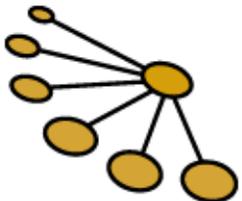# Quantum Transport:
ATOM TO TRANSISTOR

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Lecture 5: Summary/ Towards Ohm's Law
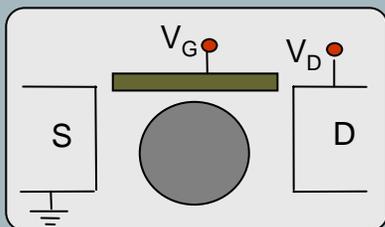Ref. Chapter 1 with emphasis on 1.4 & 1.5

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- We've been discussing the current flow through a very small device in the last few lectures.
- For this, first thing you need to do is to draw a set if energy levels and locate the equilibrium electrochemical potential.

• Current flows because of difference of agenda between the two Fermi levels in the source and drain. The cause of this difference is the applied drain voltage. Electrons get injected into the source because the energy level in the channel is lower than the one in the source. By the same token electrons go from channel to drain. While source keeps filling the channel, drain keeps emptying it; the end result is an electron flow from source to drain.

• This simple physical picture gives us an expression for the current and the number of electrons in the channel (assuming one level between µ1 and µ2): $I = -\dfrac{q}{\hbar}\dfrac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}(f_1 - f_2)$

$N = \dfrac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2}$

• We then talked about the need to include broadening which happens as a consequence of strong coupling. This sets a limit on the current. Why? Because of broadening some part of the distributed state gets out of the energy range between µ1 and µ2 which will *not* contribute to current flow. The stronger the coupling, the more spread the level will become. The stronger the voltage. The more spread the level becomes. The non-trivial maximum conductance is: $G_0 = q^2/h = 38.7\,\mu S$

• So what we had at the end of third session is like:

$$N = \int dED_\varepsilon(E)\frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2}$$

$$I = -\frac{q}{\hbar}\int dED_\varepsilon(E)\frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2}(f_1 - f_2)$$

$$D_\varepsilon(E) = \frac{\gamma/2\pi}{(E-\varepsilon)^2 + \left(\gamma/2\right)^2}$$

• In lecture 4 we saw that this gives us a reasonable current voltage characteristics only for low biases. If the bias is large, another physical point has to be taken into account. And that is the shift of the levels as a result of applied voltage.

• If the number of electrons in the channel changes, other electrons in the channel feel a different potential. In return the change of potential will change the number of electrons. This fact tells us that potential of the channel has be solved self consistently.

$$U \rightarrow N$$

$$N \rightarrow U$$
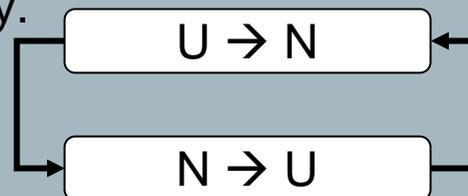
• Potential increase lowers the levels and potential decrease raises them. This can be incorporated in our model as:

$$N = \int dED_\varepsilon(E-U)\frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2}$$

$$I = -\frac{q}{\hbar}\int dED_\varepsilon(E-U)\frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2}(f_1 - f_2)$$

<u>To calculate the potential in the channel</u>

1) Calculate the potential in the channel for the case when there is no change in the number of electrons using the Laplace equation: $\vec{\nabla} \cdot \left(\varepsilon_r \vec{\nabla} V \right) = 0$ Call this potential U_L. For the case when there is a change in the number of electrons, poisson equation has to be used:

$\vec{\nabla} \cdot \left(\varepsilon_r \vec{\nabla} V \right) = -\Delta \rho / \varepsilon_0$ This adds an additional term to the Laplace potential proportional to the change in the number of electrons. We then have:

$$U = U_L + \frac{q^2}{C_E} \Delta N$$

Where the proportionality constant is called the single electron charging energy and tells us the increase in the potential energy due to addition of one electron to the channel.

• It plays an important role when it's bigger than both gamma and kT

$$q^2 / C_E >> \gamma \text{ AND } k_B T$$

• If this happens then we enter a new regime of transport which is coulomb blockade regime.

• <u>General Procedure</u>

$$N = \int dE D_\varepsilon (E - U) \frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2}$$

$$I = -\frac{q}{\hbar} \int dE D_\varepsilon (E - U) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} (f_1 - f_2)$$

$$U = U_L + \frac{q^2}{C_E} \Delta N$$

• But how can we apply this method to a bigger device with lots of levels instead of having one instead? Answer: Each of levels gets broadened and you'll have an overall density of states which is the sum of all broadened states.
• Using what we've learned we can explain why the gate is important in a transistor:
• What often determines the performance of a transistor, is how good the gate is. Why? We'll use an n-type device as an example to explain this:

• To turn the transistor off: Put a negative voltage on the gate; this will raise energy levels and   effectively goes down in the band gap. It enters the region where there are no states and so it doesn't conduct and the transistor is off.
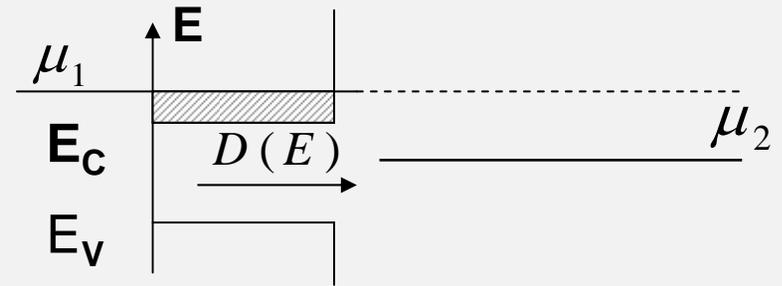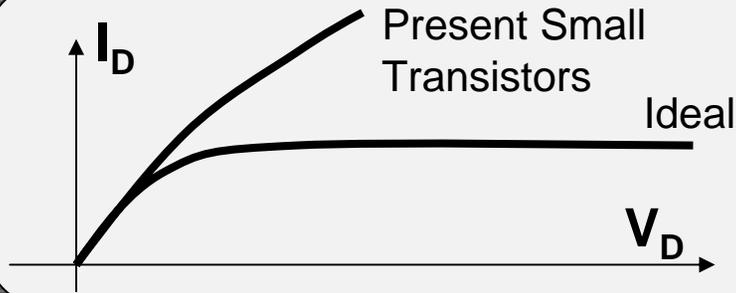
E

μ_1

E_c

$D(E)$

E_v
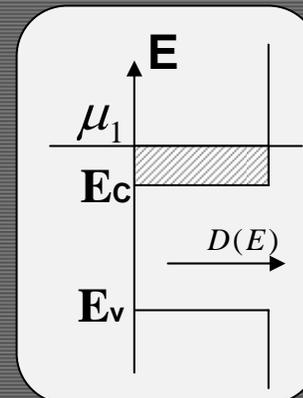
• Positive gate voltage will raise μ_1 hence more states around it and it will turn the transistor on more.

• Now what does a current voltage characteristic for a good transistor look like and what does gate have to do anything with it? This is explained on the next page.

• A good transistor is one whose current saturates so that its on-current is constant. And the value of this on-current or saturation current is determined by the gate voltage.

• Now what makes the current to saturate? To understand this consider the situation where a voltage (+ve for n-type device) is applied to the drain. This lowers the chemical potential in the drain. Our energy diagram looks like…

• By looking at this, it is a kind of obvious why the current saturates. Once µ2 drops below conduction band edge, increasing the voltage will lower it more. However the number of states between µ1 and µ2 is fixed and this is what determines the amount of current flow; since it remains unchanged the current saturates.

• But why this saturation doesn't happen for the small devices? This is answered on the next page.

• Why doesn't current saturate in a small device?

• Answer: Suppose you apply 1V to the drain. What happens in a small device is that the levels in the channel will not be held at a constant potential with respect to the source; rather they will be lowered to some extent; say half a volt. This will increase the number of states between $\mu 1$ and $\mu 2$, which in turn results in a higher conduction and current. Hence the current will not saturate.

• A good transistor is one whose channel potential remains fixed when a voltage is applied to the drain contact. How do we do this? We keep the gate really close to the channel such that it is the gate voltage which determines the channel potential not the drain's.

- Another question one could ask is about the conductance of the device:
- What is the relationship between the conductance of the device and its width and length?
- Ohm's law tells us that as the device gets longer the conductance should decrease and as it gets wider, the conductance should increase:

$$G \propto \frac{W}{L}$$

- What if we use the theory we've learned? Will the conductance change in the same way as described by the ohm's law?
- What we'll show is that the proportionality to W is obtainable but the conductance's relation to the length L needs more work.
- Let's explain all these…

- We know that more states will give us more current.
- It is also reasonable to think that larger devices have more states. For instance if you have a 2 dimensional solid, then you'd expect that D would increase as the area increases: $D \propto WL$ Or in the case of a 3-D solid we'd have: $D \propto AL$
- Wait a minute. We've just obtained something that contradicts Ohm's law. So what's wrong?
- We can easily explain why the L goes away but for the inverse proportionality of conductance and length we cannot yet describe it with the theory we have learned thus far.
- So how does the L go away?
- This is explained on the next page…

- As you make the device bigger and bigger, what happens is that γ decreases as 1/L. Why?
- Answer: Gamma represents the escape rate of the electron in the channel. The point is that the bigger the box, the lower is the chance of electron hitting the wall and getting out of the channel.
- Looking back at the equation or the current we have:

$$I = -\frac{q}{\hbar} \int dE \, D_\varepsilon (E - U) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} (f_1 - f_2)$$

$$D \propto WL \quad \text{and} \quad \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \propto 1/L$$

- When multiplied, L cancels out.
- So far our theory gives us a conductance that is independent of length L which is true for a ballistic device. Ballistic device is one for which electron transport does not have to confront scattering which is what makes electron flow more difficult and causes resistance. So the resistance that is observed is actually the contact resistance. It does not matter how long the channel is.
- Next we try to discuss the Coulomb Blockade regime. (CB regime)

$$U = U_L + \frac{q^2}{C_E} \Delta N; \qquad U_0 \equiv \frac{q^2}{C_E}$$

• As mentioned before U_0 is called the single electron charging energy and how big it is depends on how small the capacitance is.

• For instance for a capacitance of 10^{-15} F, it will be about 10^{-4} eV.

$$U_0 = \frac{16 \times 10^{-19}}{10^{-15}} \approx 10^{-4} \text{eV}$$

• Consider our conditions for the CB regime:

$$U_0 >> \gamma, k_B T$$

• At room temperature kT ~ 26meV. So the 0.1meV value derived above is not nig enough to do anything.

• If the capacitor gets even smaller, then we have to worry about it.

• Let's look at an example of what could happen if U_0 is large…

μ ——— μ

• As explained before when you have one level in the channel and then couple the channel to a contact, the level broadens out. Now in the case illustrated above, there is a high distribution of states around chemical potential and so the device should conduct well.

• But there is an important point missing here: Levels come in pairs. We have two degenerate levels: up-spin and down-spin

μ ——— μ
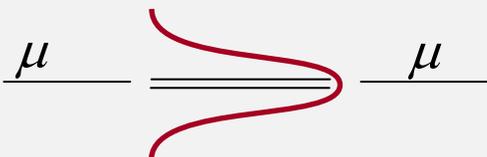
• Plain electrostatics will give the previous picture. Now under those conditions if we apply a bias, we'll have levels around chemical potential and the channel should conduct well. How ever something else happen here:

$\downarrow$ or $\uparrow$

μ ——— $\uparrow$ or $\downarrow$ U_0 ——— μ
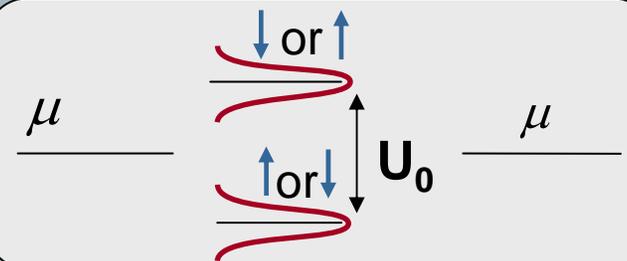
• When a level gets filled, it does not feel any potential due to itself. So it remains where it was. How ever the other level feels the potential and floats up. Depending on which level gets filled the picture looks one of the two ways illustrated above.

• So in general we have this entire different regime of transport under the situation:
$$U_0 >> \gamma, k_B T$$

• Over all notice that what allows us to treat this problem in a simple matter is that an electron in the channel feels come average potential due to its surroundings which is the U appearing in the equations discussed earlier. This method of doing things is called mean field theory and it's the bases of our understanding of current flow and all electronic properties.

• The real big multi-electron problem is virtually insolvable. There is just so much involved and cannot be done exactly. Approximation is always used. The way you validate things is to compare with experiment. Any real calculation has some points that are ignored. The better match and accordance between theory and experiment, the better the theory.

• From next lecture, we'll stop talking about current flow temporarily. Instead we'll spend several weeks discussing energy levels in a given device. These energy levels are the eigenvalues of the Hamiltonian in the Schrödinger equation. We'll start next lecture by describing what Schrödinger equation is.