

ECE616 Homework #4
Out: Tuesday, October 18, 2011
Due: Tuesday, November 1, 2011

Announcements:

Quiz 2 will be on Thursday, Oct 20 during the 1st half of the class. It will cover Chapter 3 on Measurement. You are allowed 1 formula sheet (8 1/2 x 11 one sided) and it is also open book. (Remember - there will not be much time for this quiz).

Topic: Pulse Measurement; Dispersion

Text: Ultrafast Optics

Chapter 3, Sections 3.5-3.6, 3.8; Chapter 4, Section 4.1

- 1) Text 3.9
 - 2) Text 3.10
 - 3) See attached
 - 4) See attached
 - 5) Text 4.1
 - 6) Text 4.2
- 3) In this problem your task is to use PG-FROG (polarization gating FROG) data to retrieve the amplitude and phase profile of an ultrashort pulse. The data will be available on Blackboard.

The polarization gating FROG signal is given by

$$I_{FROG}(\omega, \tau) = \left| \int dt e(t) |e(t - \tau)|^2 e^{-i\omega\tau} \right|^2$$

For the retrieval please use the basic iterative Fourier transform algorithm discussed in the text.

After retrieving the pulse, please do the following:

- (a) Make plots of the intensity and the phase of the pulse, also the real part and imaginary part of the electric field, **as functions of time**. For my convenience please normalize the peak intensity of your retrieved pulse to 1, and express phase in radians.
- (b) Make plots of the power spectrum, the phase, and the real and imaginary parts of the spectrum, **as functions of frequency (in THz)**. Also, determine and specify a polynomial fit to the spectral phase.

- (c) What is the intensity FWHM pulse duration and the time-bandwidth product?
- (d) Give data on your FROG-trace error vs. number of iterations (a log plot would be helpful.) What is the lowest FROG-error you can get?

For real traces with noise, the final FROG error is usually below 1%. Here there is no noise, so your error may be much lower (how much I'm not sure). Note, however, in deciding when your algorithm has converged, it's not enough to just look at the FROG-trace error. You should also compare the supplied FROG-data and your computer generated FROG-data visually to make sure they agree closely.

- (e) Run the algorithm several times, using different initial guesses. Are your results consistent for different runs? Do you get the same pulse every time? Indicate the results of your comparison in some appropriate way. This is a simple check on the uniqueness of your retrieved pulse.
- (f) Try corrupting the original FROG data with additive noise. For various amplitudes of additive noise, run the retrieval algorithm and report on the final FROG error and the resemblance of the retrieved waveform to that you obtained in the noise-free case. Your results should help elucidate how the ability to retrieve the actual waveform degrades as the signal to noise ratio decreases.

Notes and hints:

- (1) The data is available on Blackboard with filename PGFROGdata.mat. Download and then in Matlab, type the following commands

```
load PGFROGdata
levels = [1, 0.85, 0.7, 0.5, 0.3, 0.17, 0.1];
figure(1);
colormap(1-gray);
contourf(tau,f,I_PGFROG,levels);
axis([-2 2 -2 2]);
grid on;
xlabel('\tau (ps)');
ylabel('Frequency (THz)');
```

You should get the same image as the “Expanded view of dataset” plot which is attached for your reference.

In the PGFROGdata.mat file there are three arrays:

f (1x256) : the frequencies of the PGFROG data
tau (1x129) : the time delays, tau, of the FROG data
I_PGFROG (256x129) : the actual FROG data (normalized to a maximum value of unity.)

The array element $I_PGFROG(7,25)$ contains data corresponding to $f(7)$ and $\tau(25)$. The array f is uniformly spaced in units of THz, and the array τ is uniformly spaced in units of ps.

- (2) Possible ambiguities that will arise are the overall phase factor, the time position of the pulse, and the pulse amplitude. These ambiguities are not important, except as they affect your algorithm. You do have to be careful with the time position of your pulse, to make sure the pulse does not disappear at one of the edges of your array. Two possible solutions: (a) shift the pulse back to zero in each loop. (b) When forming the time shifted gating pulse, use a *circular* shifting function prior to multiplying the pulse and the gate. This can be accomplished for example using the Matlab function “wshift”.
- (3) You may wish to use a random estimate for the initial $e(t)$, generated with the Matlab function of “rand”.
- (4) To find the spectral phase and generate a polynomial fit, you may wish to use the following procedure: Select a frequency range where the magnitude of the retrieved spectrum is significantly greater than zero (e.g., a few percent of the peak value). Calculate the spectral phase of this subset using the Matlab functions “angle” and “unwrap”. Then fit the spectral phase to a polynomial using the Matlab function “polyfit”. Try different orders of polynomials.
4. Self-referencing spectral interferometry (or SPIDER), discussed in section 3.8.2, is another popular pulse measurement technique (not discussed in class, sorry). As one step in SPIDER, one splits the unknown input pulse into a replica of identical pulses with delay τ . As described in text, the algorithm for extracting the spectral phase $\psi(\omega)$ from the SPIDER data requires exact knowledge of the delay τ .
 - a) The actual delay is τ_0 , but the experimentalist believes (erroneously) that the delay is $\tau_0 + \delta\tau$, where $\delta\tau$ is a small error ($|\delta\tau| \ll \tau_0$). Assume that all the other conditions outlined in the text are satisfied. The erroneous delay value $\tau_0 + \delta\tau$ is used to extract the spectral phase from SPIDER data. Derive a formula for the error induced in the estimate of spectral phase.
 - b) Now we consider a concrete example. The pulse to be measured is a bandwidth-limited Gaussian with duration of lps (intensity FWHM). The actual delay is $\tau_0 = 10\text{ps}$, and the spectral shear is $\Omega = 2\pi(0.04\text{THz})$. Assume delay errors of (i) $\delta\tau = 0$ (no error), (ii) $\delta\tau = +100\text{ fs}$, and (iii) $\delta\tau = +1\text{ ps}$. Work out the spectral phase function computed by the experimentalist in each of the three cases. What are the FWHM intensity durations corresponding to each of the three cases?

PG-FROG Data

