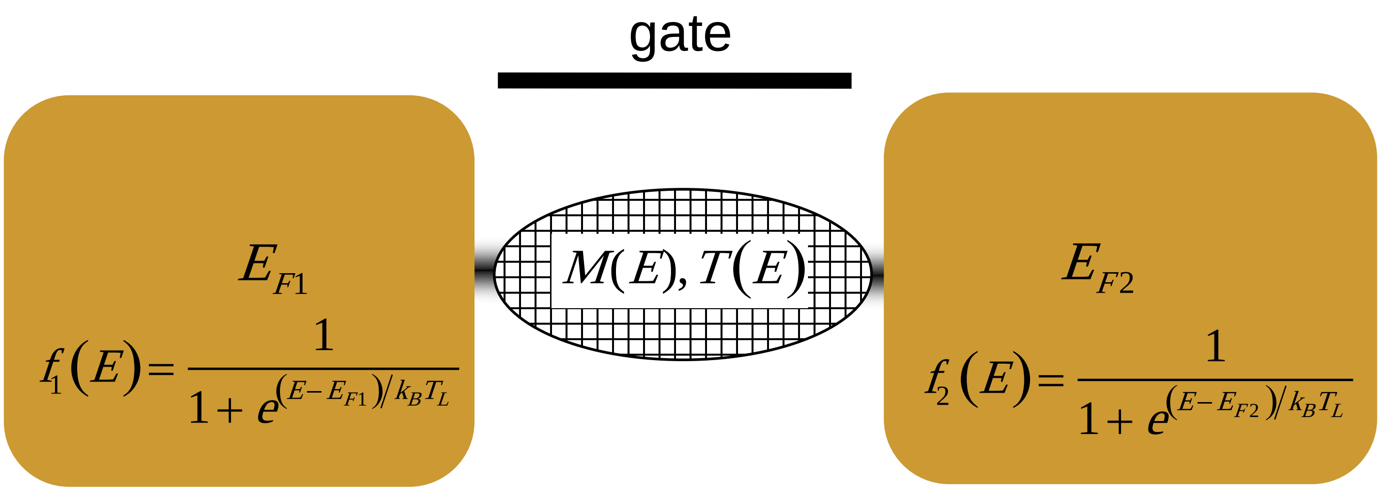
Lecture 6: Ballistic Model

Mark Lundstrom

Electrical and Computer Engineering
Network for Computational Nanotechnology
and
Birck Nanotechnology Center
Purdue University
West Lafayette, Indiana USA

Landauer approach to transport

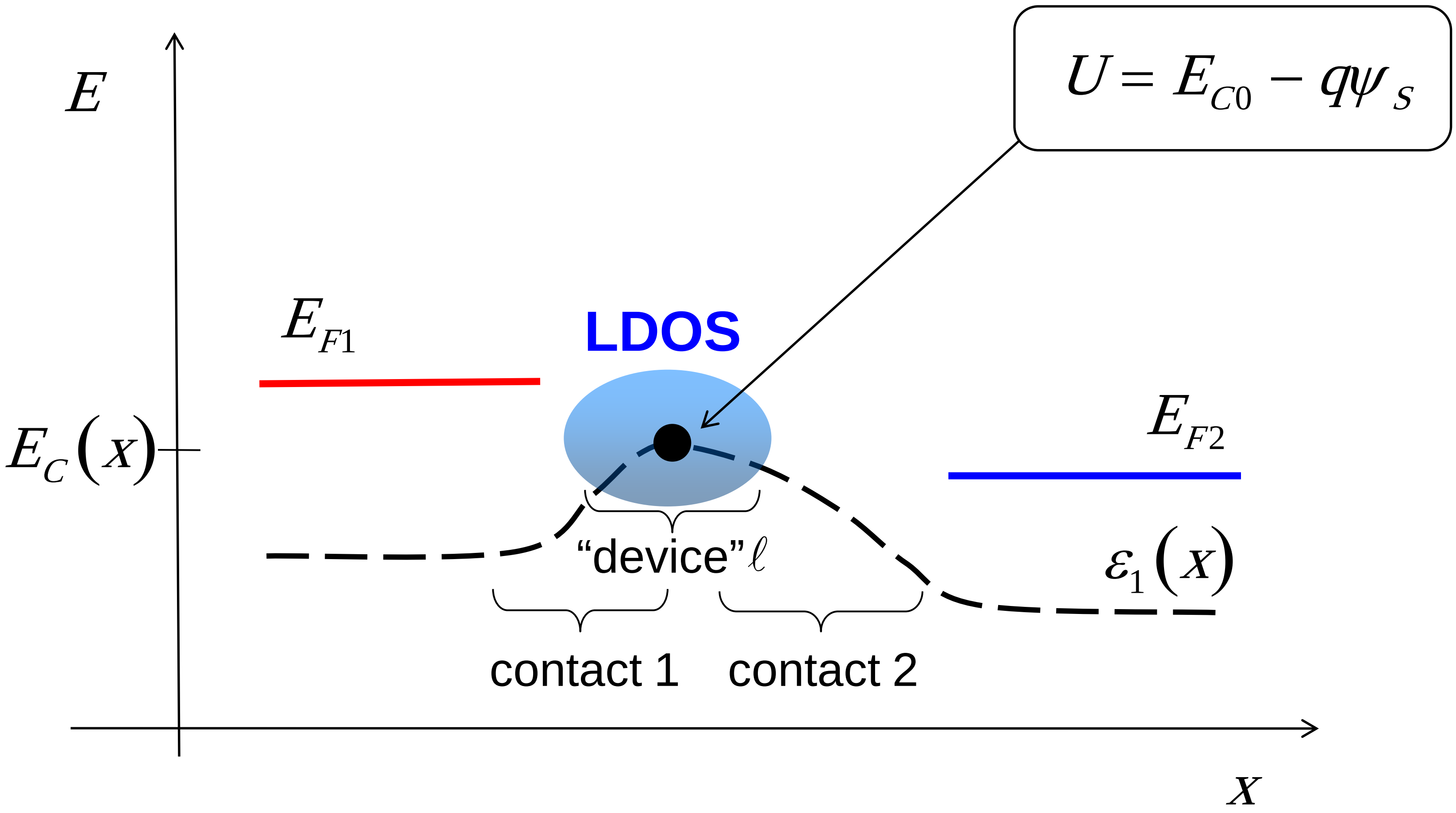


$$I = \frac{2q}{h} \int T(E) M(E) \left(f_1 - f_2 \right) dE$$ any drain bias

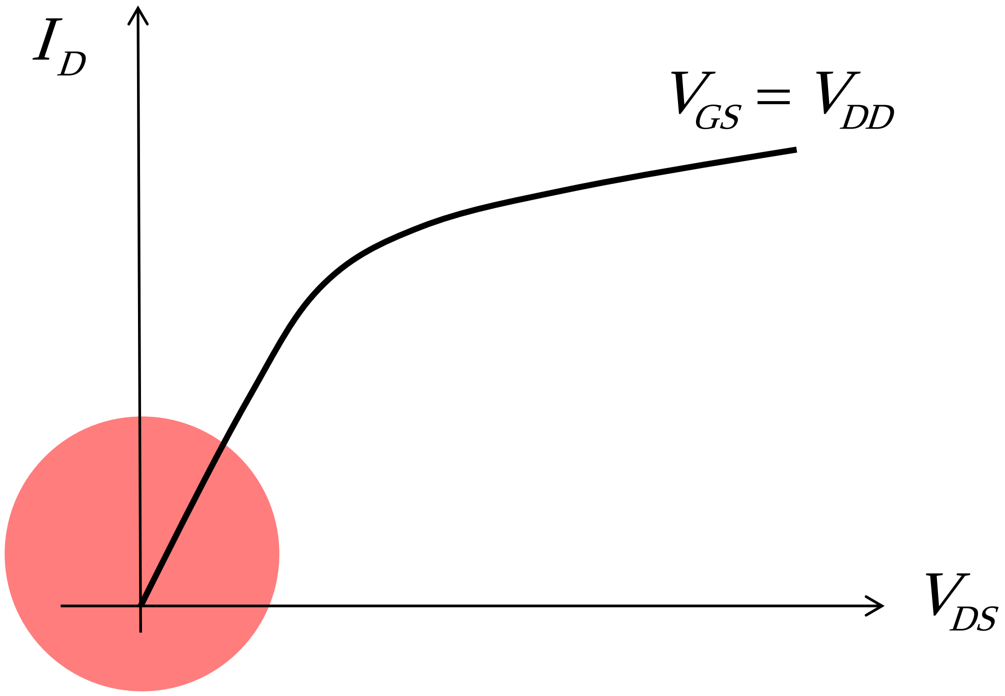
$$I = GV$$

$$G = \frac{2q^2}{h} \int T(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$ low drain bias

“top of the barrier model”



ballistic MOSFET: linear region

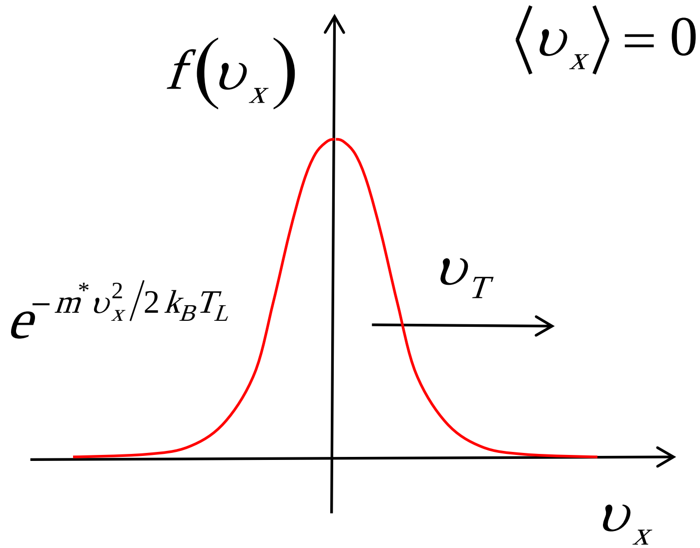


near-equilibrium $f_1 \approx f_2$

$$I_D = G_{CH} V_{DS}$$

linear region with MB statistics

$$G_{CH} = \frac{2q^2}{h} \int_{E_C}^{\infty} T(E) M_{2D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$



$$\upsilon_T = \sqrt{2 k_B T_L / \pi m^*} = \langle \upsilon_x^+ \rangle$$

$$M_{2D}(E) = g_V W \frac{\sqrt{2m^* (E - E_C)}}{\pi \hbar}$$

$$T(E) = 1$$

$$f_0(E) = \frac{1}{1 + e^{(E - E_F / k_B T_L)}} = e^{(E_F - E / k_B T_L)}$$

$$n_S = N_{2D} e^{(E_F - E_C)/k_B T_L}$$

$$N_{2D} = \left(g_V \frac{m^*}{\pi \hbar^2} k_B T_L \right)$$

linear region with MB statistics

$$G_{CH} = \frac{2q^2}{h}\int_{E_C}^{\infty} T(E) M_{2D}(E)\left(-\frac{\partial f_0}{\partial E}\right) dE$$

$$-\partial f_0 / \partial E = f_0 / k_B T_L$$

$$G_{CH} = \frac{2q^2}{h}\int_{E_C}^{\infty} 1 \left[g_V W \frac{\sqrt{2m^*(E - E_C)}}{\pi\hbar} \right]\left(\frac{f_0}{k_B T_L}\right) dE$$

$$G_{CH} = W(qn_S)\frac{\upsilon_T}{2(k_B T_L / q)}$$

$$G_{CH} = WQ_n(V_{GS})\frac{\upsilon_T}{2(k_B T_L / q)}$$

$$M_{2D}(E) = g_V W \frac{\sqrt{2m^*(E - E_C)}}{\pi\hbar}$$

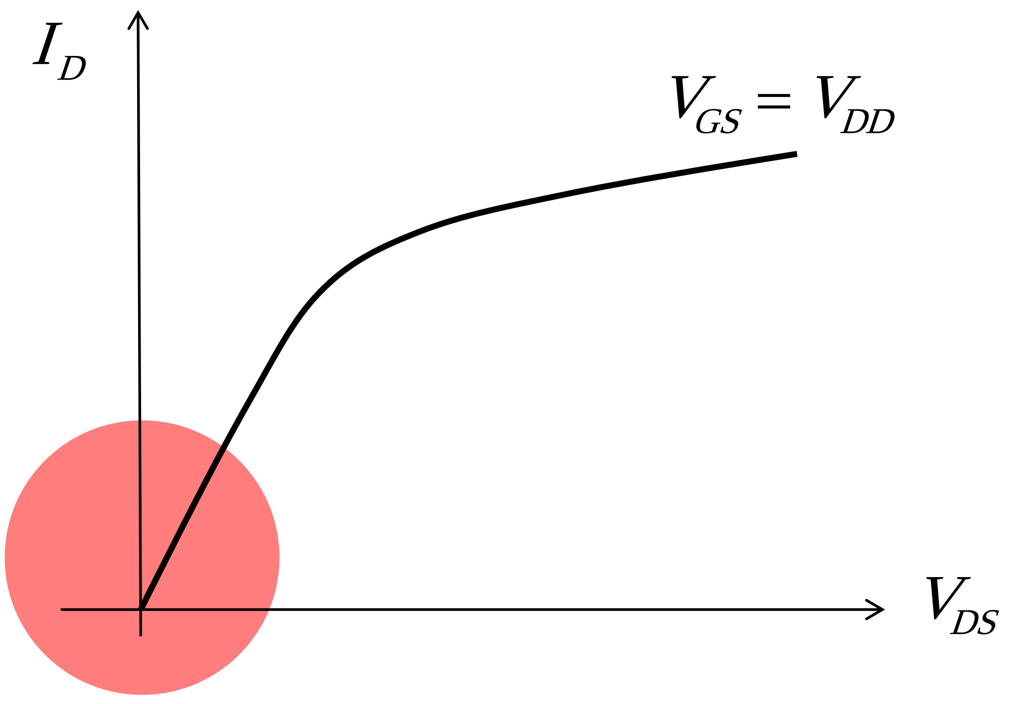
$$T(E) = 1$$

$$f_0(E) = e^{(E_F - E/k_B T_L)}$$

$$n_S = N_{2D} e^{(E_F - E_C)/k_B T_L}$$

$$N_{2D} = \left(g_V \frac{m^*}{\pi\hbar^2} k_B T_L\right)$$

ballistic MOSFET: linear region



near-equilibrium $f_1 \approx f_2$

$$I_D = G_{CH} V_{DS}$$

$$I_D = W C_{inv} \frac{\upsilon_T}{2 k_B T_L / q} \left(V_{GS} - V_T \right) V_{DS}$$

aside…

$$G_{CH} = \frac{2q^2}{h}\int_{E_C}^{\infty} M_{2D}(E)\left(-\frac{\partial f_0}{\partial E}\right) dE = \frac{2q^2}{h}\langle M_{2D}\rangle$$

$$\langle M_{2D}\rangle \equiv \int_{E_C}^{\infty} M_{2D}(E)\left(-\frac{\partial f_0}{\partial E}\right) dE$$

$$\langle M_{2D}\rangle = W\frac{h}{4}\upsilon_T\left(\frac{n_S}{k_B T_L}\right)$$

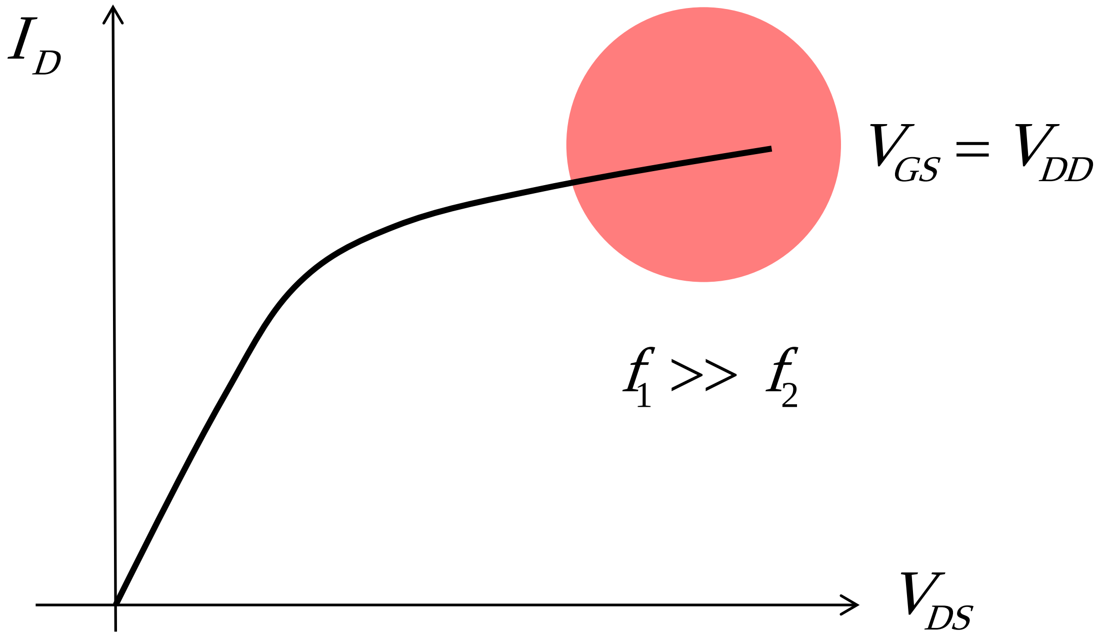
non-degenerate:

$$\langle M_{2D}\rangle \propto n_S$$

degenerate (T_L = 0K):

$$\langle M_{2D}\rangle = M(E_F)$$

ballistic MOSFET: saturated region



$$I_D = \frac{2q}{h}\int T(E)M(E)(f_1 - f_2)dE \rightarrow I_D = \frac{2q}{h}\int T(E)M(E)\, f_1\, dE$$

saturated region with MB statistics

$$I_D = \frac{2q^2}{h}\int_{E_C}^{\infty} T(E) M_{2D}(E) f_1(E) dE$$

$$M_{2D}(E) = g_V W \frac{\sqrt{2m^*(E-E_C)}}{\pi\hbar}$$

$$T(E) = 1$$

$$f_0(E) = e^{(E_F - E/k_B T_L)}$$

$$\upsilon_T = \sqrt{2k_B T_L / \pi m^*} = \langle \upsilon_x^+ \rangle$$

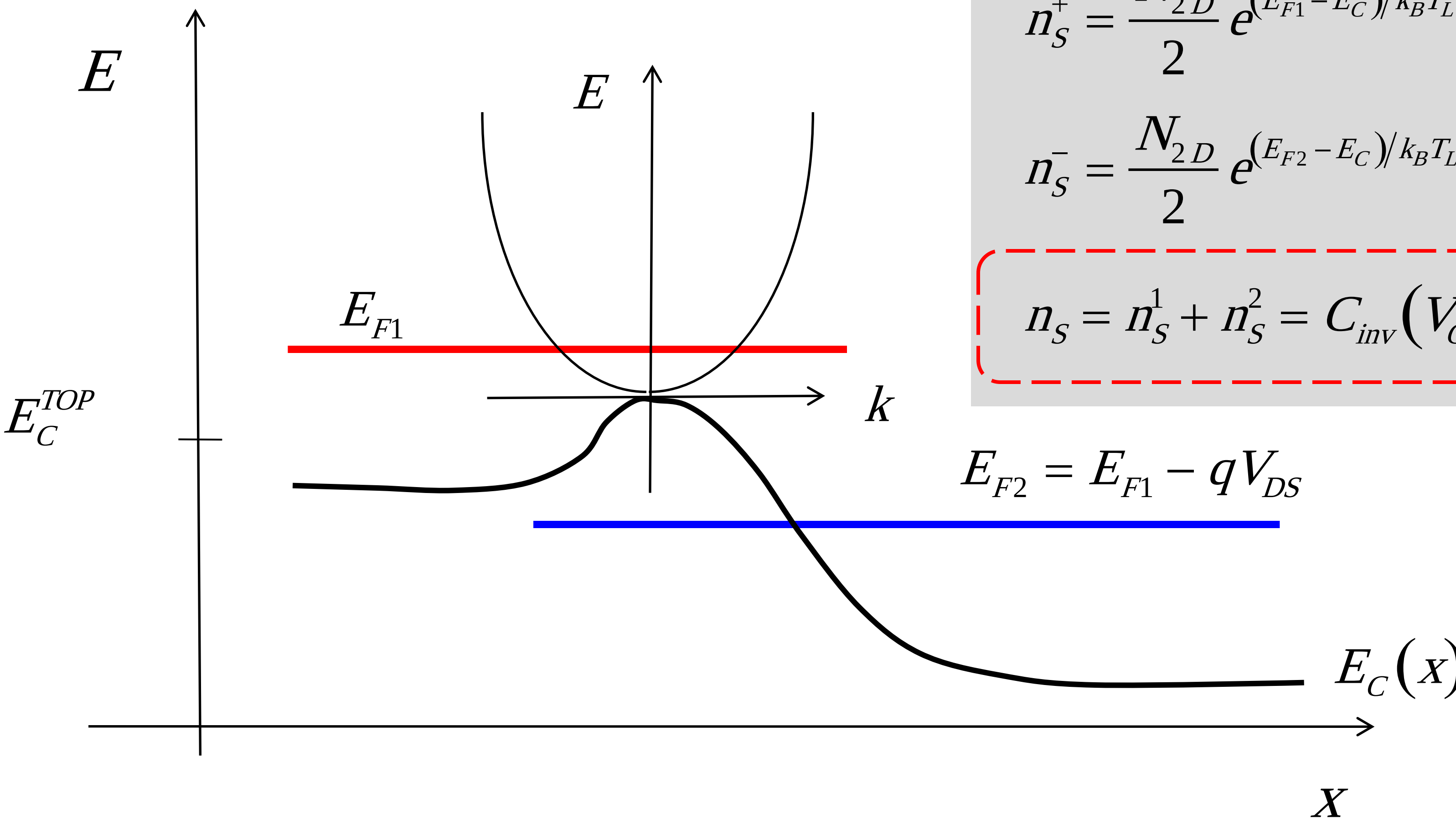
$$n_S = \frac{N_{2D}}{2} e^{(E_F - E_C)/k_B T_L}$$

carrier density at the top of the barrier

$$n_S^{+} = \frac{N_{2D}}{2} e^{(E_{F1} - E_C)/k_B T_L}$$

$$n_S^{-} = \frac{N_{2D}}{2} e^{(E_{F2} - E_C)/k_B T_L}$$

$$n_S = n_S^1 + n_S^2 = C_{inv}\left(V_{GS} - V_T\right)$$



saturated region with MB statistics

$$I_D = \frac{2q^2}{h} \int_{E_C}^{\infty} T(E) M_{2D}(E) f_1(E) dE$$

$$I_D = \frac{2q^2}{h} \int_{E_C}^{\infty} 1 \left[g_V W \frac{\sqrt{2m^* (E - E_C)}}{\pi\hbar} \right] f_0(E) dE$$

$$I_D = W q n_S \upsilon_T$$

$$I_D = W Q_n (V_{GS}) \upsilon_T$$

$$M_{2D}(E) = g_V W \frac{\sqrt{2m^* (E - E_C)}}{\pi\hbar}$$

$$T(E) = 1$$

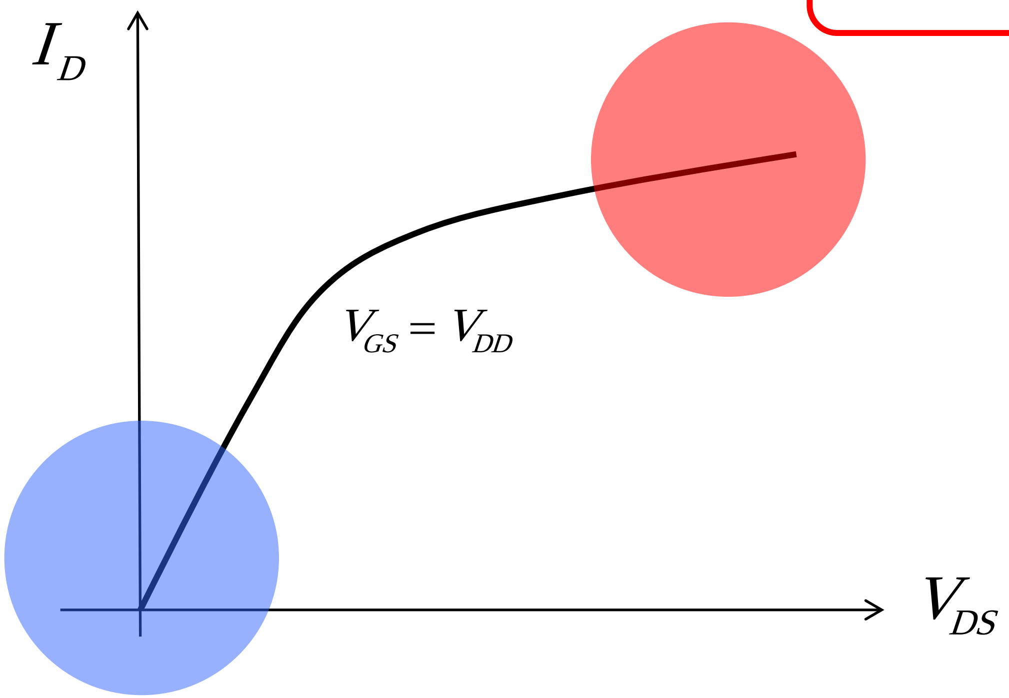
$$f_0(E) = e^{(E_F - E / k_B T_L)}$$

$$\upsilon_T = \sqrt{2 k_B T_L / \pi m^*} = \langle \upsilon_x^+ \rangle$$

$$n_S = \frac{N_{2D}}{2} e^{(E_F - E_C)/k_B T_L}$$

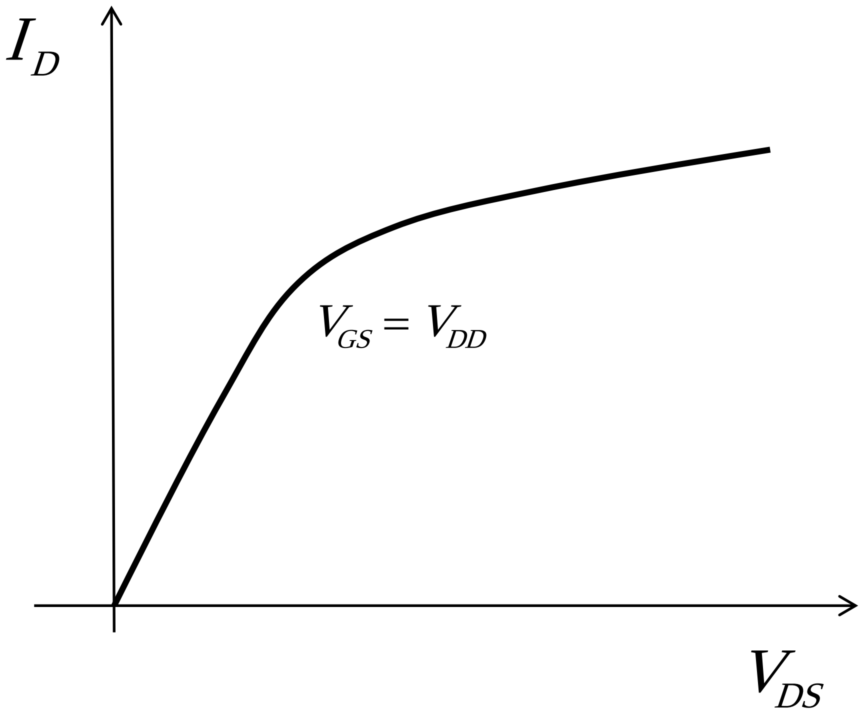
ballistic MOSFET:

$$I_D = W C_{inv} \upsilon_T \left(V_{GS} - V_T \right)$$



$$I_D = W C_{inv} \frac{\upsilon_T}{2 k_B T_L / q} \left(V_{GS} - V_T \right) V_{DS}$$

ballistic MOSFET: full Vds range



$$I_D = \frac{2q}{h}\int T(E) M_{2D}(E)\left(f_1 - f_2\right) dE$$

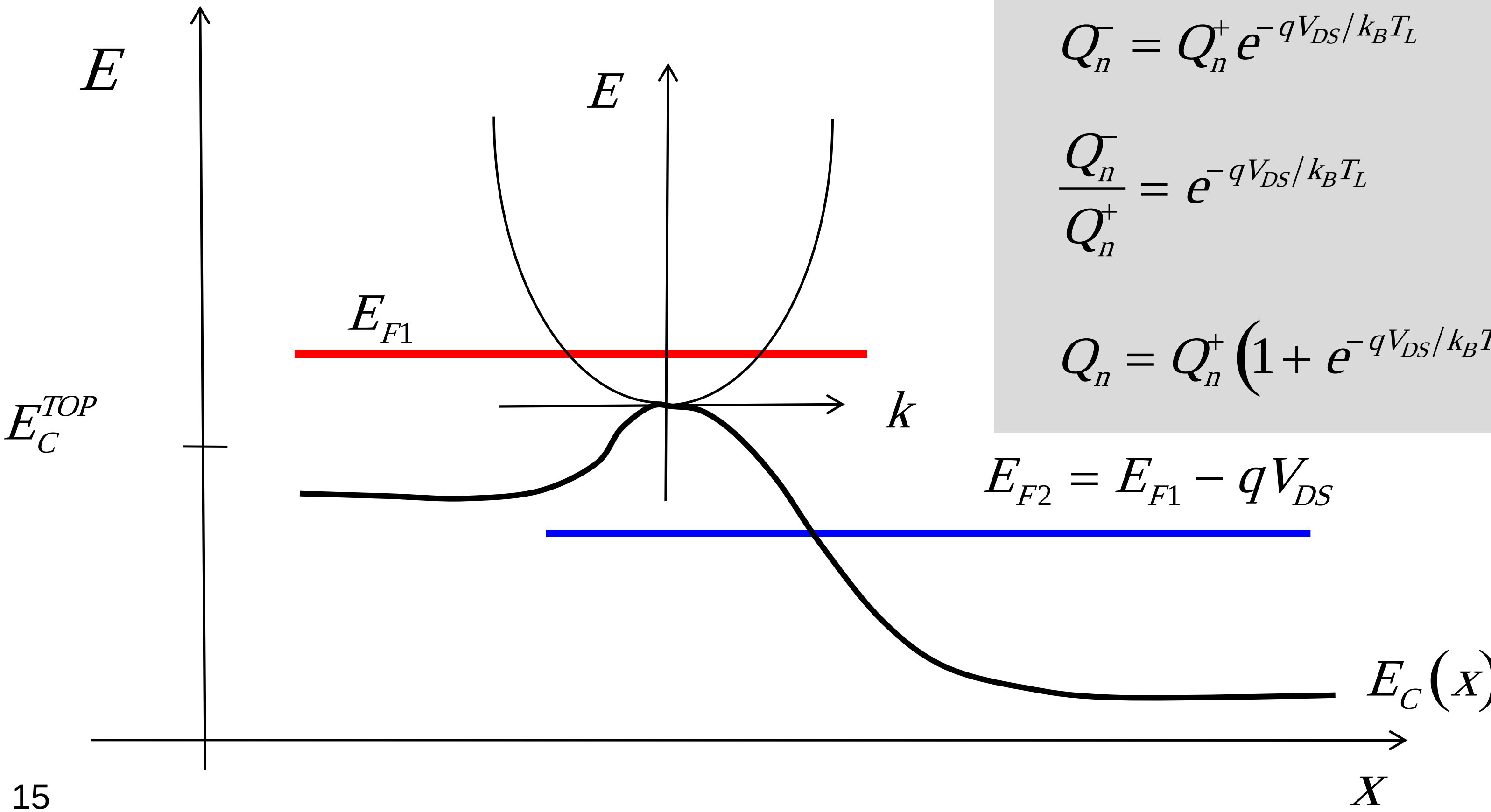
$$I_D = I_{D1} - I_{D2}$$

$$I_{D1} = WQ_n^+\left(V_{GS}\right)\upsilon_T$$

$$I_{D2} = WQ_n^-\left(V_{GS}\right)\upsilon_T$$

$$I_D = W\upsilon_T\left(Q_n^+\left(V_{GS}\right) - Q_n^-\left(V_{GS}\right)\right) = W\upsilon_T Q_n^+\left(V_{GS}\right)\left(1 - \frac{Q_n^-\left(V_{GS}\right)}{Q_n^+\left(V_{GS}\right)}\right)$$

charge balance at the top of the barrier



$$Q_n = Q_n^+ + Q_n^-$$

$$Q_n^- = Q_n^+ e^{-qV_{DS}/k_BT_L}$$

$$\frac{Q_n^-}{Q_n^+} = e^{-qV_{DS}/k_BT_L}$$

$$Q_n = Q_n^+ \left(1 + e^{-qV_{DS}/k_BT_L}\right)$$

ballistic MOSFET: full Vds range

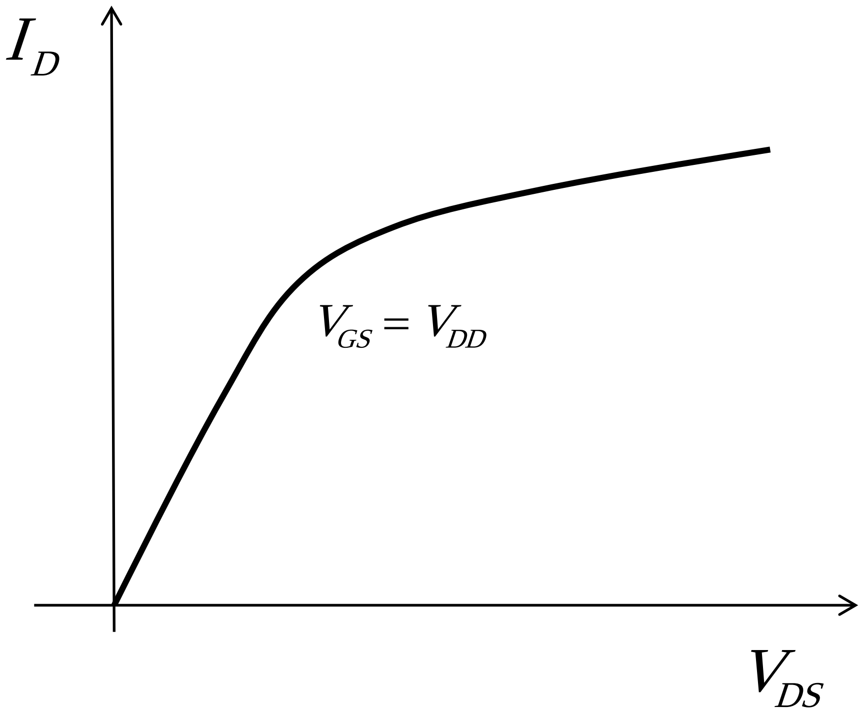
$$I_D = W\upsilon_T\left(Q_n^+\left(V_{GS}\right) - Q_n^-\left(V_{GS}\right)\right) = W\upsilon_T Q_n^+\left(V_{GS}\right)\left(1 - \frac{Q_n^-\left(V_{GS}\right)}{Q_n^+\left(V_{GS}\right)}\right)$$

$$\frac{Q_n^-}{Q_n^+} = e^{-qV_{DS}/k_BT_L}$$

$$Q_n = Q_n^+\left(1 + e^{-qV_{DS}/k_BT_L}\right)$$

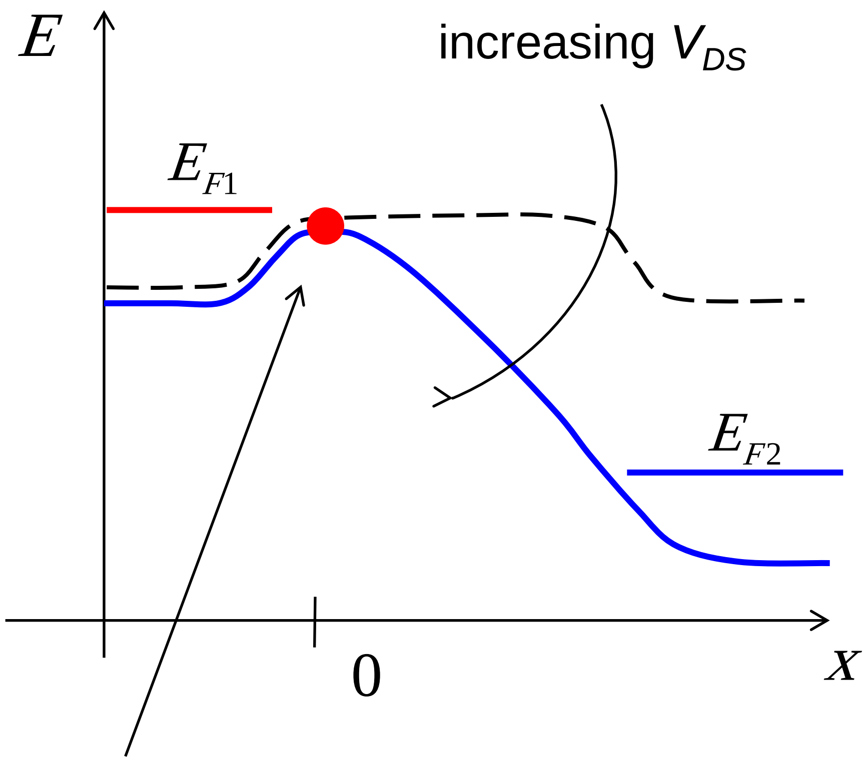
$$I_D = W\upsilon_T Q_n\left(V_{GS}\right)\left(\frac{1 - e^{-qV_{DS}/k_BT_L}}{1 + e^{-qV_{DS}/k_BT_L}}\right)$$

full range ballistic model



$$I_D = W\upsilon_T Q_n\left(V_{GS}\right)\left(\frac{1-e^{-qV_{DS}/k_BT_L}}{1+e^{-qV_{DS}/k_BT_L}}\right)$$

average velocity at the top of the barrier



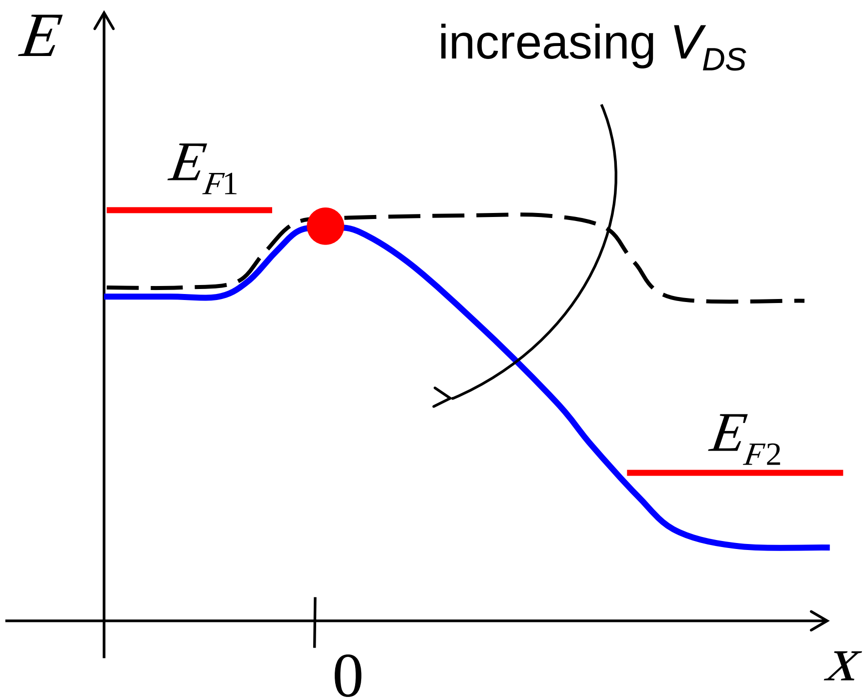
$$I_D = WQ_n(0)\upsilon_T \frac{\left(1 - e^{-qV_{DS}/k_BT_L}\right)}{\left(1 + e^{-qV_{DS}/k_BT_L}\right)}$$

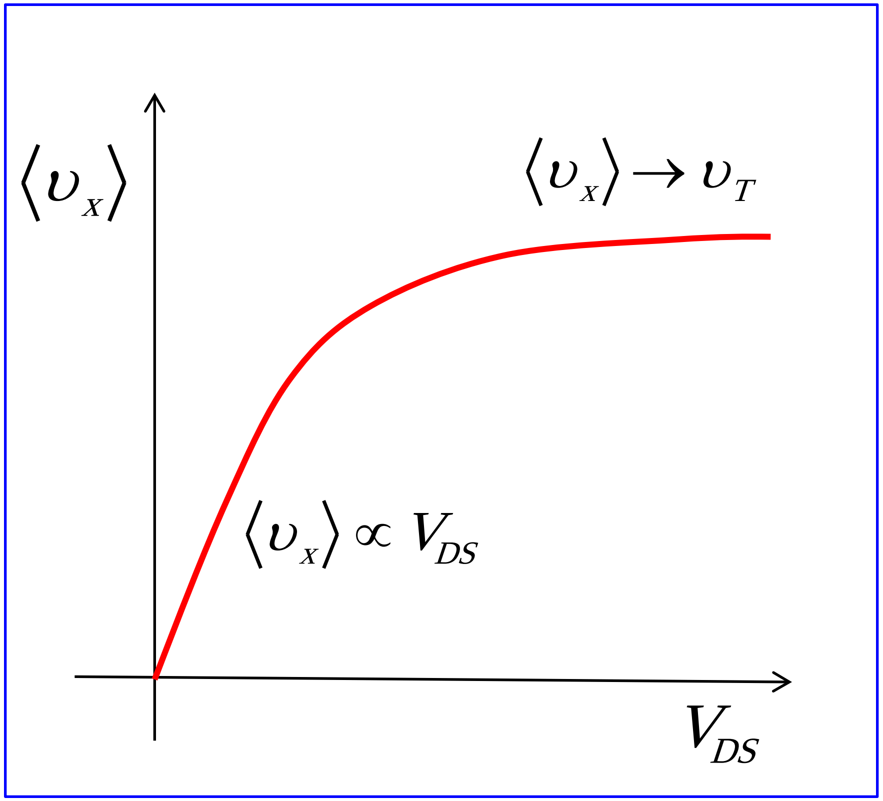
$$\langle \upsilon(0) \rangle = \frac{I_D}{WQ_n}$$

$$\langle \upsilon(0) \rangle = \upsilon_T \frac{\left(1 - e^{-qV_{DS}/k_BT}\right)}{\left(1 + e^{-qV_{DS}/k_BT}\right)}$$

velocity at the top of the barrier?

velocity vs. V_{DS}

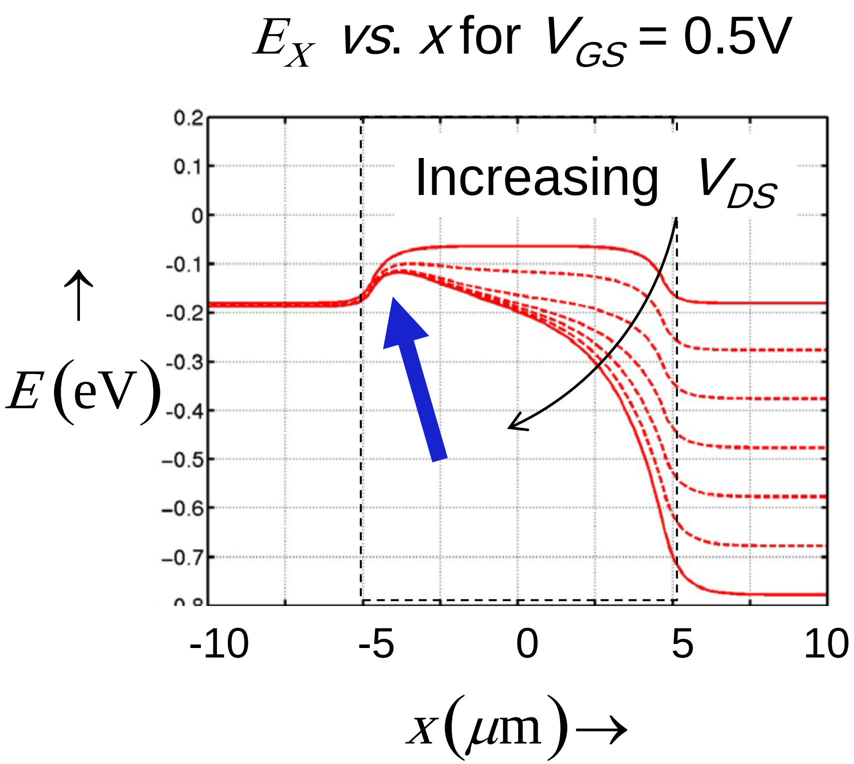


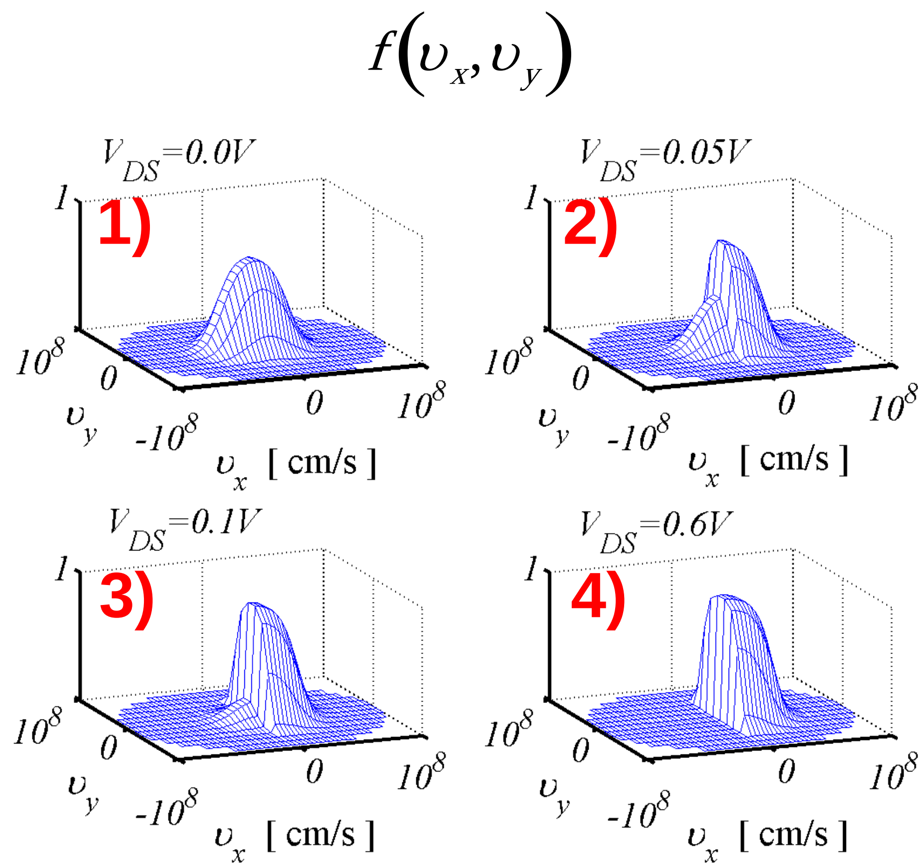


$$\langle \upsilon(0) \rangle = \upsilon_T \frac{\left(1 - e^{-qV_{DS}/k_BT}\right)}{\left(1 + e^{-qV_{DS}/k_BT}\right)}$$

Velocity ***saturates*** in a ballistic MOSFET but at the top of the barrier, where *E*-field = 0.

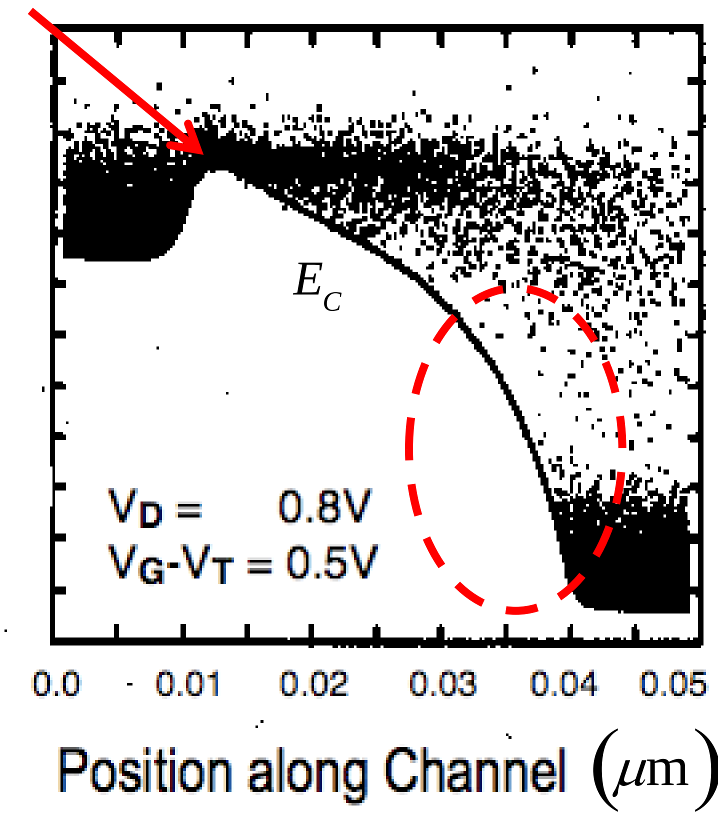
filling states at the top of the barrier

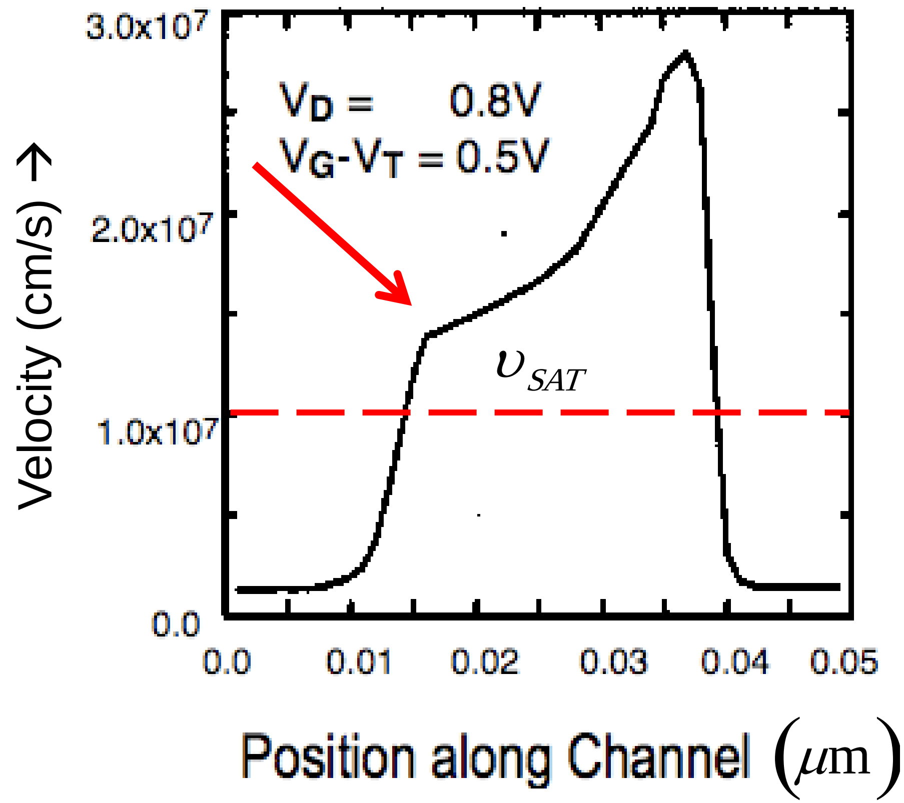




(Numerical simulations of an L = 10 nm double gate Si MOSFET from J.-H. Rhew and M.S. Lundstrom, *Solid-State Electron.*, **46**, 1899, 2002)

non-local transport in a nanoscale MOSFET

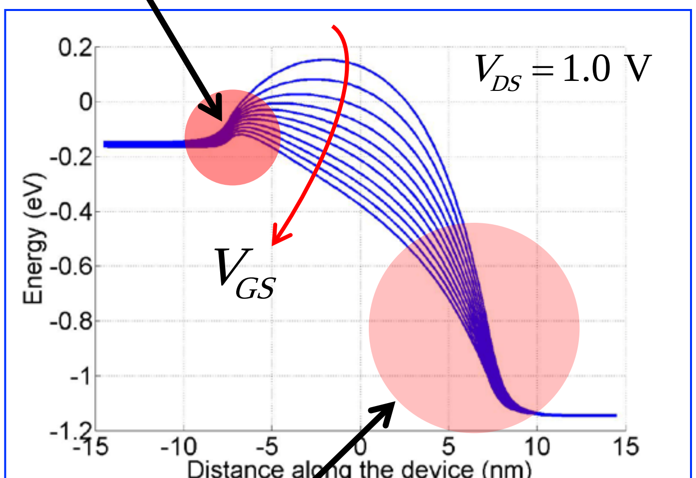




D. Frank, S. Laux, and M. Fischetti, Int. Electron Dev. Mtg., Dec., 1992.

velocity saturation in a ballistic MOSFETs

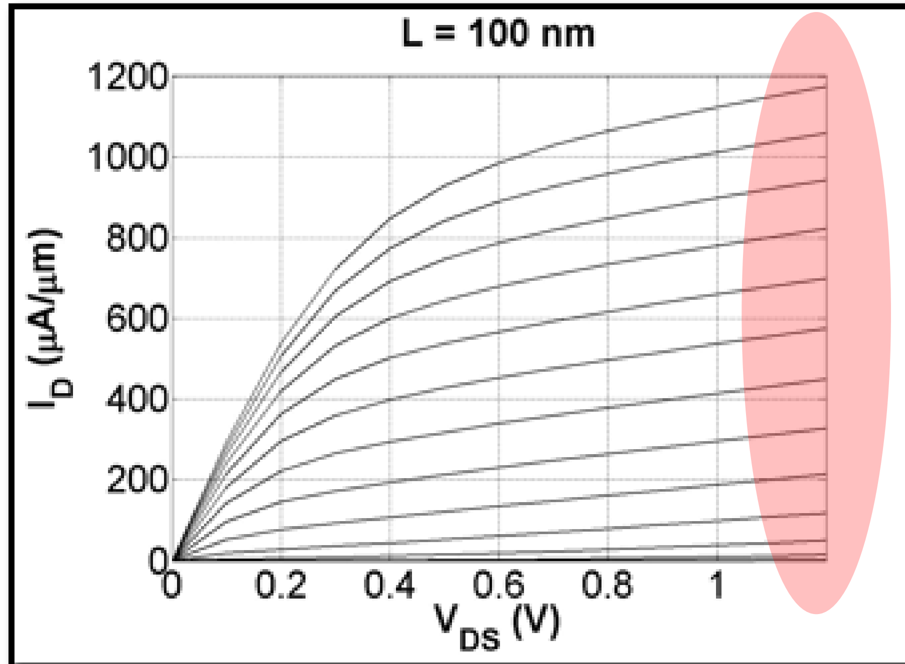
$\upsilon = \upsilon_T \approx 1.2 \times 10^7 \text{ cm/s}$



$\upsilon = \upsilon_{sat} \approx 1.0 \times 10^7 \text{ cm/s}$

2007 N-MOSFET

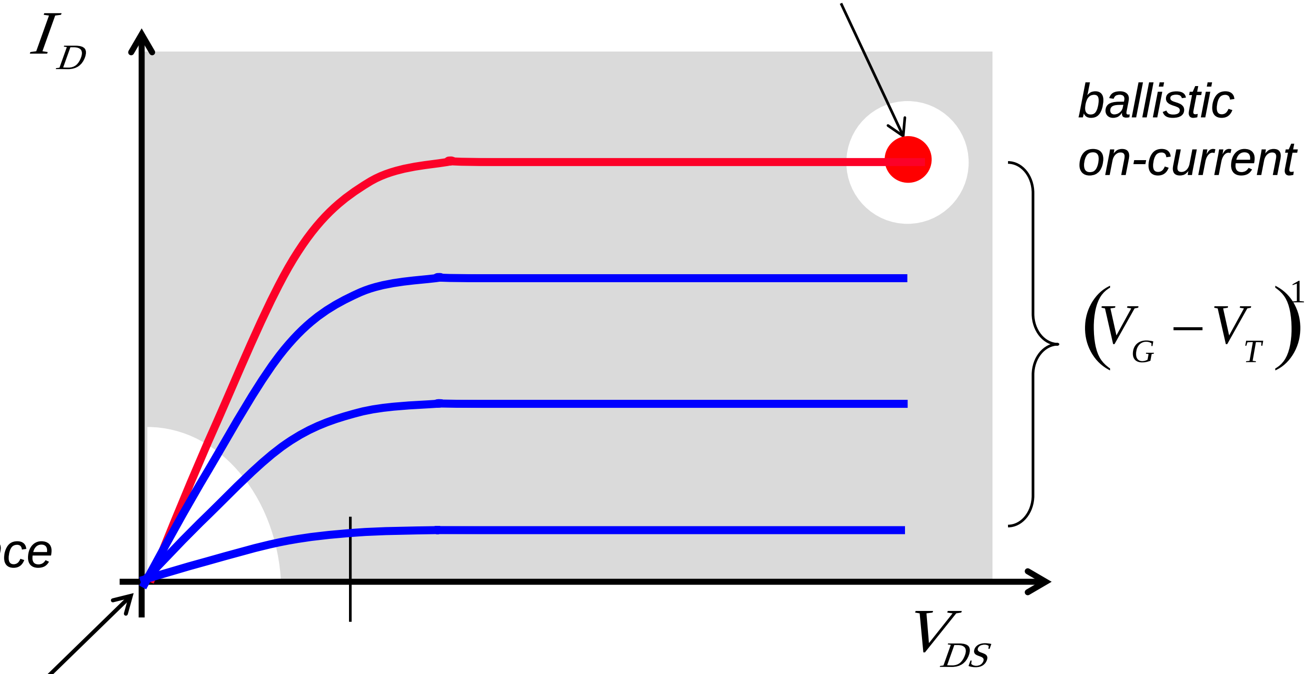
velocity saturation



(Courtesy, Shuji Ikeda, ATDF, Dec. 2007)

the ballistic MOSFET (MB statistics)

$$I_D(\text{on}) = W \upsilon_T C_{ox} \left(V_{GS} - V_T \right)$$



ballistic
channel resistance

$$I_D = G_{CH} V_{DS} = W C_{ox} \frac{\upsilon_T}{\left(2 k_B T_L / q \right)} \left(V_{GS} - V_T \right) V_{DS}$$

K. Natori, *JAP*, **76**, 4879, 1994.

the ballistic MOSFET (MB vs. FD statistics)

Maxwell-Boltzmann statistics

$$n_S^+(0) = \int_{E_C}^{\infty} \frac{D_{2D}(E)}{2} f_0(E)\,dE$$

$$= \frac{N_{2D}}{2} e^{(E_{F1}-E_C)/k_B T_L}$$

$$= \frac{N_{2D}}{2} e^{\eta_F}$$

$$\eta_{F1} = \frac{E_{F1} - E_C}{k_B T_L}$$

Fermi-Dirac statistics

$$n_S^+(0) = \int_{E_C}^{\infty} \frac{D_{2D}(E)}{2} f_0(E)\,dE$$

$$= \frac{N_{2D}}{2} \mathcal{F}_0(\eta_{F1})$$

"Fermi-Dirac integral of order 0"

Fermi-Dirac integrals

$$\mathcal{F}_j(\eta_F) \equiv \Gamma(j+1) \int_0^{+\infty} \frac{\eta^j \, d\eta}{1+e^{\eta-\eta_F}}$$

Raseong Kim and Mark Lundstrom, “Notes on Fermi-Dirac Integrals (3rd Ed.), 2008. http://nanohub.org/resources/5475/

Ballistic MOSFET IV

Maxwell-Boltzmann statistics:

$$I_D = WQ_n(0)\upsilon_T \frac{\left(1 - e^{-qV_{DS}/k_BT_L}\right)}{\left(1 + e^{-qV_{DS}/k_BT_L}\right)}$$

Fermi-Dirac statistics:

$$I_D = WQ_n(0)\upsilon_{inj}(\eta_{F1})\left[\frac{1 - \mathcal{F}_{1/2}(\eta_{F2})/\mathcal{F}_{1/2}(\eta_{F1})}{1 + \mathcal{F}_0(\eta_{F2})/\mathcal{F}_0(\eta_{F1})}\right]$$

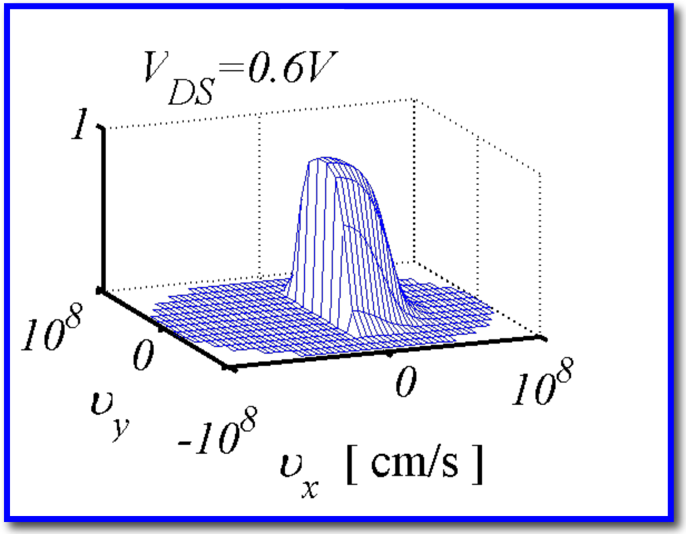
$$\upsilon_{inj}(\eta_{F1}) = \upsilon_T \frac{\mathcal{F}_{1/2}(\eta_{F1})}{\mathcal{F}_0(\eta_{F1})} \quad \upsilon_T = \sqrt{\frac{2k_BT_L}{\pi m^*}}$$

"ballistic injection velocity"

ballistic injection velocity

$$I_D = WQ_n(0)\upsilon_{inj}(\eta_{F1})\left[\frac{1-\mathcal{F}_{1/2}(\eta_{F2})/\mathcal{F}_{1/2}(\eta_{F1})}{1+\mathcal{F}_0(\eta_{F2})/\mathcal{F}_0(\eta_{F1})}\right]$$

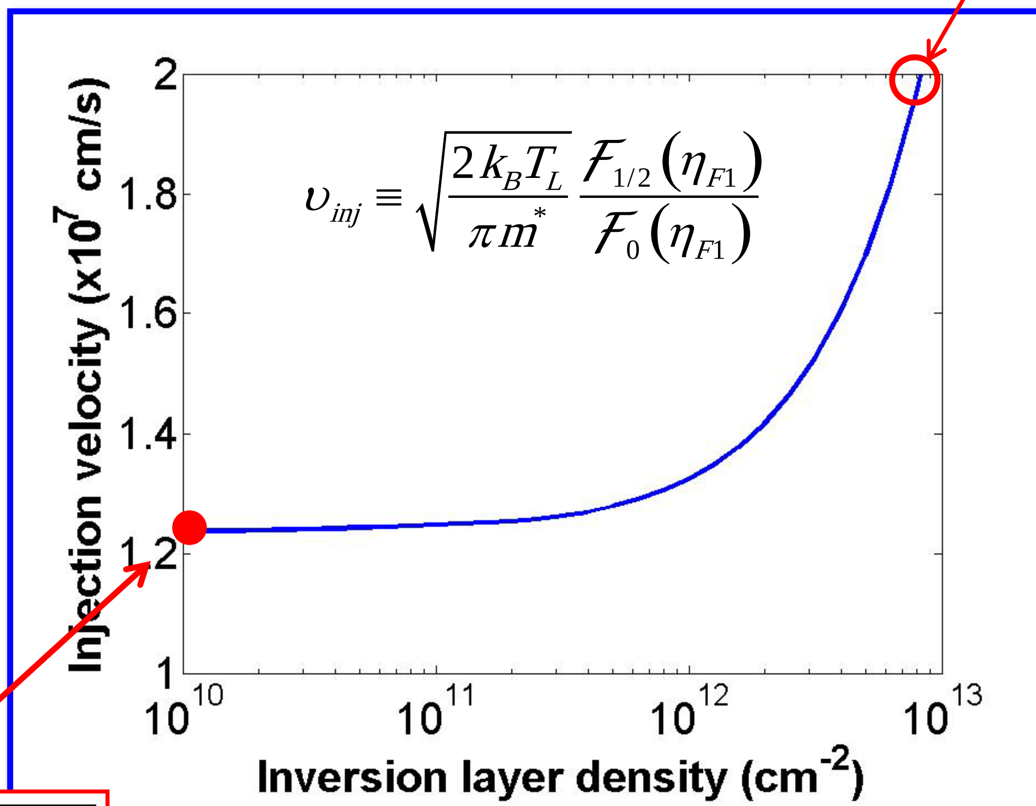
$$\rightarrow WQ_n(0)\upsilon_{inj}(\eta_{F1})$$



$\langle \upsilon_x(0) \rangle$ for high drain bias.

ballistic injection velocity

$$\tilde{\upsilon}_T \rightarrow (4/3\pi)\upsilon_F$$



$$\upsilon_F = \frac{\hbar k_F}{m^*}$$

$$\upsilon_T = \sqrt{2k_BT/\pi m^*}$$

wrap-up

1) Ballistic IV characteristics

2) Velocity saturation in a ballistic MOSFET

3) Ballistic injection velocity

references

For a careful development of the IV characteristics for nanoscale MOSFETs, see:

Mark Lundstrom and Jing Guo, *Nanoscale Transistors: Physics, Modeling, and Simulation*, Springer, New York, 2006.