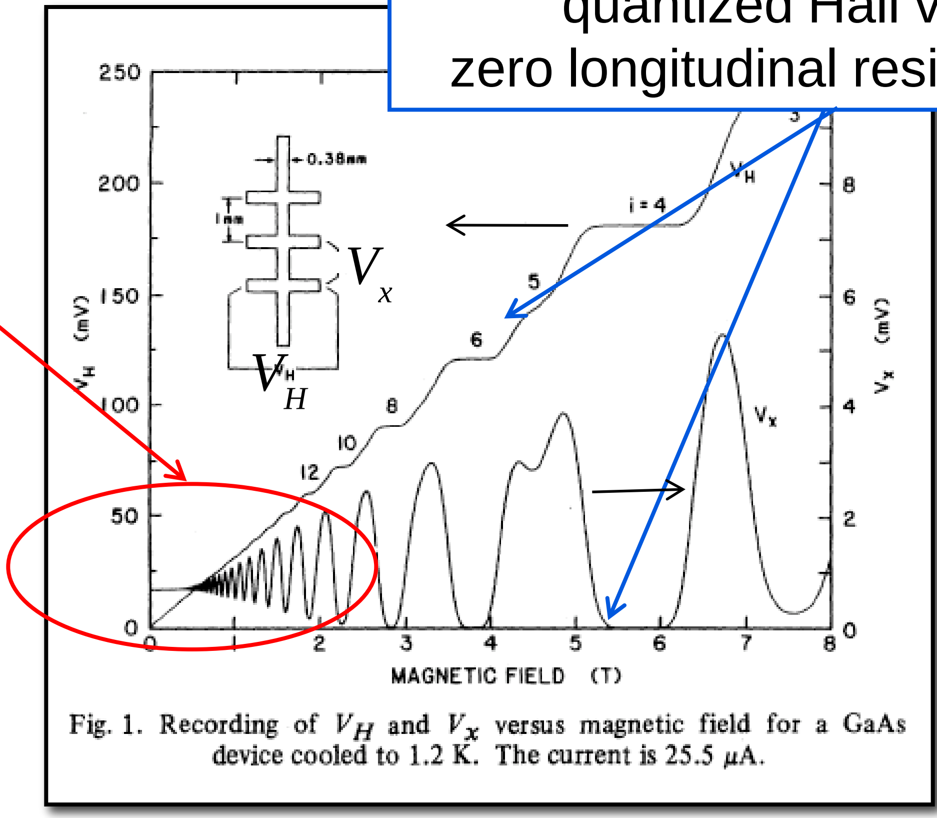
$$\mu B > 1 \rightarrow \mu > 10{,}000\ \text{cm}^2/\text{V-s} \quad \left(B = 1\,\text{T}\right)$$

"modulation-doped semiconductors"

SdH oscillations

Longitudinal magneto-resistance

"Shubnikov-deHaas (SdH) oscillations"



quantized Hall voltage
zero longitudinal resistance, R_{xx}

Fig. 1. Recording of V_H and V_x versus magnetic field for a GaAs device cooled to 1.2 K. The current is 25.5 μA.

M.E. Cage, R.F. Dziuba, and B.F. Field, "A Test of the Quantum Hall Effect as a Resistance Standard," *IEEE Trans. Instrumentation and Measurement*," Vol. IM-34, pp. 301-303, 1985

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summary

1) Hall bar or van der Pauw geometries allow measurement of both resistivity and Hall effect from which the Hall concentration and Hall mobility can be deduced.

2) Temperature-dependent measurements provide information about the dominant scattering mechanisms.

3) Care must be taken to exclude thermoelectric effects.

4) High B-field measurements provide additional information, but also require high B-fields or high mobilities.

5) Measurements of the Seebeck coefficient and electronic heat conductivity require special considerations.

for more about low-field measurments

D.K. Schroder, *Semiconductor Material and Device Characterization, 3rd Ed.*, IEEE Press, Wiley Interscience, New York, 2006.

D.C. Look, *Electrical Characterization of GaAs Materials and Devices*, John Wiley and Sons, New York, 1989.

M.E. Cage, R.F. Dziuba, and B.F. Field, “A Test of the Quantum Hall Effect as a Resistance Standard,” *IEEE Trans. Instrumentation and Measurement*,” Vol. IM-34, pp. 301-303, 1985

L.J. van der Pauw, “A method of measuring specific resistivity and Hall effect of discs of arbitrary shape,” *Phillips Research Reports*, vol. 13, pp. 1-9, 1958.

Lundstrom, *Fundamentals of Carrier Transport*, Cambridge Univ. Press, 2000. Chapter 4, Sec. 7

questions

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