Lecture 10: Scattering Model

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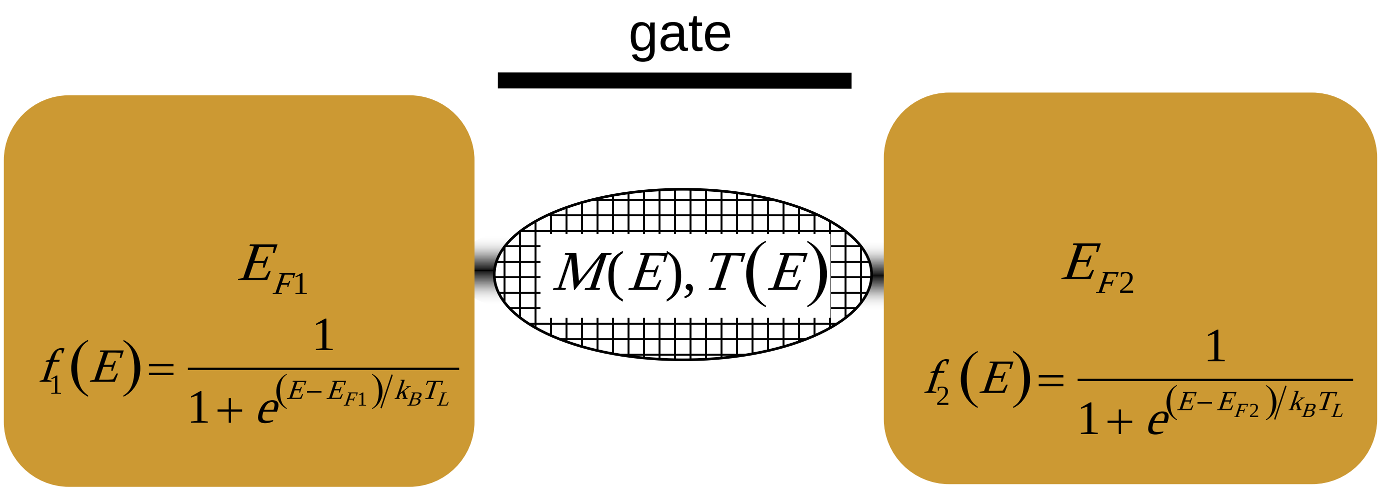
Landauer approach to transport



$$I_D = \frac{2q}{h} \int T(E) M(E) \left(f_1 - f_2 \right) dE$$

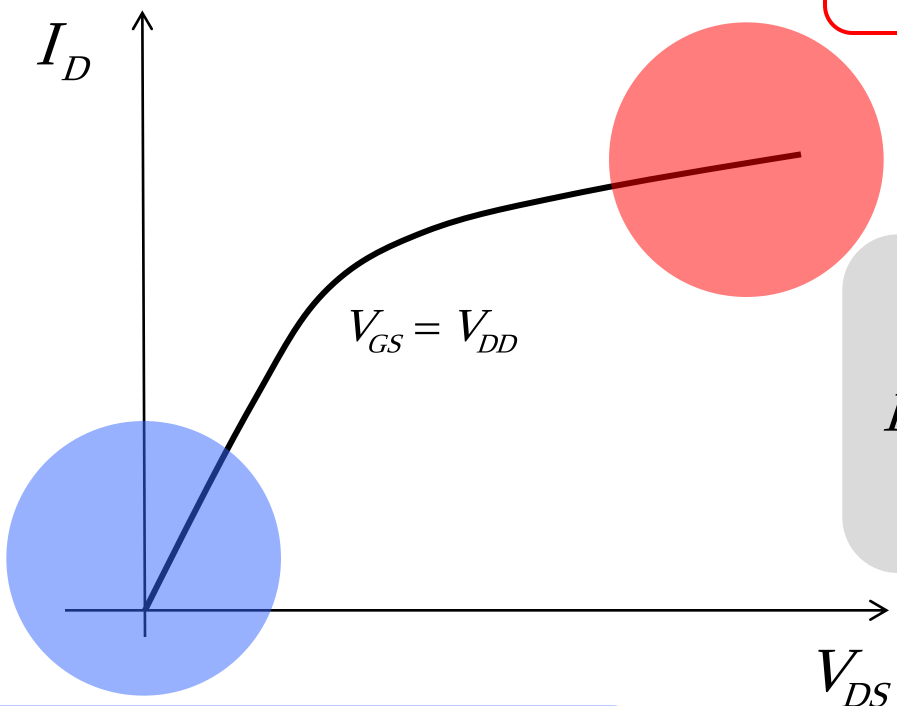
any drain bias

$$I_D = \left[\frac{2q^2}{h} \int T(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE \right] V$$

low drain bias

ballistic MOSFET (MB)

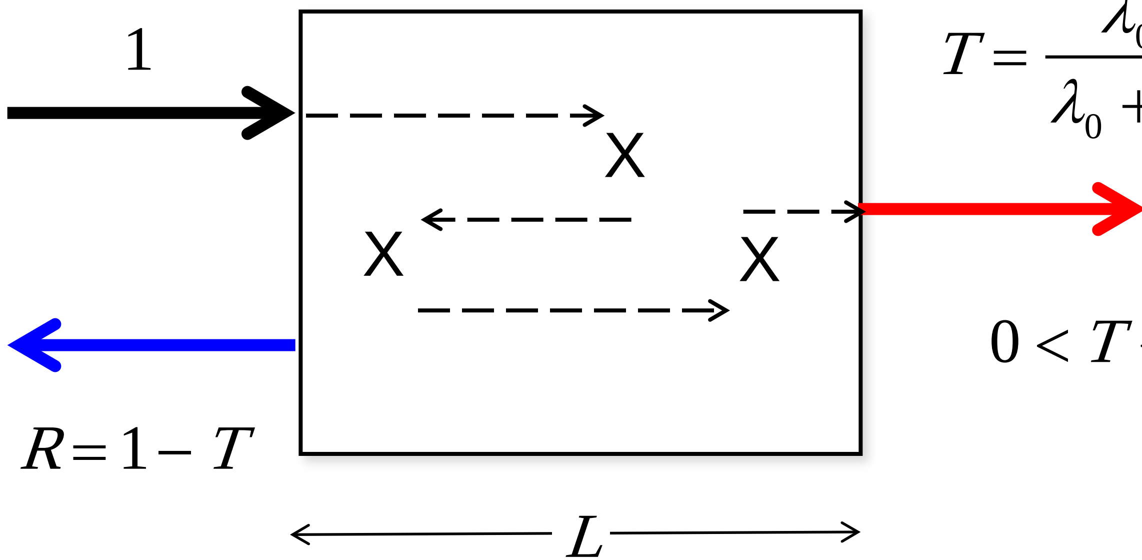
$$I_D = WQ_n\left(V_{GS}, V_{DS}\right)\upsilon_T$$



$$I_D = WQ_n\left(V_{GS}, V_{DS}\right)F_{SAT}\left(V_D\right)\upsilon_T$$

$$I_D = WQ_n\left(V_{GS}, V_{DS}\right)\frac{\upsilon_T}{2k_BT_L/q}V_{DS}$$

transmission



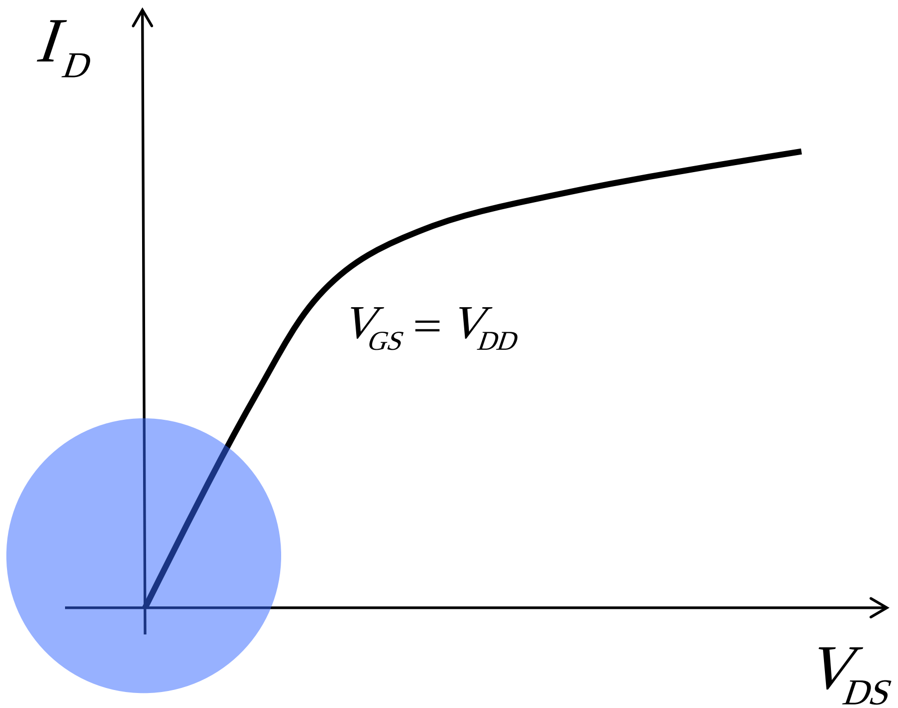
$$T = \frac{\lambda_0}{\lambda_0 + L}$$

$$0 < T < 1$$

λ_0 is the mean-free-path for backscattering

$$I_D \rightarrow T I_D\,?$$

scattering: linear region



$$T = \frac{\lambda_0}{\lambda_0 + L}$$

$$I_D = WQ_n\left(V_{GS}, V_{DS}\right)\frac{\upsilon_T}{2k_BT_L/q}V_{DS} \rightarrow \boxed{I_D = TWQ_n\left(V_{GS}, V_{DS}\right)\frac{\upsilon_T}{2k_BT_L/q}V_{DS}}$$

example

For a reasonable carrier mobility and a state-of-the-art channel length of 22 nm (2012), what is *T*?

$$\mu_{eff} \approx 200 \text{ cm}^2/\text{V-s}$$

$$\mu_n = \frac{\upsilon_T \lambda_0}{2\left(k_B T/q\right)}$$

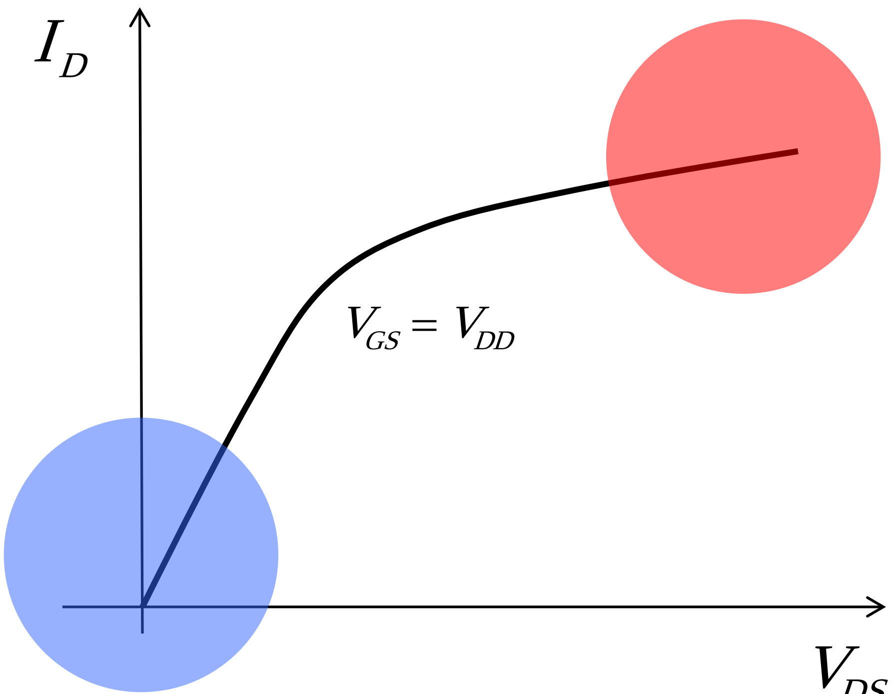
$$\lambda_0 \approx 9 \text{ nm}$$

$$T \approx \frac{\lambda_o}{L + \lambda_o} \approx 0.3$$

ballistic MOSFET (MB)

$$I_D = TWQ_n\left(V_{GS}, V_{DS}\right)\upsilon_T$$

?



$$I_D = TWQ_n\left(V_{GS}, V_{DS}\right)\frac{\upsilon_T}{2k_BT_L/q}V_{DS}$$

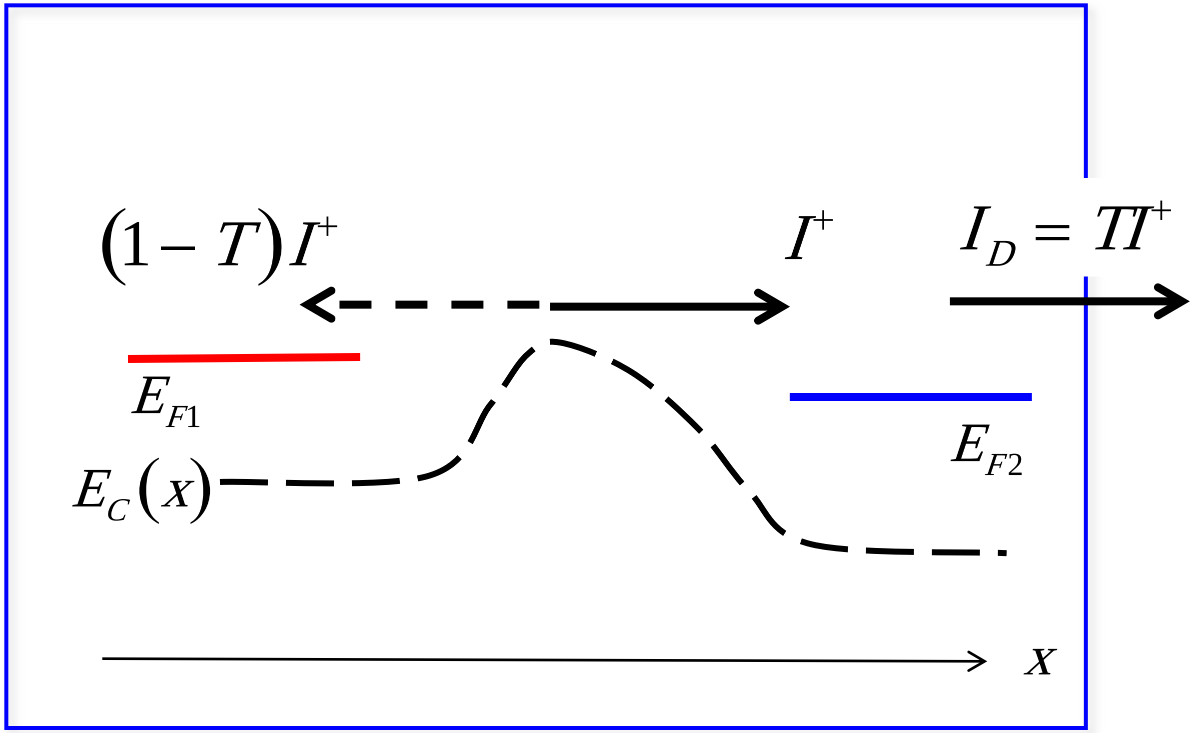
on-current and transmission

$$Q_n(0) = \frac{I_{BALL}^+}{W\upsilon_T}$$

$$Q_n(0) = \frac{I^+ + (1-T)I^+}{W\upsilon_T}$$

$$I^+ = \frac{I_{BALL}^+}{(2-T)}$$

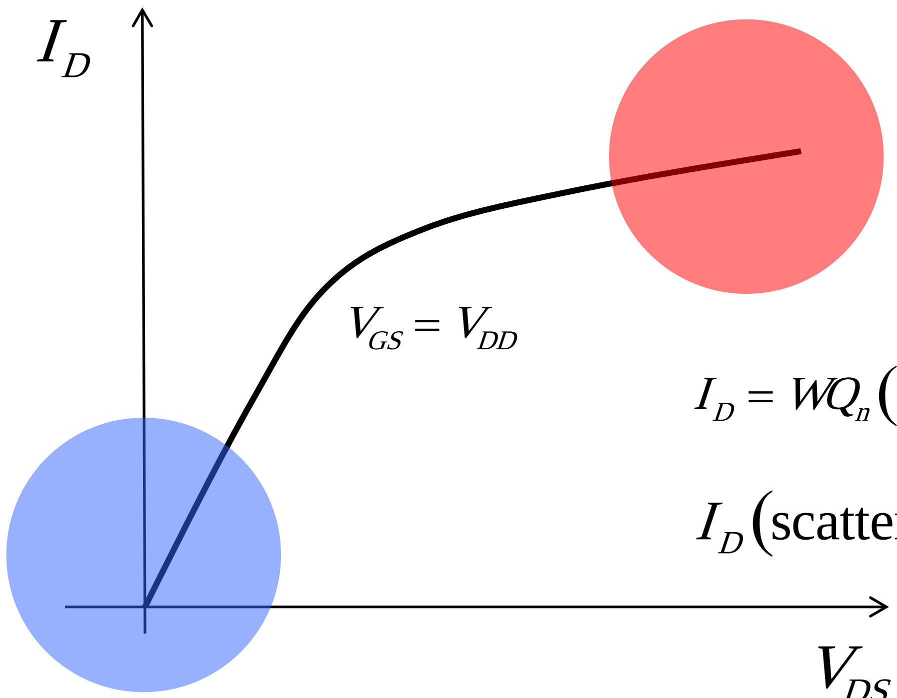
$$I_{ON} = \left(\frac{T}{2-T}\right) I_{BALL}^+$$



ballistic MOSFET (MB)

$$I_D = \left(\frac{T}{2-T}\right) W Q_n\left(V_{GS}, V_{DS}\right)\upsilon_T$$

✓



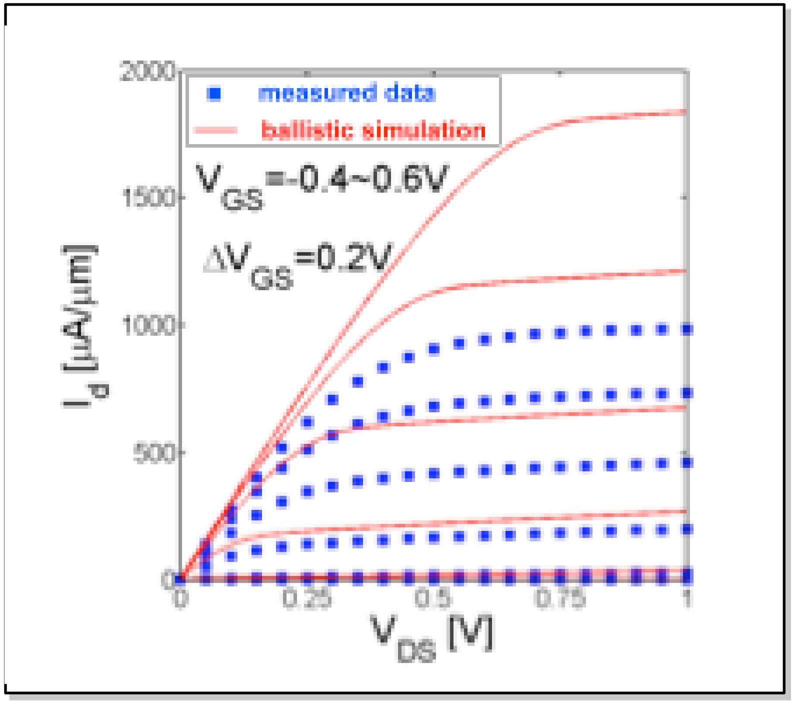
$$I_D = WQ_n\left(V_{GS}, V_{DS}\right)T\left(\frac{1-e^{-qV_{DS}/k_BT}}{\left(2-T\right)+Te^{-qV_{DS}/k_BT}}\right)\upsilon_T$$

$$I_D\left(\text{scattering}\right) \neq TI_D\left(\text{ballistic}\right)$$

$$\boxed{I_D = TWQ_n\left(V_{GS}, V_{DS}\right)\frac{\upsilon_T}{2k_BT_L/q}V_{DS}}$$

recall: ETSOI MOSFET

$$L_G = 40 \text{ nm}$$



$$I_{Dlin}/I_{ballistic} \approx 0.2$$

$$I_{ON}/I_{ballistic} \approx 0.6$$

- Si MOSFETs deliver > one-half of the ballistic on-current. (Similar for the past 15 years.)
- MOSFETs operate closer to the ballistic limit under high V_{DS}.

A. Majumdar, Z. B. Ren, S. J. Koester, and W. Haensch, "Undoped-Body Extremely Thin SOI MOSFETs With Back Gates," *IEEE Transactions on Electron Devices*, **56**, pp. 2270-2276, 2009.

Device characterization and simulation: Himadri Pal and Yang Liu, Purdue, 2010.

transmission under low and high V_{DS}

$$I_{Dlin} / I_{ballistic} \approx 0.2$$

$$I_{Dlin} / I_{ballistic} = T_{lin}$$

$$I_{ON} / I_{ballistic} \approx 0.6$$

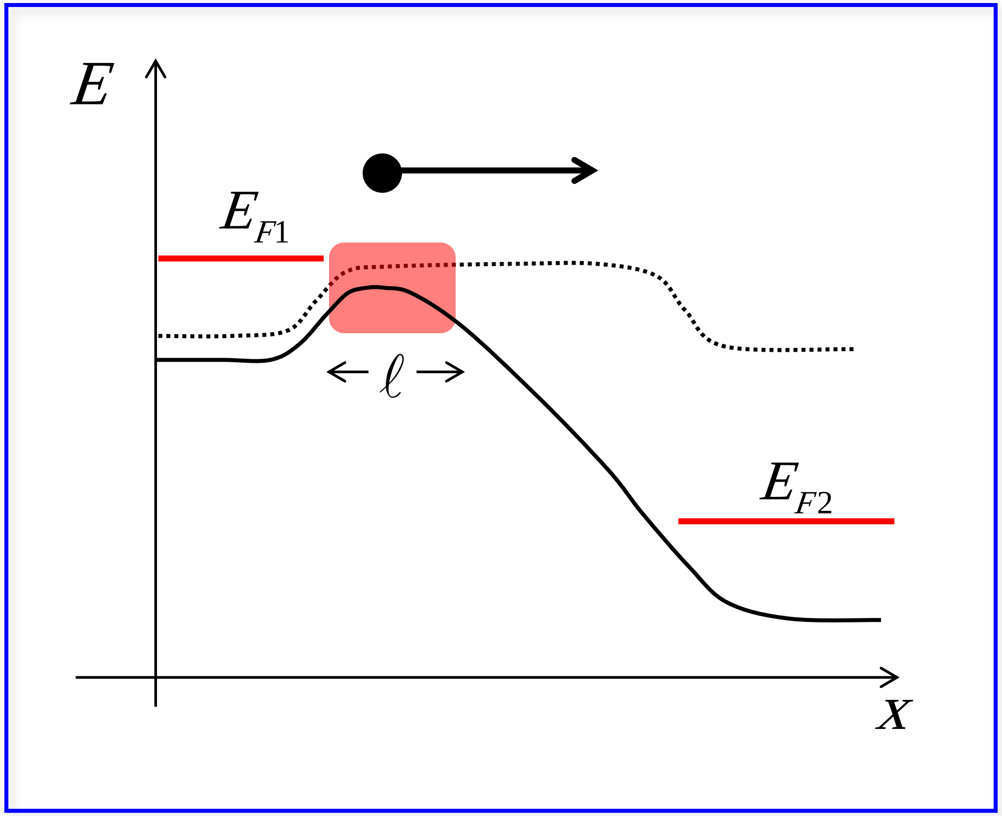
$$I_{ON} / I_{ballistic} \approx T_{sat} / \left(2 - T_{sat}\right)$$

$$I_{Dlin} = T W Q_n\left(V_{GS}, V_{DS}\right) \frac{\upsilon_T}{2 k_B T_L / q} V_{DS}$$

$$I_{ON} = \left(\frac{T}{2 - T}\right) W Q_n\left(V_{GS}, V_{DS}\right) \upsilon_T$$

$T_{lin} \approx 0.2$ $\qquad$ $T_{sat} \approx 0.7$ $\qquad$ ***Why?***

scattering under high V_{DS}



$$T_{lin} = \frac{\lambda_0}{\lambda_0 + L}$$

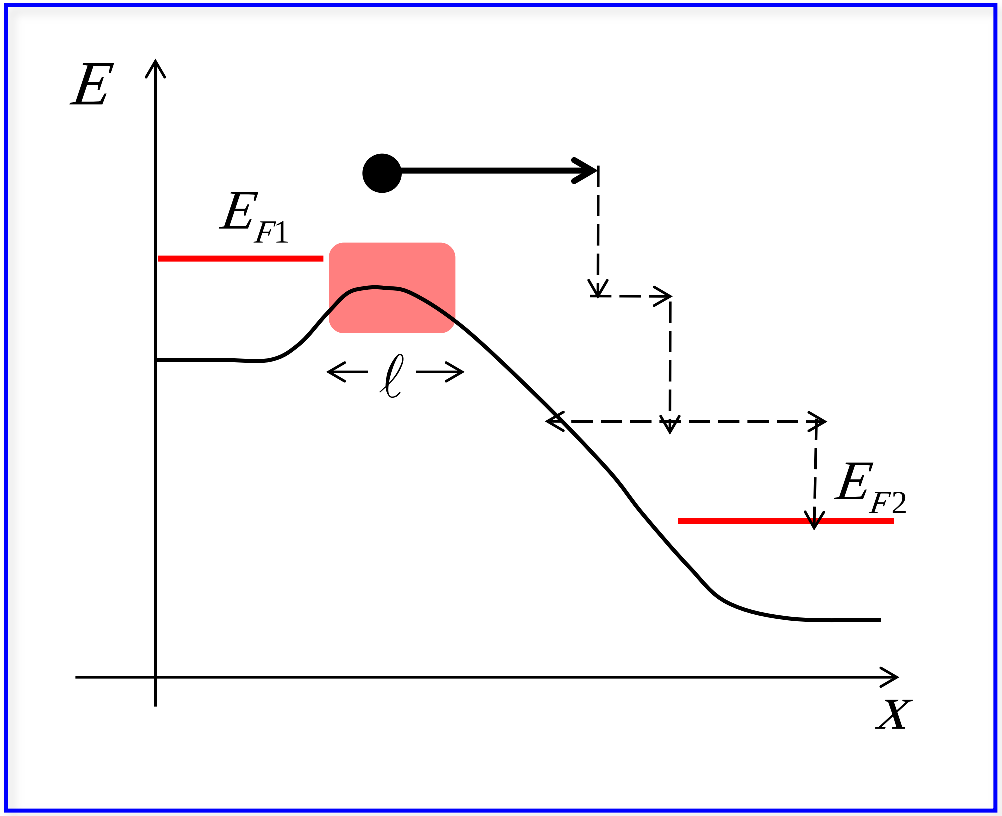
$$L \rightarrow \ell$$

$$T_{sat} = \frac{\lambda_0}{\lambda_0 + \ell}$$

$$\ell << L$$

$$T_{sat} > T_{lin}$$

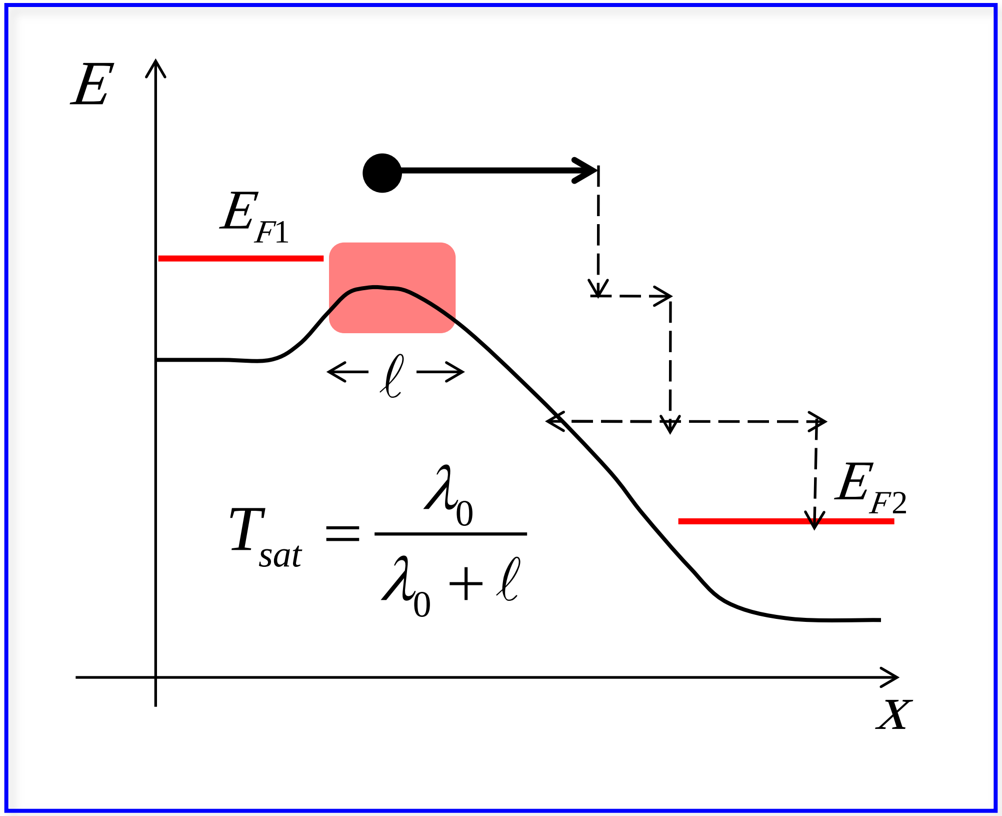
operation near the "ballistic limit"



$$T_{sat} = \frac{\lambda_0}{\lambda_0 + \ell}$$

Operation near the ballistic limit current just means that $T_{sat} \rightarrow 1$, it does not imply that there is little scattering.

is mobility relevant at the nanoscale?



- mobility is related to the near-eq. MFP
- backscattering in the critical region is also controlled by the near-eq. MFP.
- mobility determines the on-current
- but the MFP near the drain is very short.

connection to traditional model: linear

$$I_D = \frac{W}{L}\mu_{eff}C_{inv}\left(V_{GS}-V_T\right)V_{DS}$$

$$I_D = T\left(WC_{inv}\left(V_{GS}-V_T\right)\frac{\upsilon_T}{\left(2k_BT_L/q\right)}\right)V_{DS} \qquad T = \frac{\lambda_0}{\lambda_0 + L}$$

$$I_D = \frac{W}{L+\lambda_0}\mu_{eff}C_{inv}\left(V_{GS}-V_T\right)V_{DS}$$

$$I_D = \frac{W}{L}\mu_{app}C_{inv}\left(V_{GS}-V_T\right)V_{DS}$$

$$1/\mu_{app} = 1/\mu_{eff} + 1/\mu_B$$

connection to traditional model: linear region

$$I_D = \frac{W}{L} \mu_{eff} C_{inv} \left(V_{GS} - V_T \right) V_{DS}$$

$$I_D = \frac{W}{L} \mu_{app} C_{inv} \left(V_{GS} - V_T \right) V_{DS}$$

"Mathiessen's Rule"

$$\frac{1}{\mu_{app}} = \frac{1}{\mu_{eff}} + \frac{1}{\mu_B}$$

μ_{eff} : measured

$$\mu_B = \frac{\upsilon_T L}{2 k_B T_L / q}$$

connection to traditional model: saturation

$$I_D = WC_{inv}\upsilon_{sat}\left(V_{GS} - V_T\right)$$

$$I_D = WC_{inv}\upsilon_T\left(\frac{T}{2-T}\right)\left(V_{GS} - V_T\right) \qquad T = \frac{\lambda_0}{\lambda_0 + \ell}$$

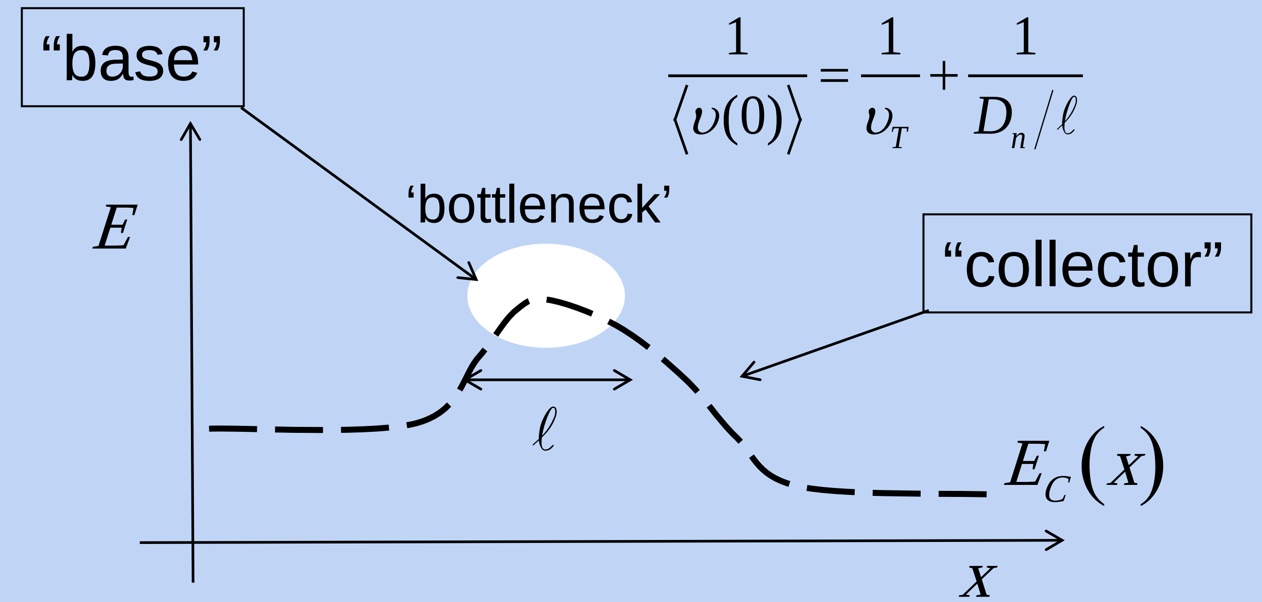
$$I_D = W\left[\frac{1}{\upsilon_T} + \frac{1}{\left(D_n/\ell\right)}\right]^{-1} C_{inv}\left(V_{GS} - V_T\right)$$

how do we interpret this result?

current transmission in a MOSFET

$$I_D = W\langle \upsilon(0)\rangle C_{inv}\left(V_{GS} - V_T\right)$$

$$\frac{1}{\langle \upsilon(0)\rangle} = \frac{1}{\upsilon_T} + \frac{1}{D_n/\ell}$$



VS model for the MOSFET

$$I_D = W Q_n\left(V_{GS}, V_{DS}\right)\left\langle \upsilon(0)\right\rangle$$

$$\left\langle \upsilon(0)\right\rangle = \left\{ \frac{V_D / V_{DSAT}}{\left[1 + \left(V_D / V_{DSAT}\right)^{\beta}\right]^{1/\beta}} \right\} \upsilon_{SAT} = F_{SAT}\left(V_{DS}\right)\upsilon_{SAT}$$

$$V_{DSAT} = \frac{\upsilon_{SAT} L}{\mu_{eff}}$$

υ_{SAT}

μ_{eff}

a more physical VS model for a MOSFET

1) $\mu_{eff} \rightarrow \mu_{app}$ $$\frac{1}{\mu_{app}} = \frac{1}{\mu_{eff}} + \frac{1}{\mu_B}$$

2) $$\upsilon_{sat} \rightarrow \left(\frac{T_{sat}}{2 - T_{sat}}\right)\upsilon_{inj}^{ball} = \left[\frac{1}{\upsilon_{inj}^{ball}} + \frac{1}{D_n/\ell}\right]^{-1}$$

wrap-up

1) Scattering reduces the MOSFET current.

2) For small V_{DS}, the effective of scattering is stronger than for high V_{DS}.

3) In the end, the expressions for I_D look remarkably similar to the traditional expressions, but with parameters that are more physical at the nanoscale.

references

For more discussion of scattering in nanoscale MOSFETs, see:

Mark Lundstrom and Jing Guo, *Nanoscale Transistors: Physics, Modeling, and Simulation*, Springer, New York, 2006.