Quantum Transport:

Atom to Transistor

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Lecture 39: “Physics” of Ohm’s Law

Ref. Chapter 11.2

Network for Computational Nanotechnology

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Conductance

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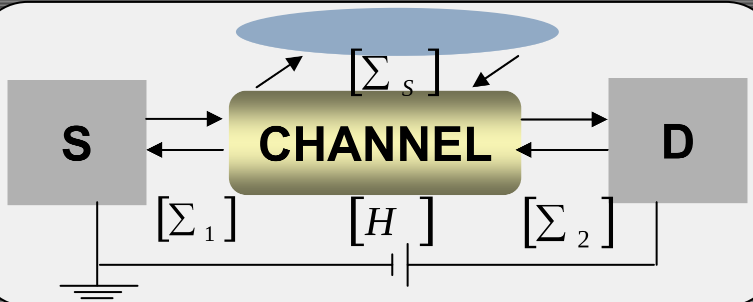

• What we've discussed thus far in this class was current flow through a small device using the Hamiltonian and the self energy matrices. For large devices we also had to take into account the scattering processes.

• Today we want to see how one eventually gets Ohm's law as the device gets bigger and bigger.

• One of our early results was that:

$$I \propto \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} D$$

• Naturally then you would think that conductance will depend on this product.

• When you think of a conductor that has length L and area S, Ohm's law predicts that conductance is proportional to S/L.

• For our expression of current, we know that DOS will increase as the volume is increased. This is the same as saying D is proportional to S*L=V.

• On the other hand, you may think since the Gamma represents the escape rate at the two contacts, it shouldn't depend on size. How ever for longer devices, electron wave function is more spread over the channel; hence escaping into contacts becomes less easy. So Gamma should be proportional to 1/L; hence current would become proportional to **S**.

Dependence of Conductance on Scattering

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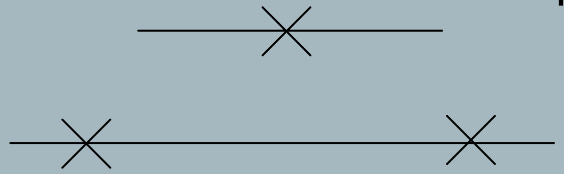
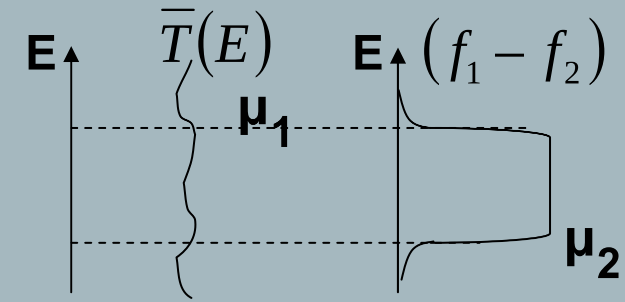
- What $I \propto \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} D$ predicts is not what Ohm's law states. This is due to the fact that we've considered the device to be ballistic i.e. there is no scattering inside the device.
- How ever, our expectation is that if we make the device long enough, and include all factors, then Ohm's law should follow.
- To investigate this, we consider two conductors; one with 1 scatterer and the other with 2 . Then we ask the question that how the two conductors compare.



- For current we have:

$$I = \frac{2q}{h} \int dE \overline{T}(E)(f_1 - f_2)$$



Landuaer Formula

$$I \sim \overbrace{\frac{2q}{h} T(\mu) \times qV}^{G}$$

Example: Two Scatterers

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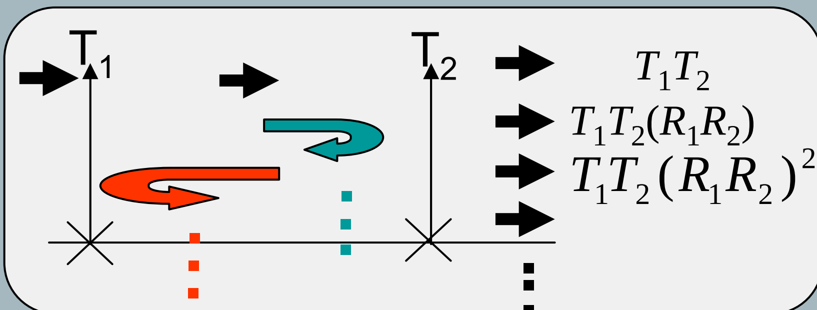
• Thinking of electrons as particles, for transmission through 2 scatterers we have:



$$T = T_1T_2\left(1 + x + x^2 + \ldots\right);\quad x = R_1R_2$$

• This is the Geometric series:

$$T = \frac{T_1T_2}{1 - R_1R_2};\qquad \begin{aligned} T_1 + R_1 &= 1 \\ T_2 + R_2 &= 1 \end{aligned}$$

$$\frac{1}{T} = \frac{1-(1-T_1)(1-T_2)}{T_1T_2} = \frac{T_1 + T_2 - T_1T_2}{T_1T_2}$$

$$\Rightarrow \frac{1}{T} = \frac{1}{T_1} + \frac{1}{T_2} - 1$$

• Now the question is how transmission is related to the length of a long conductor.

$$T(L) = ?$$

• Our function must satisfy the above relationship. If you separate your conductor into two sections of equal length we should have:

$$\frac{1}{T(2L)} = \frac{1}{T(L)} + \frac{1}{T(L)} - 1$$

• This equation will satisfy the relationship $\boxed{T(L) = \dfrac{\lambda}{L + \lambda}}$

$$\frac{2L + \lambda}{\lambda} = \frac{L + \lambda}{\lambda} + \frac{L + \lambda}{\lambda} - 1$$

Mean Free Path

- λ is called the mean free path and its definition is:

A conductor that has a length of λ, has a transmission of ½.

- But does the relation we have for Transmission lead to Ohm's law?
- For conductance we have:

$$G = \frac{2q^2}{h}\frac{\lambda}{L+\lambda}$$

- This looks like Ohm's law except for the term **λ** in the denominator. Ohm's law predicts infinite conductance when L=0 but this tells us that there is maximum conductance due to the contact resistance. Of course when L gets large (>100 **λ)**, this looks more like Ohm's law.

- For resistance we have:

$$R = \frac{h}{2q^2}\left(1 + \frac{L}{\lambda}\right)$$

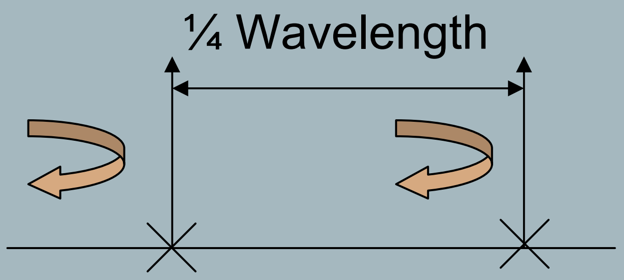
- Again as L tends to 0 , resistance does not become 0 but it will approach a constant value due to contact resistance. For large L we get Ohm's law.
- For a 2D conductor, we use the concept of sub-bands where increasing the number of modes will increase the conductance and reduce the resistance.

$$R = \frac{h}{2q^2}\frac{1}{M}\left(1 + \frac{L}{\lambda}\right); G = \frac{2q^2}{h}M\frac{\lambda}{L+\lambda}$$

- The number of modes will roughly increase as the area. So we actually are getting the Ohm's law.

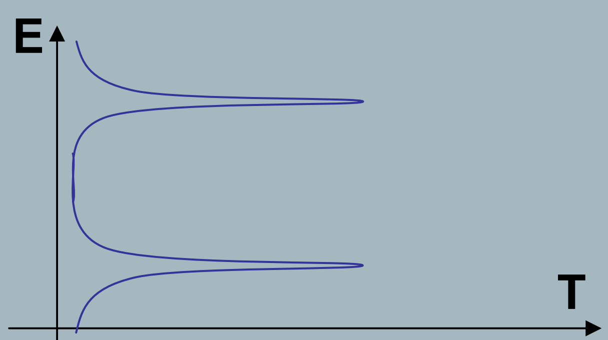
Electrons as Waves

• This picture is not as easy if we think of electrons as waves and our relationship will not compensate for all the physics that is inherent in the problem.



• Consider the case were we have two scatterers; from particle point of view and using the relationship that we've derived for transmission, transmission should reduce by a factor of 2 when we go from one to two scatterers. How ever this is not the case.

• Looking at the problem Quantum mechanically, it is interference between the waves that changes the picture. For example if the distance between the two scatterers is quarter of a wavelength, the two reflections cancel each other and this way we can even get more transmission
(Constructive interference) for some certain energies relative to one scatterer.



• So what do we get if we calculate the transmission Quantum Mechanically?

Weak Localization

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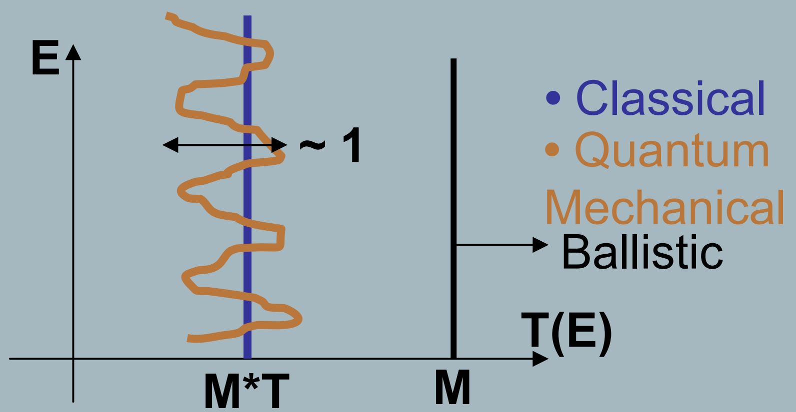
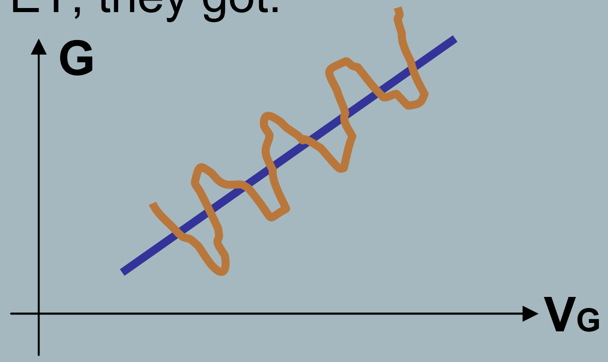
- Let's say we'd calculate the transmission through a conductor with 50 modes. If the conductor was ballistic, transmission would be the same as number of modes (here it is 50).



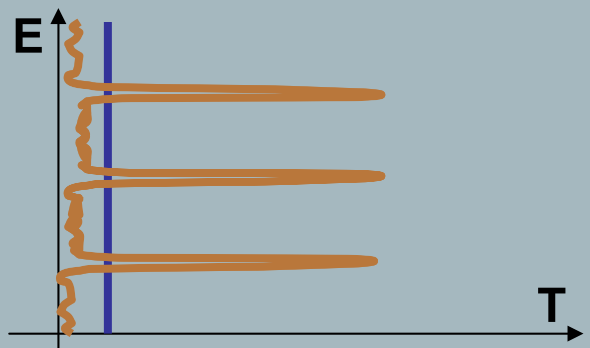
- Whereas for a classical approach for a device that has scattering we get a constant transmission value as a function of energy, Quantum mechanical approach gives us fluctuations with height of order 1.

- The conduction then would fluctuate by order of $2q^2/h$. This is what people have observed experimentally.
- Measuring the conductance as a function of gate voltage for an FET, they got:



- Note that the quantum mechanical calculation gives a little lower average than the classical transmission because of what's called weak localization.

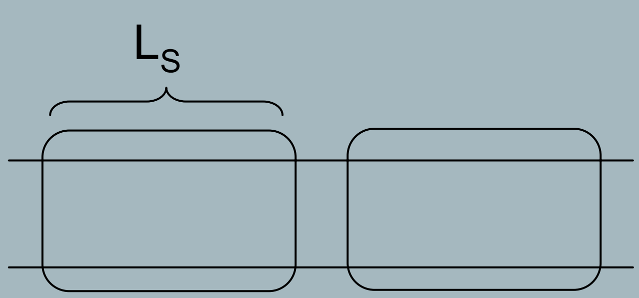
Regime Of Strong Localization



• Now what happens if the transmission per mode was, say, 1/100 having 50 modes? We'd have a transmission of 0.5 as the product of MT. Then a fluctuation of order one would result in a negative transmission which is conceptually impossible. So what happens?

• People believe that Quantum mechanical transmission would be 0 except that at some certain energies it would have sharp peaks. This is what's called **regime of strong localization**.

• This theory predicts that if you make any wire long enough you should get to this regime but this is not what happens in real world. This is due to treating the problem quantum mechanically without any phase breaking. And this is something that does NOT happen in the real world. Here is how to explain it:



• In a wire there are regions with length of order L_S (Scattering length), which function like a quantum mechanical entity. But these regions do not have phase coherence with each other; hence cannot have destructive interference.

Power Dissipation: Where does the heat go?

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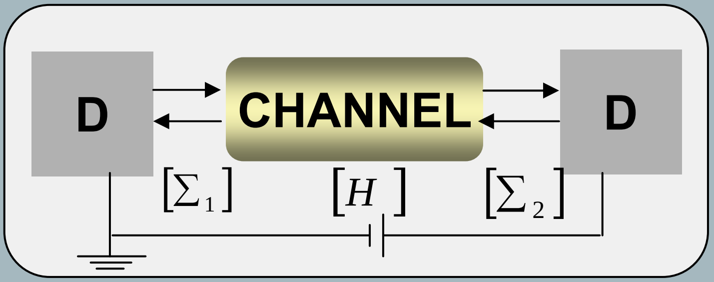
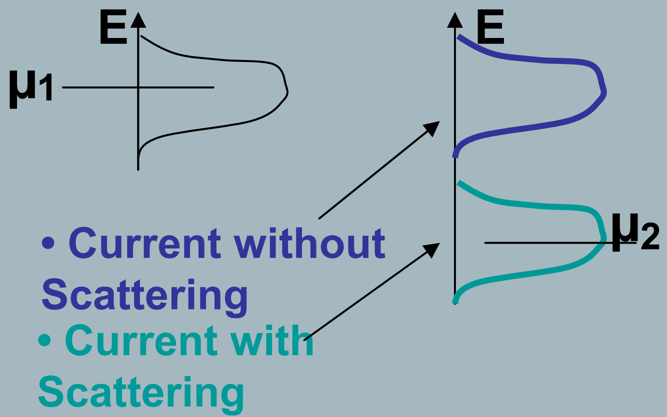


Power Dissipation

- When we think of resistance we mentally associate a certain amount of energy loss to it which we can describe as I^2R.
- For a small ballistic device we have a minimum resistance of $h/2q^2$.
- One might think that this is associated with the channel; however there are no scatterers and there are no electron phonon interactions. For resistance to occur electron must loose its energy to something. So what happens to I^2R?

Consider the current flow as a function of energy:



- If the energy currents on the left and the right contacts were the same, that would mean no energy was left behind. How ever with scattering process, the current at right contact flows at a lower energy. So the energy currents are not equal. For our model without scattering, the two energy currents are equal.

Experiments on CNT's

Observations From Experiment and Interpreting the Results

- In some experiments, the amount of current flow in carbon nanotubes results in big enough energy dissipation that should burn the device if the dissipation was happening in the channel. But this does not happen i.e. the device does not burn for high currents.

- It is believed that the heat is dissipated in the contacts. However, how much heat is dissipated in each contact remains to be established.