Homework 2

Due in class on September 8, 2011

1. Prove all the Fourier transform properties discussed in class:
   a) Shift theorem
   b) Parseval's theorem
   c) Similarity theorem
   d) Convolution theorem
   e) Correlation theorem
   f) If F is the Fourier transform of *f*, what is the value F(0)? This is the "central ordinate" theorem.

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2. Calculate the Fourier transform of the following functions:
   a) $f(x) = \exp\left[-x^2/(2a^2)\right]$
   b) $\Pi(x/a)$, where Π is the rectangular function, defined as:
   $$\Pi(x/a) = \begin{cases} 1, \; if \; -a/2 < x < a/2 \\ 0, \; rest \end{cases}$$
   c) $f(x) = \sin(kx)$
   d) $f(x) = \cos(kx)$
   e) $f(x) = \exp(-x/a)$ and $f(x) = \exp(-|x|/a)$, a>0

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3. Solve problem 2 for 2D functions (e.g., $f(x,y) = \exp\left[-\left(x^2 + y^2\right)/(2a^2)\right]$, $f(x,y) = \cos(k_x x + k_y y)$).

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4. Calculate the autocorrelation of the following functions
   a) $f(x) = \exp\left[-x^2/(2a^2)\right]$
   b) $\Pi(x/a)$
   c) $f(x) = \sin(kx)$

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5. Let *f*:***R*** ➔*C* be a function that has the property f(x)=0 for x<0. Show that the real and imaginary parts of its Fourier transform, F(w), are related. *Hint: express f in terms of a step (Heaviside) function and look up its Fourier transform.* Discuss as much as you can the implications of this important result. Look up "complex analytic signals".

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