

**ECE-656: Fall 2011**

**Lecture 33:**

**Heterostructures**

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# L32 outline

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- 1) Review of L31
- 2) Carrier temperature and heat flux
- 3) Heterostructures**
- 4) Summary

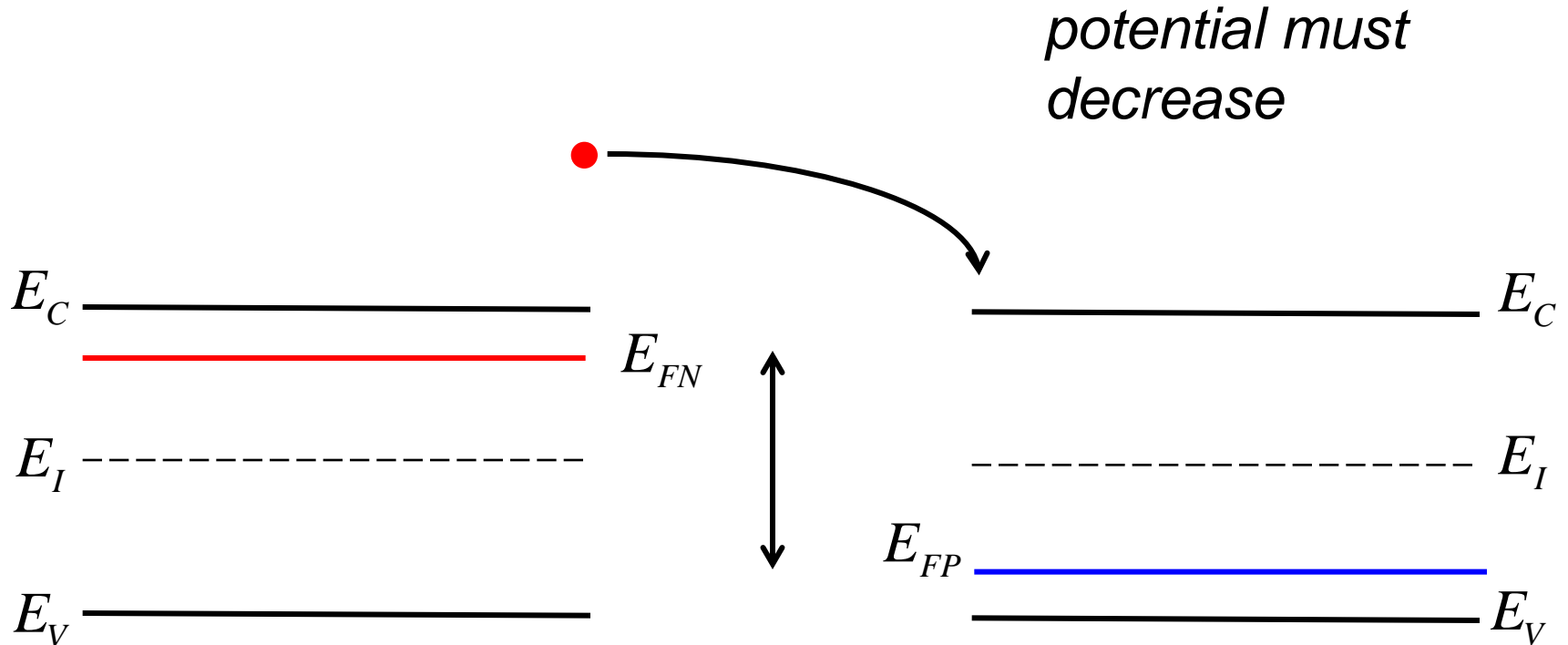
(Reference: Chapter 5, Lundstrom, FCT)



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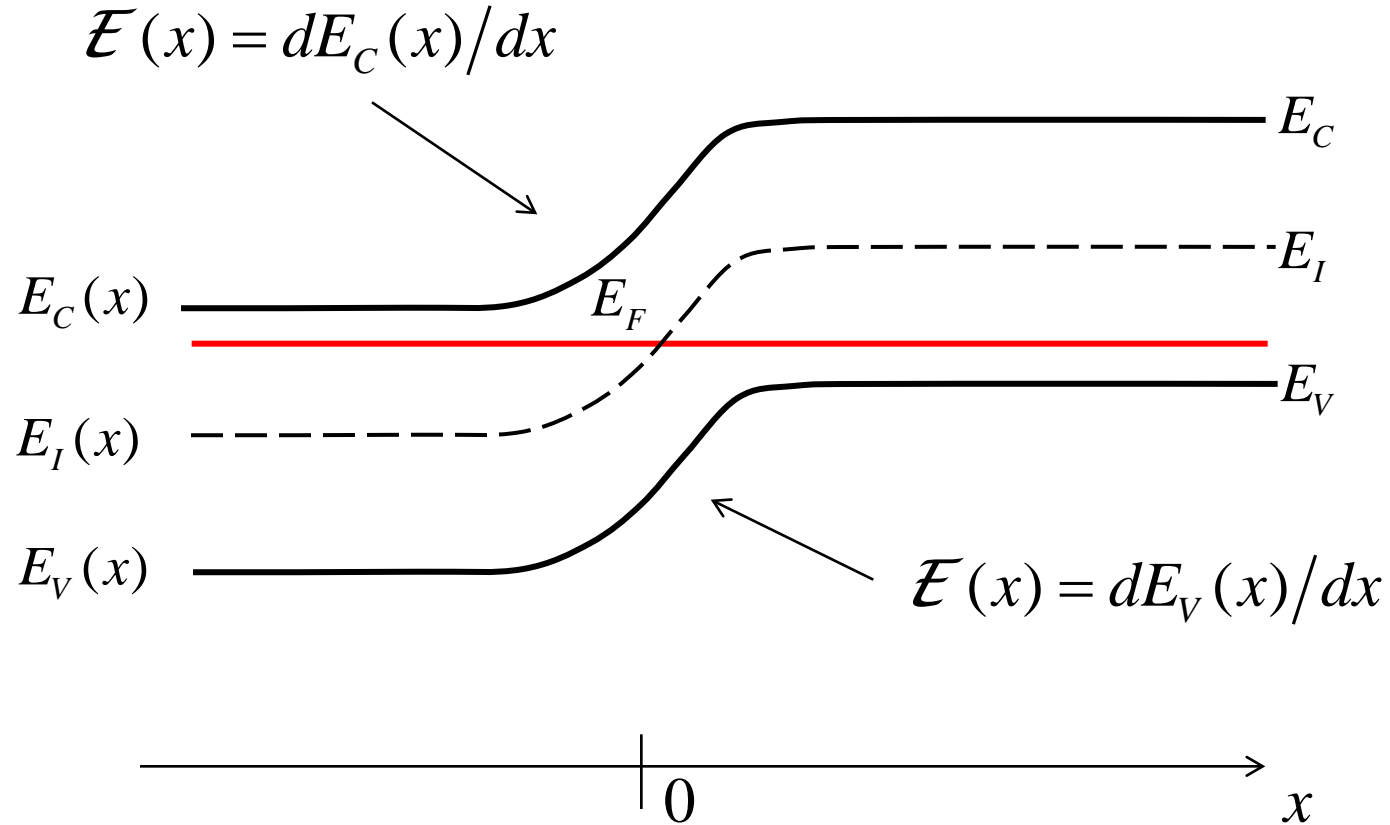
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# review: pn homojunctions

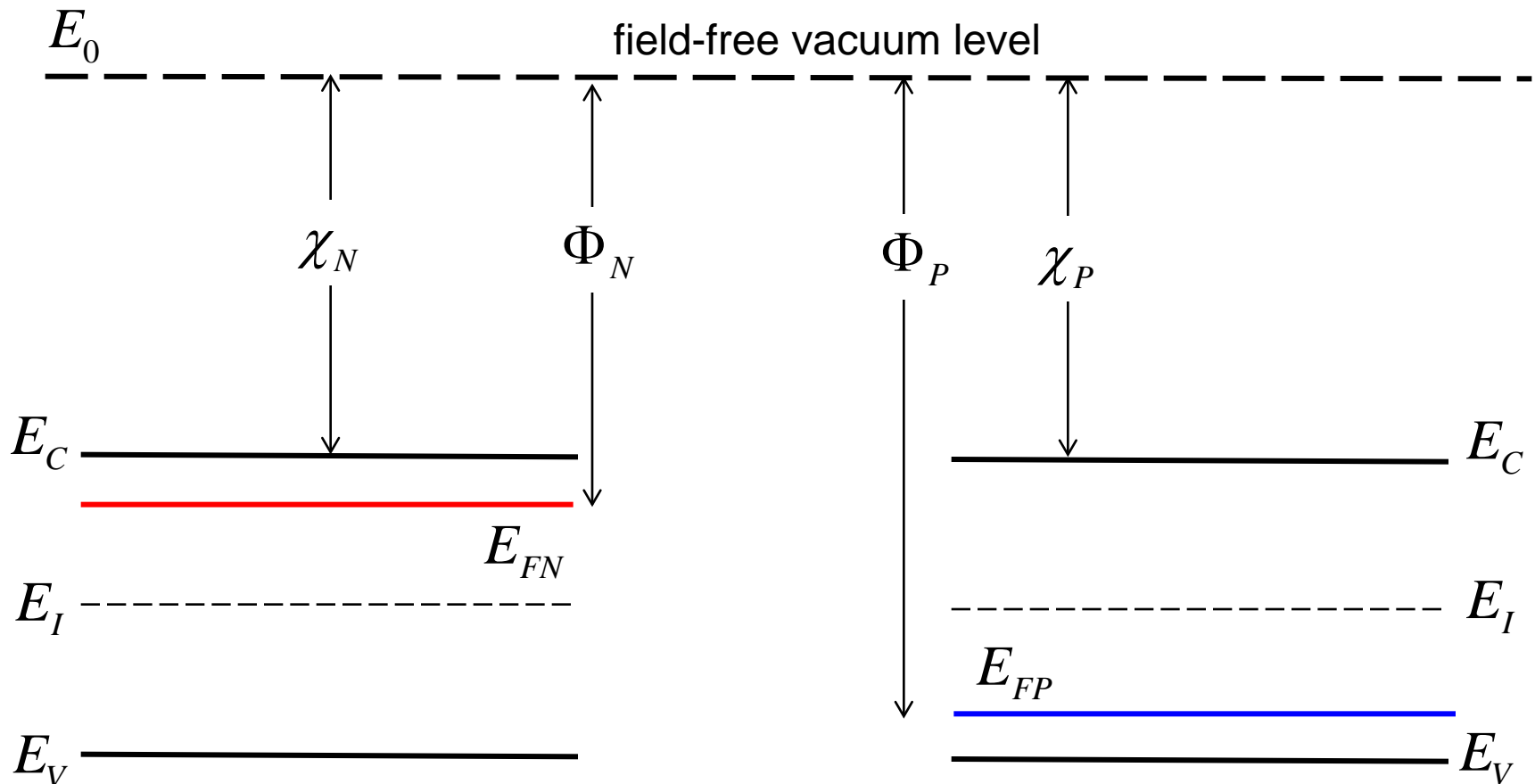


$$qV_{BI} = (E_{FN} - E_{FP})/q$$

# review: pn homojunctions



# reference for the energy bands



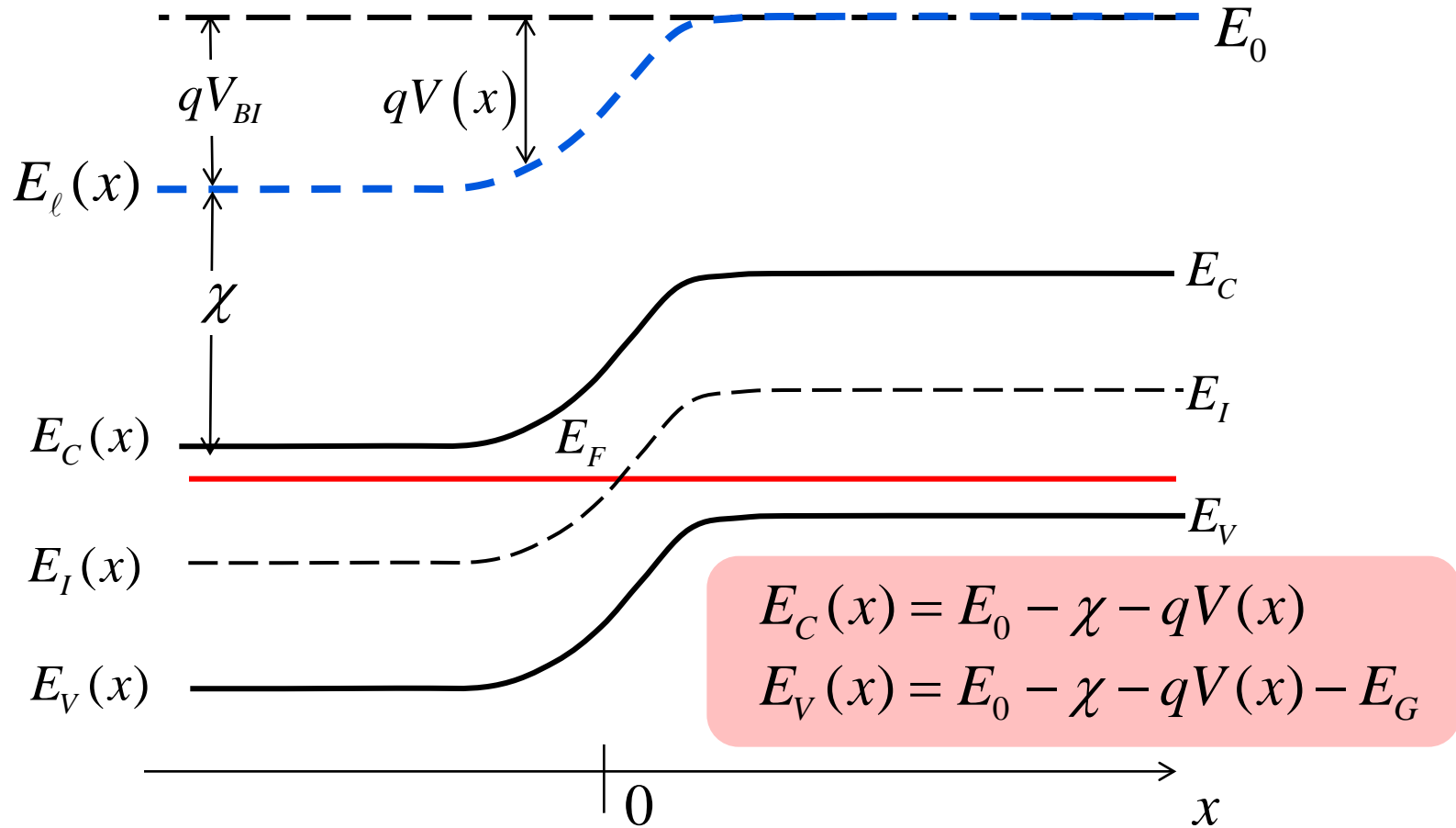
$$E_C = E_0 - \chi$$

$$E_V = E_0 - \chi - E_G$$

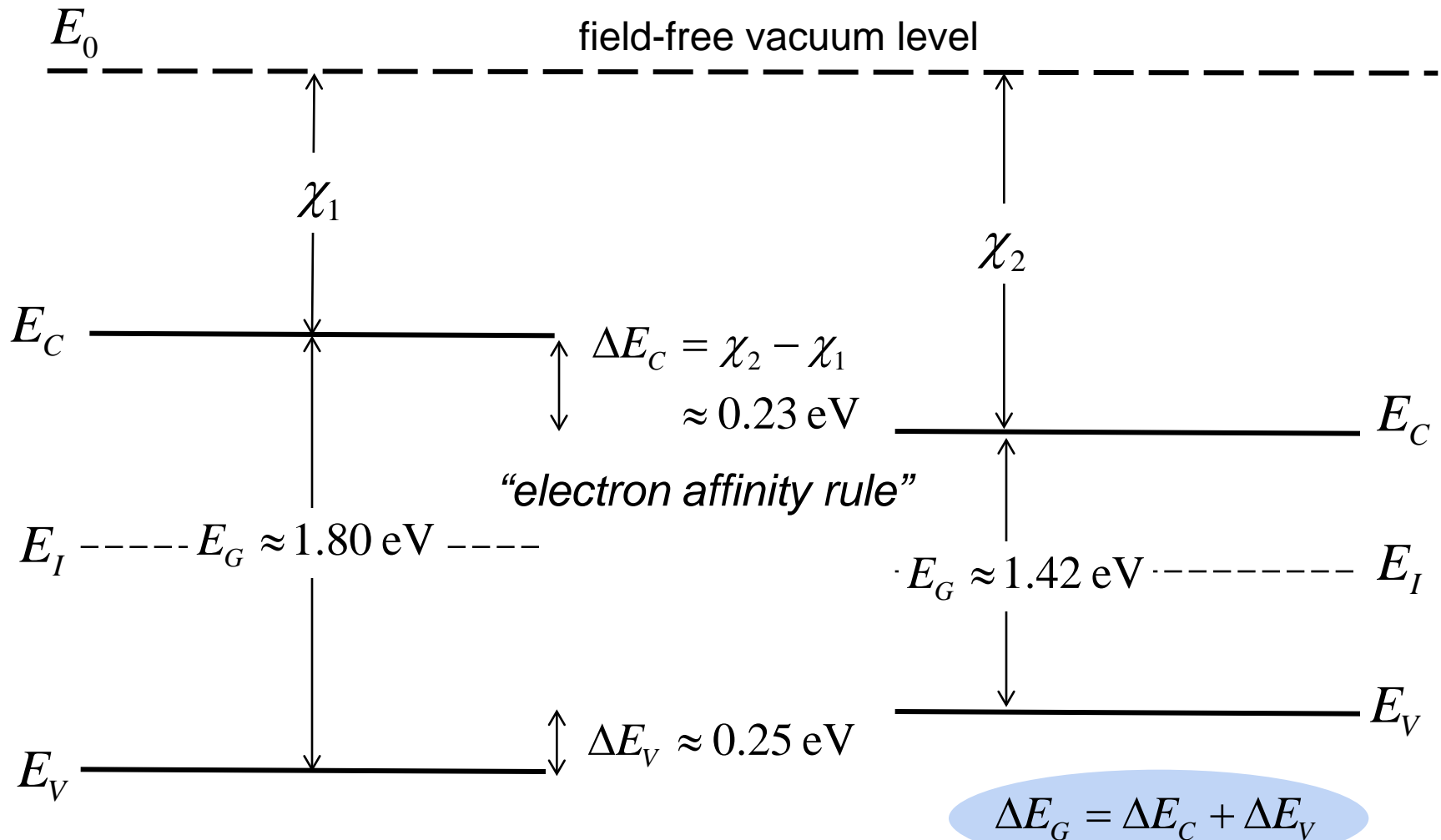
$$qV_{BI} = (\Phi_P - \Phi_N)$$

Lundstrom ECE-656 F11

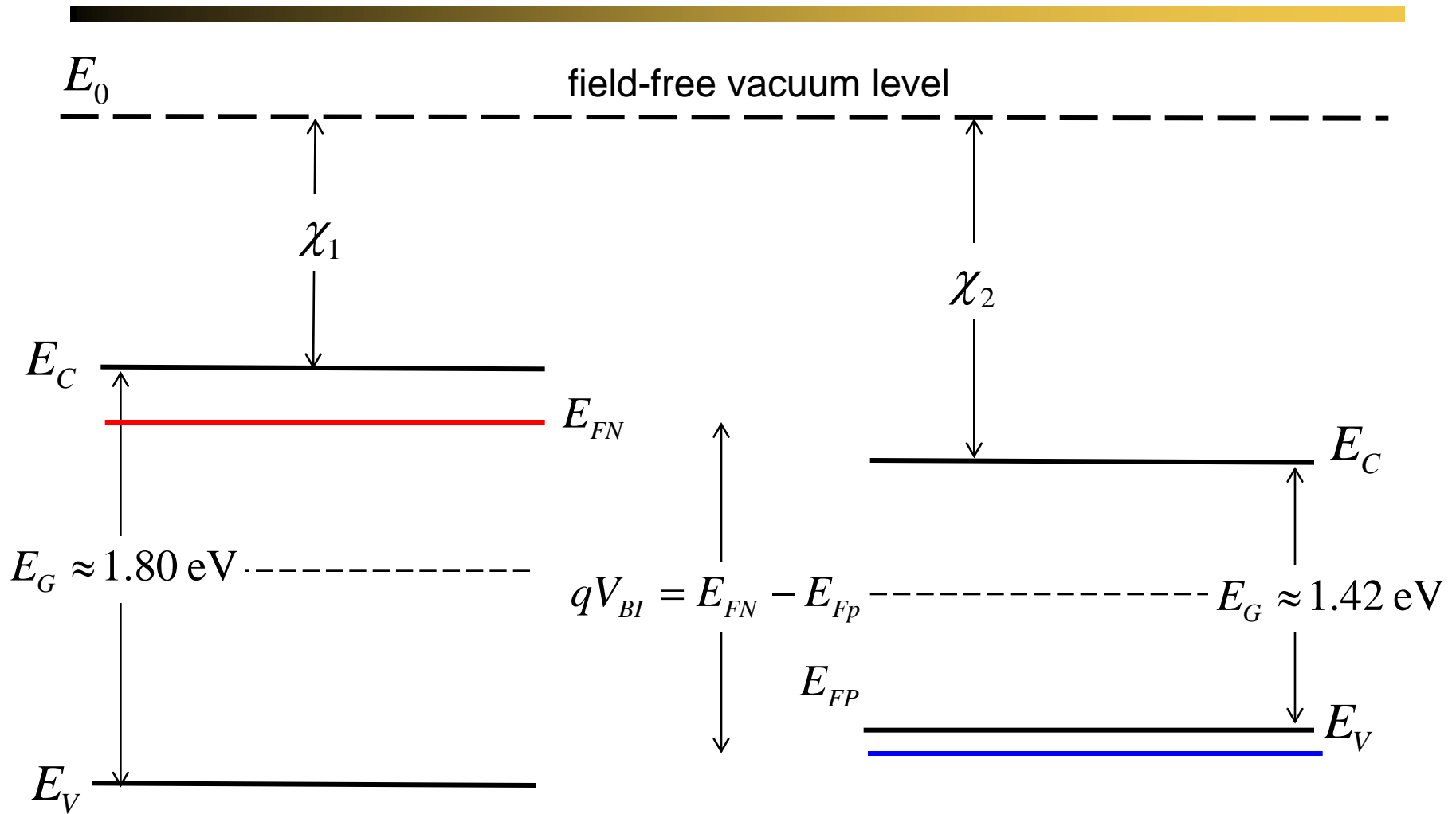
# local vacuum level



# Al<sub>0.3</sub>Ga<sub>0.7</sub>As : GaAs (Type I HJ)

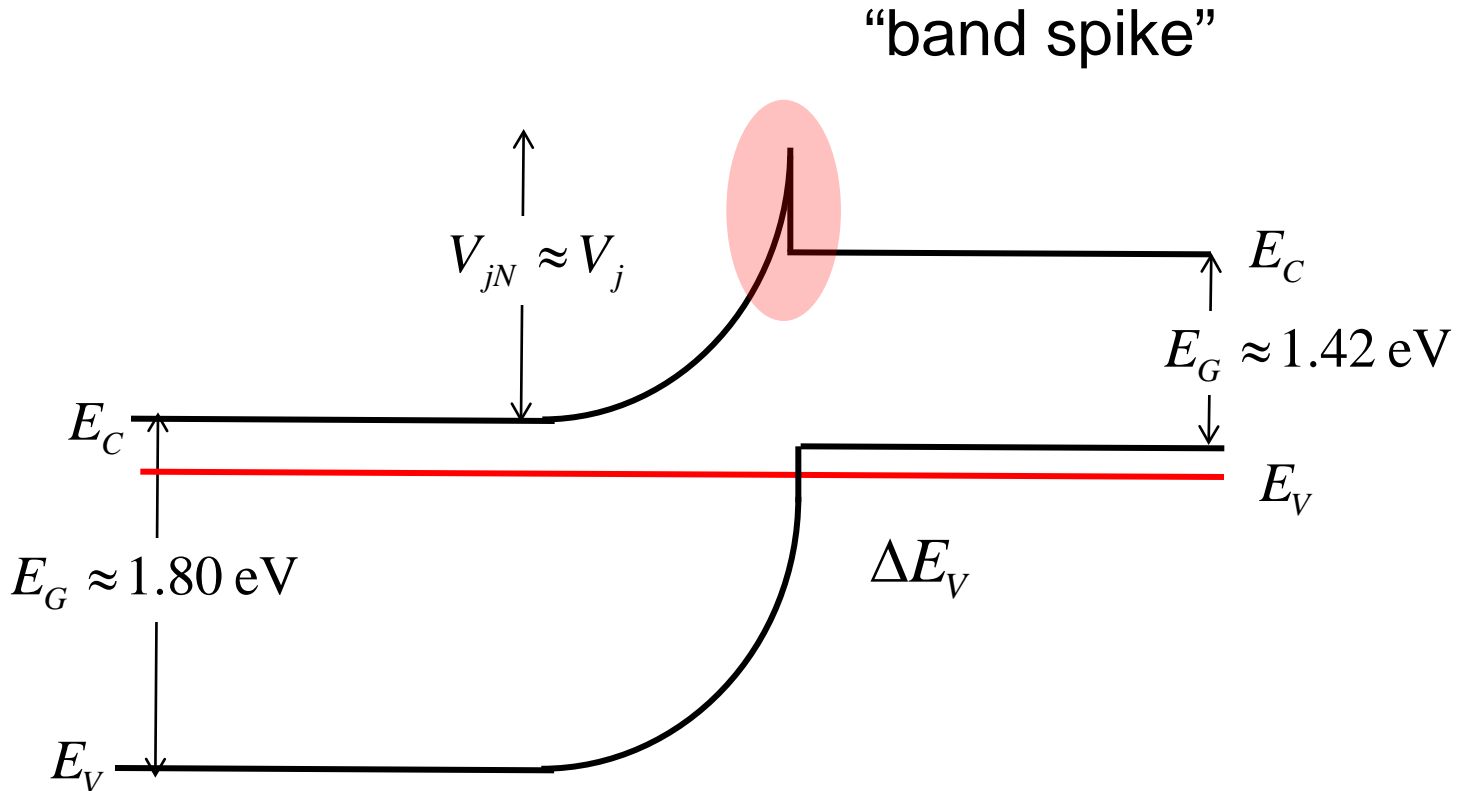


# N-Al<sub>0.3</sub>Ga<sub>0.7</sub>As : p<sup>+</sup>-GaAs (Type I HJ)

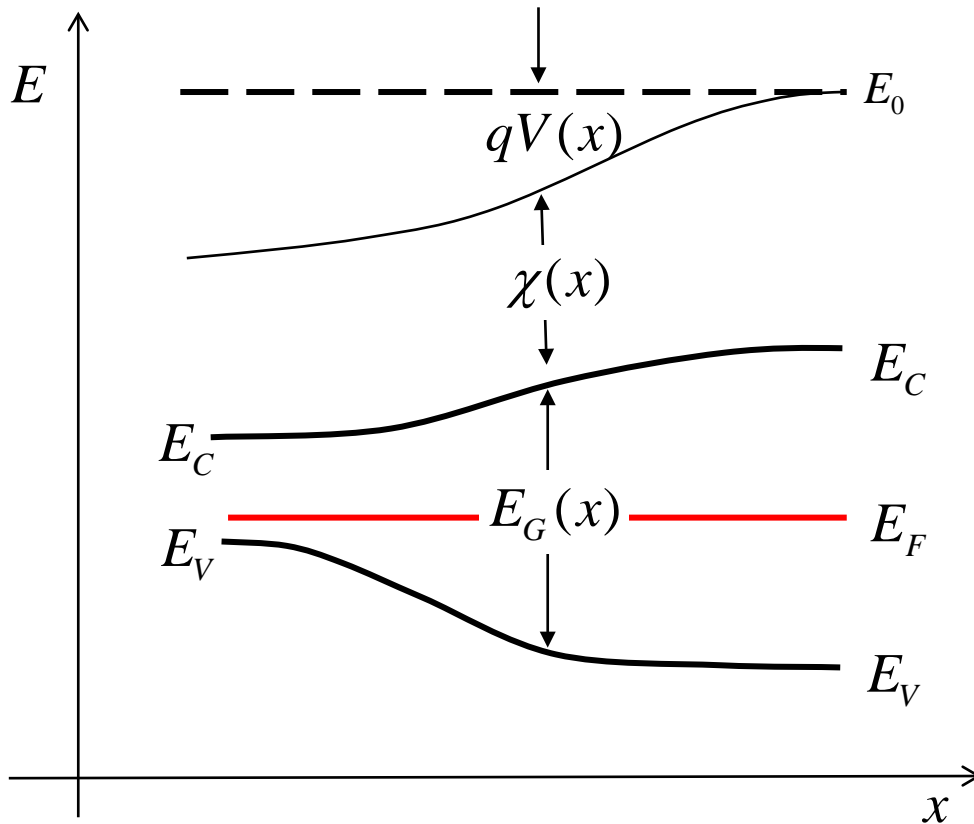




# N-Al<sub>0.3</sub>Ga<sub>0.7</sub>As : p<sup>+</sup>-GaAs (Type I HJ)



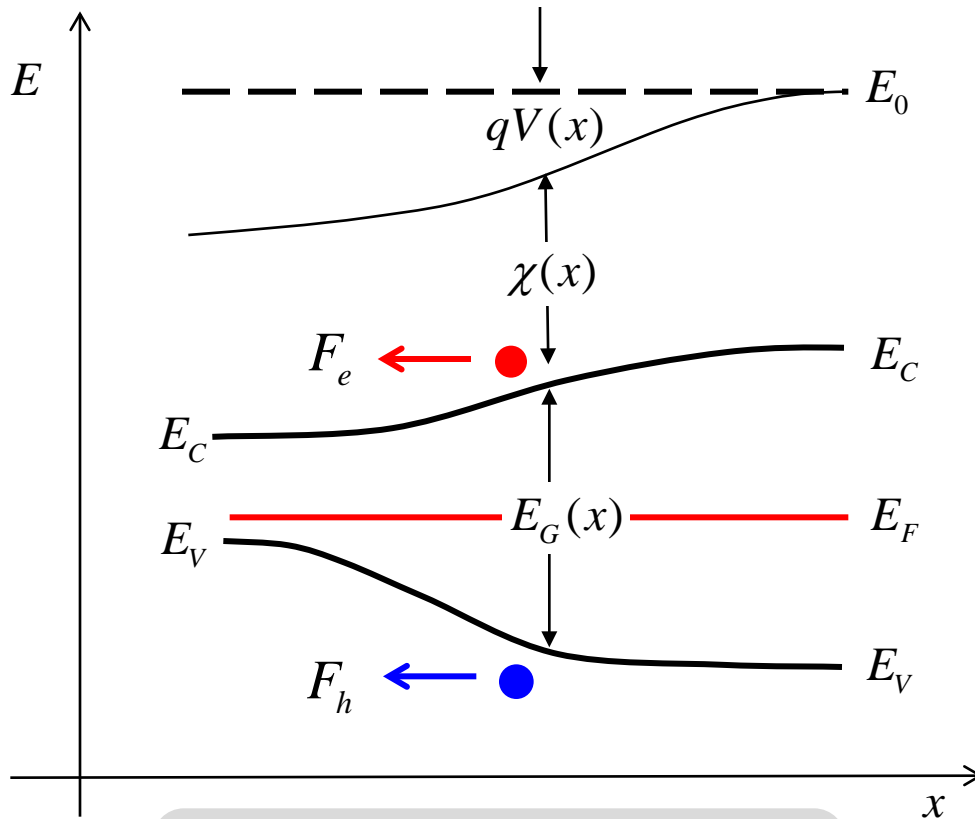
# general, graded heterostructure



$$E_C(x) = E_0 - \chi(x) - qV(x)$$

$$E_V(x) = E_C(x) - E_G(x)$$

# “quasi-electric fields”



$$E_C(x) = E_0 - \chi(x) - qV(x)$$

$$E_V(x) = E_C(x) - E_G(x)$$

$$F_e = -\frac{dE_C}{dx} = q \frac{dV}{dx} + \frac{d\chi}{dx}$$

$$F_e = -q\mathcal{E}(x) - q\mathcal{E}_{QN}(x)$$

$$\mathcal{E}_{QN} \equiv -\frac{1}{q} \frac{d\chi}{dx}$$

$$F_h = +\frac{dE_V}{dx} = -q \frac{dV}{dx} - \frac{d(\chi + E_G)}{dx}$$

$$F_h = +q\mathcal{E}(x) + q\mathcal{E}_{QP}(x)$$

$$\mathcal{E}_{QP} \equiv -\frac{1}{q} \frac{d(\chi + E_G)}{dx}$$

# BTE

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$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla_r f + \frac{d\vec{p}}{dt} \cdot \nabla_p f = 0$$

$$\frac{d\vec{p}}{dt} = \frac{d(\hbar\vec{k})}{dt} = -\nabla_r E_C(\vec{r}) = -q\vec{\mathcal{E}}(\vec{r})$$

(constant effective mass)

These equations do not hold when the effective mass is position dependent. Lundstrom, FCT, Sec. 5.8.

# alternative approach: hole current

$$J_p = p\mu_p \frac{dF_p}{dx} \quad p = N_V(x) e^{(E_V - F_p)/k_B T_L} \quad F_p = E_V(x) - k_B T_L \ln(p/N_V)$$

$$\frac{dF_p}{dx} = \frac{dE_V(x)}{dx} - k_B T_L \left[ \frac{1}{p} \frac{dp}{dx} - \frac{1}{N_V} \frac{dN_V}{dx} \right]$$

$$J_p = p\mu_p \left[ \frac{dE_V(x)}{dx} + \frac{k_B T_L}{N_V} \frac{dN_V}{dx} \right] - k_B T_L \mu_p \frac{dp}{dx}$$

$$\frac{dE_V(x)}{dx} = \frac{d}{dx} [E_0 - \chi(x) - qV(x) - E_G(x)] = q(\mathcal{E}(x) + \mathcal{E}_{QP})$$

# hole and electron currents

$$J_p = pq\mu_p \left[ \mathcal{E} + \mathcal{E}_{QP} + \frac{k_B T_L}{q} \frac{1}{N_V} \frac{dN_V}{dx} \right] - qD_p \frac{dp}{dx}$$

*“DOS effect”*

$$J_p = nq\mu_n \left[ \mathcal{E} + \mathcal{E}_{QN} - \frac{k_B T_L}{q} \frac{1}{N_C} \frac{dN_C}{dx} \right] + qD_n \frac{dn}{dx}$$

***quasi-electric fields***

$$\mathcal{E}_{QP} \equiv -\frac{1}{q} \frac{d(\chi + E_G)}{dx} \quad \mathcal{E}_{QN} \equiv -\frac{1}{q} \frac{d\chi}{dx}$$

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(Reference: Chapter 5, Lundstrom, FCT)



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# balance equation summary

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- 1) The four balance equations can be reduced to two continuity equations and two constitutive relations.
- 2) We can write them as two equations in two unknowns.
- 3) The unknowns are  $n$  and  $W$  or  $n$  and  $T_e$ .



# the simplified equations

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$$\frac{\partial n}{\partial t} = \frac{1}{q} \nabla \cdot \vec{J}_n$$

cont. eqn. for electrons

$$\vec{J}_n = nq\mu_n \vec{\mathcal{E}} + \frac{2}{3} \mu_n \nabla W$$

current equation

$$\frac{\partial W}{\partial t} = -\nabla \cdot \vec{F}_W + \vec{J}_n \cdot \vec{\mathcal{E}} - \frac{(W - W_0)}{\langle \tau_E \rangle}$$

continuity eqn. for energy

$$\vec{F}_W = ?$$

current eqn. for energy

# energy current equation

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First approach:

$$\vec{F}_W = -\frac{2}{3} \mu_E W \vec{\mathcal{E}} - \langle \tau_{F_W} \rangle \nabla \cdot \vec{X} \quad X_{ij} \approx \frac{5}{3} \frac{k_B T_e}{q} \mu_E W \delta_{ij}$$

Second approach:

$$\vec{F}_W = W \vec{v}_d + n k_B T_e \vec{v}_d + \vec{Q} \quad \vec{Q} \approx -\kappa_e \nabla T_e$$

# questions?

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