Quantum Transport:
Atom to Transistor

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04.30.2004

Lecture 41: Coulomb Blockade

Ref. Chapter 3.4

Network for Computational Nanotechnology

NCN nanoHUB online simulations and more

When is transport in the CB regime?

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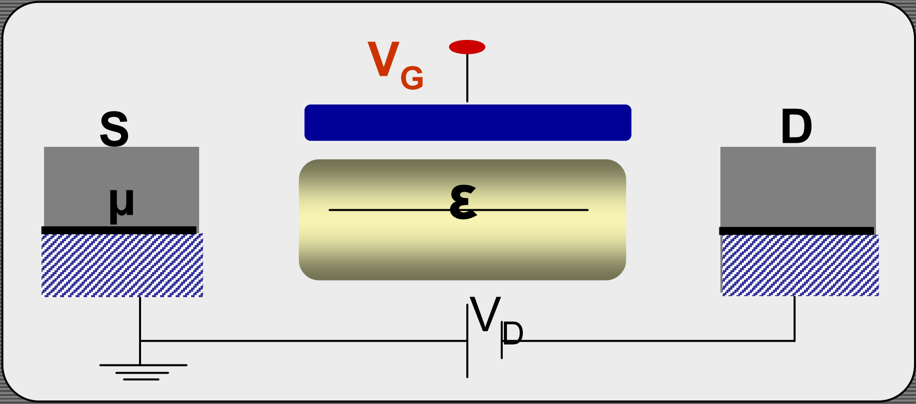

• What we've discussed in this course is the self consistent regime of transport which is often used in transistors and transistor like devices (i.e. Good conductors). How ever in memory type devices conductors are weakly coupled to the contacts; U_0 becomes comparable or lager than Gamma and hence coulomb blockade must be considered for the regime of transport.

• The way that we calculate the current is to solve the following equations self consistently.

$$U = U_L + U_0 \Delta N$$

$$N = \int dE D(E-U) \frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2}$$

• The current will be:

$$I = \int dE D(E-U) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} (f_1 - f_2)$$

• There are two types of experiments that are usually done on these devices.

• 1) Apply a very small drain voltage. And measure the number of electros or conductance as a function of gate voltage. 2) The other is to held V_G constant and measure the current as drain voltage changes.

One Level Device

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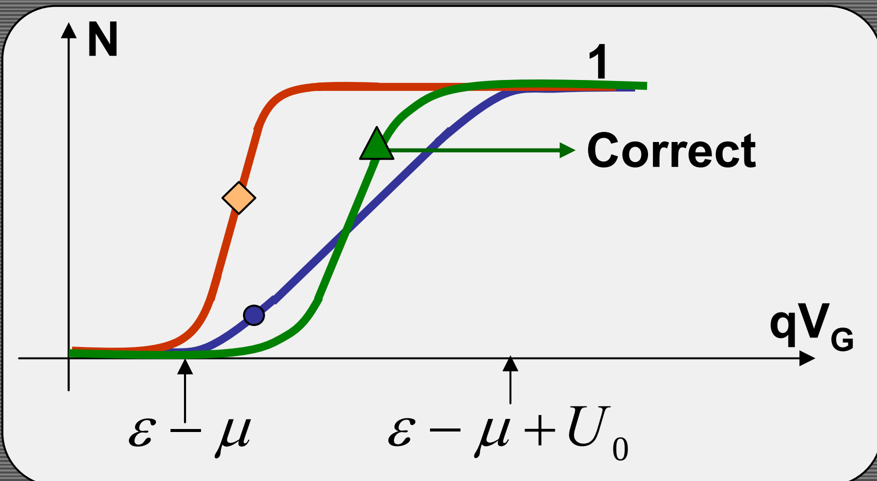
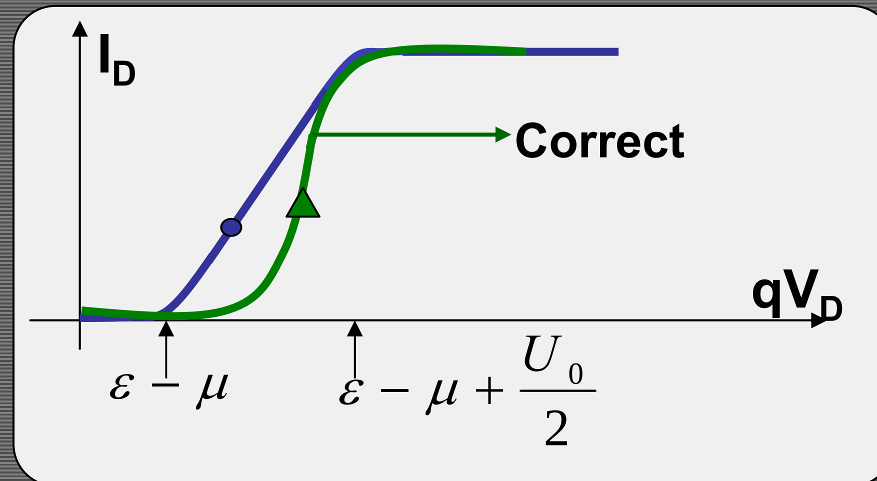
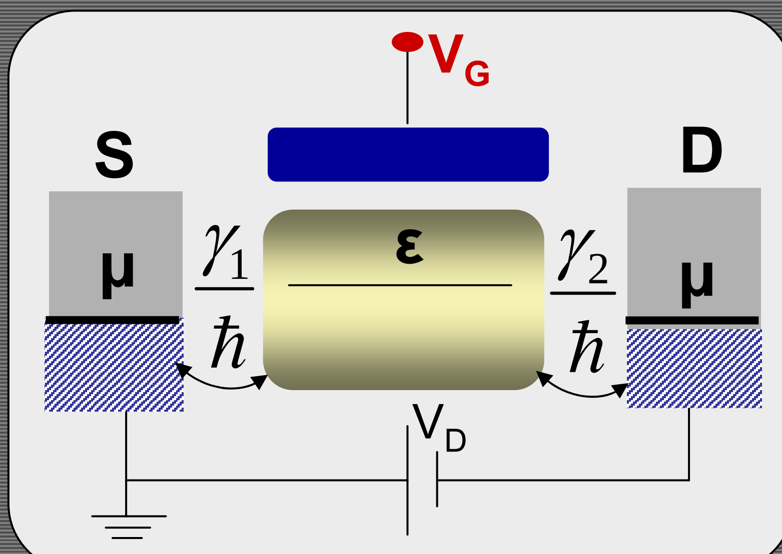




◇ Self Consistent Solution is not used

● Solved Self Consistently

▲ Experiment

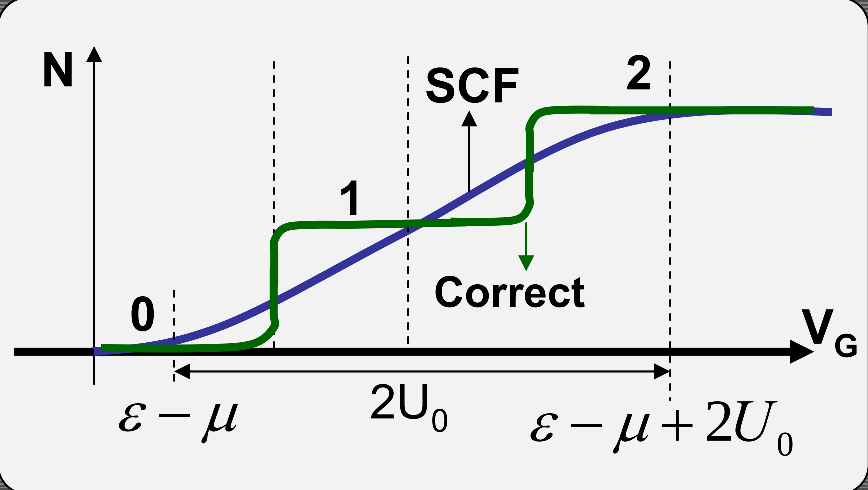


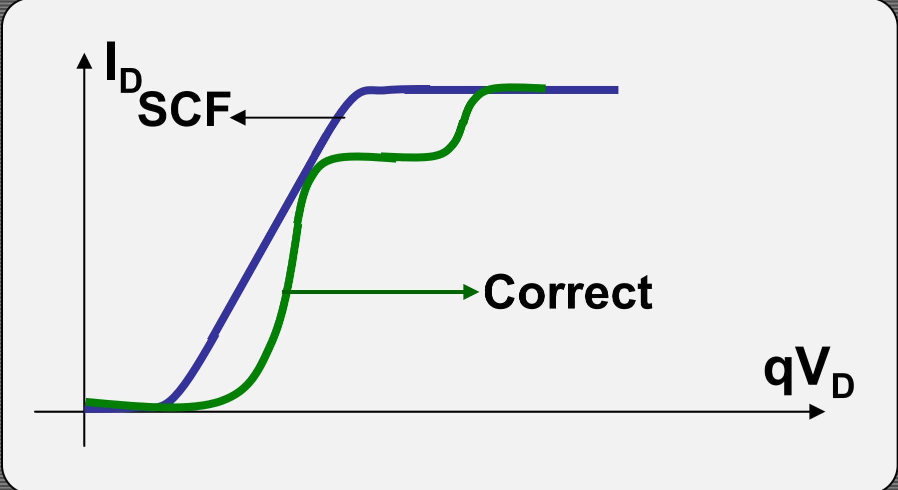
● Self Consistent Theory

▲ Experiment

Device with two Degenerate Levels

- To obtain the correct results we have to think of levels as one big system. See next page to see how this view point works.





Level to System

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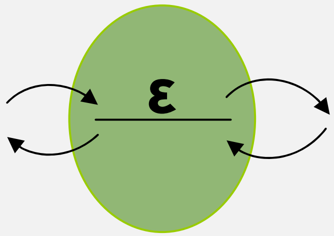
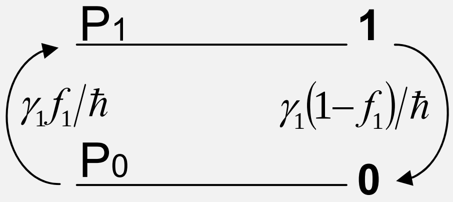


$P_1 + P_0 = 1$



• When an electron comes in, the system goes form 0 state to 1 state. When an electron goes out the system goes from 1 to 0 state.

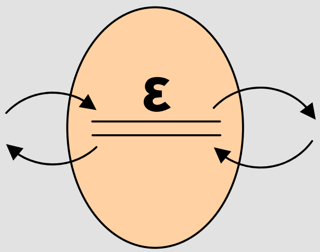
• At steady state, the rate at which an electron comes in must be equal to the rate at which electron goes out; therefore:

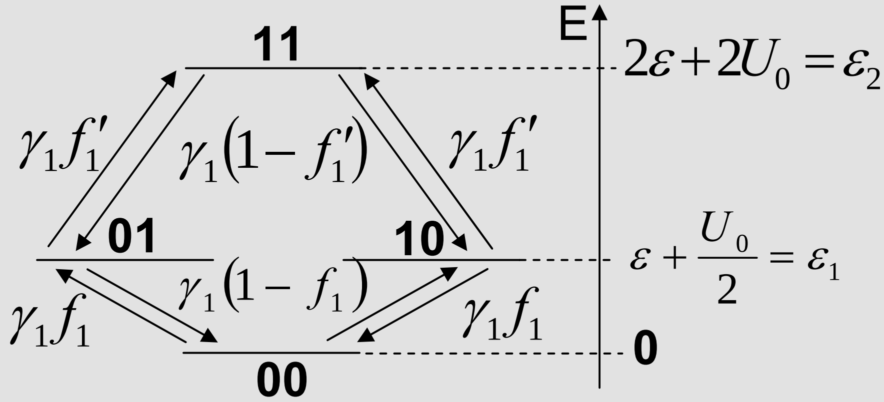
$$\frac{P_1}{P_0} = \frac{(\gamma_1/\hbar) f_1}{(\gamma_1/\hbar)(1-f_1)} = \frac{f_1}{1-f_1} \qquad \frac{a}{b}=\frac{c}{d} \Rightarrow \frac{a}{a+b} = \frac{c}{c+d} \Rightarrow \frac{P_1}{P_0+P_1} = \frac{f_1}{1} \Rightarrow \boxed{P_1 = f_1}$$

• This is exactly what we would expect. We got the same answer from two different pictures.

• <u>A note about energy of state 1</u>: The energy of the whole system changes by ε plus the amount of stored energy in the capacitor when an electron enters the system. The amount of energy in the capacitor can be found by looking at the energy stored in the capacitor that is formed by the electron and the positive charges on the gate. This energy is $Q^2/2C$. For this case, it becomes $q^2/2C$. Since we've defined U_0 as q^2/C_E, the energy stored in the capacitor is $U_0/2$; hence the total energy of state 1 becomes: $\varepsilon + U_0/2$

Two level Multi Particle Picture





$$f_1 = \frac{1}{e^{(\varepsilon_1-\mu)/kT}+1} \Rightarrow \frac{1}{f_1} = e^{(\varepsilon_1-\mu)/kT}+1 \Rightarrow \frac{1-f_1}{f_1} = e^{(\varepsilon_1-\mu)/kT} \Rightarrow \frac{P_{01}}{P_{10}} = \frac{f_1}{1-f_1} = e^{(\varepsilon_1-\mu)/kT}$$

- General Principle of Equilibrium Statistical Mechanics

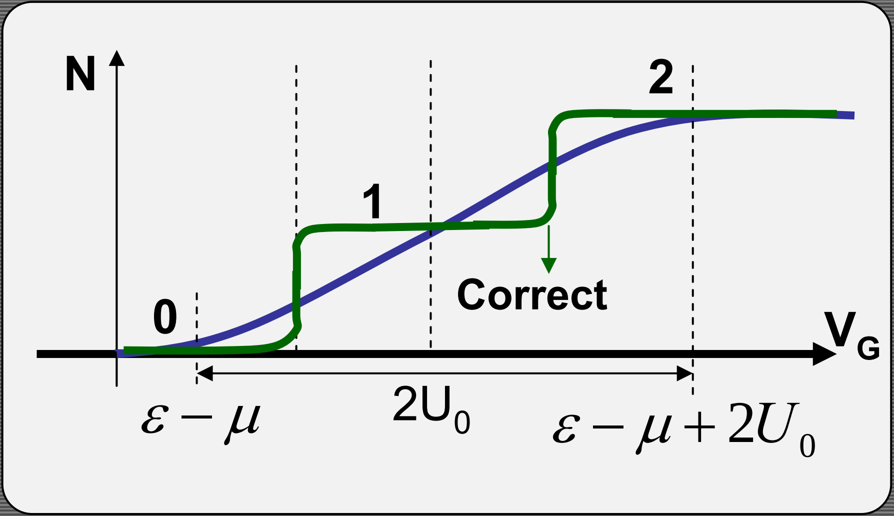
$$P_\alpha = \frac{1}{Z} e^{(E_\alpha - \mu N_\alpha)/kT}$$

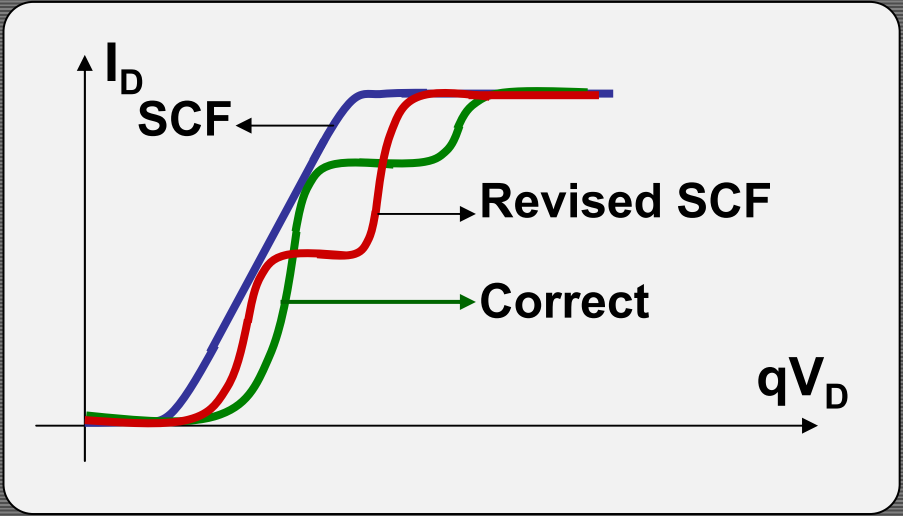
- Given any system the probability that a particular state is occupied is proportional to the exponential term in the above equation. Z is called the Partition Function and can be found by setting the sum of all probabilities equal to 1.

What's the problem with using Multi Particle method?

- Are there any issues or problems with using this method?
- Yes; the size of it. The number of possible states goes as 2^N where N is the number of states. It can easily be seen that this quickly goes out of hand. This is one of the reasons we don't use this method.
- Is there any other problem with it?
- Yes, we don't know how to incorporate broadening into this picture. For small broadening it will be fine but for large broadening you can't use it.
- This is why people try to come up with a simple one particle picture. One thought is to modify the self consistent field such that it would mimic the correct results. See next page for an example.

Modified SCF





- If we wanted to get the correct plot, we could say that the level in the channel is at $\varepsilon + \frac{U_0}{2}$ instead of being at ε. Then you could argue that every level only feels the potential due to electrons in other levels not the ones in itself. So then the potential is not one quantity that all levels feel; rather for calculating the potential for each level (i) the contributions due to electrons in that level should be left out. The potential becomes:

$U_i = U_L + U_0(\Delta N - \Delta n_i)$. But this is not really a solution over all because the I-V curve will not come out to be right.

Summary

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Summary

• If the size gets big it is hard to use the multi electron picture. So people try to modify the one electron picture to get the correct results. They take out the self interaction from the self consistent field. This works for equilibrium; how ever for non-equilibrium we yet don't know of a simple way of doing things right.

• Be aware that there is this whole other regime of transport. What we've talked in this course is self consistent theory which works when U_0 is much smaller than γ or kT. If U_0 gets larger or becomes comparable to γ and kT , then Coulomb blockade has to be considered for the regime of transport and the right way to do it is to use the multi electron picture.

• In the intermediate regime a lot of strange things happen and no one really has a good understanding of how it really works.