ECE 595, Section 10
Numerical Simulations
Lecture 13: Programming with FFTW

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Outline

- Recap from Wednesday
- Rationale for FFTW
- Planning DFTs
- Executing DFTs
  - Basic interface
  - Advanced interface
- Application examples

Recap from Wednesday

- Real FFTs
- Multidimensional FFTs
- Applications:
  - Correlation measurements
  - Filter diagonalization method

Rationale for FFTW

- In past, most codes focused exclusively on data sets of length 2^m
- Required padding can → 2x runtime
- Processing pure real data can → 2x runtime
- Ignoring symmetry/anti-symmetry → 2x runtime
- How do we account for all of these possibilities with a single software package?

Planning in FFTW

- “Most people don’t plan to fail; they fail to plan” – John L. Beckley
- Planning our FFT’s before we perform them can make an enormous difference
- FFTW uses a set of short codes, or “codelets,” which can be called as needed by the planner
- FFTW also compares the different possibilities using dynamic programming

Planning in FFTW

- Execution time can be found in different ways:
  - Estimate: uses heuristics to roughly determine
  - Measure: makes direct test runs with multiple candidate plans
- Execution time may not be directly related to the number of operations
- Instruction-level parallelism can play a critical role in enhancing performance – for example: SIMD

Planning in FFTW

```
#include <fftw3.h>
...
{
    fftw_complex *in, *out;
    fftw_plan p;
    ...
    in = (fftw_complex*) fftw_malloc(sizeof(fftw_complex) * N);
    out = (fftw_complex*) fftw_malloc(sizeof(fftw_complex) * N);
    p = fftw_plan_dft_1d(N, in, out, FFTW_FORWARD, FFTW_ESTIMATE);
    fftw_execute(p);
```

Sign in exponent

Method of estimating execution time

1D Real DFT's

```
#include <fftw3.h>
...
{
    double *in, *final;
    fftw_complex *out;
    fftw_plan p1, p2;
    ...
    p1 = fftw_plan_dft_r2c_1d(N, in, out, FFTW_MEASURE);
    p2 = fftw_plan_dft_c2r_1d(N, out, final, FFTW_MEASURE);
    fftw_execute(p1);
    fftw_execute(p2);
```

Transform forward

Transform back

Method of estimating execution time

Multidimensional Real DFTs

```
#include <fftw3.h>
...
{
    double *in, *final;
    fftw_complex *out;
    fftw_plan p1, p2;
    ...
    p1 = fftw_plan_dft_r2c_2d(n0, n1, in, out, FFTW_PATIENT);
    p2 = fftw_plan_dft_c2r_2d(n0, n1, out, final, FFTW_PATIENT);
    fftw_execute(p1);
    fftw_execute(p2);
```

2D forward transform

2D backwards transform (un-normalized)

Method of estimating execution time

Multidimensional Complex DFT’s

```
#include <fftw3.h>
...
{
    double fftw_complex *in, *out, *final;
    fftw_plan p1, p2;
    ...
    p1 = fftw_plan_dft_2d(n0, n1, in, out, FFTW_EXHAUSTIVE);
    p2 = fftw_plan_dft_2d(n0, n1, out, final, FFTW_EXHAUSTIVE);
    fftw_execute(p1);
    fftw_execute(p2);
```

2D forward transform

2D backwards transform (un-normalized)

Method of estimating execution time

Learning from Your Experience

- **Wisdom** allows one to compute good plans once and save them to disk:

  ```
  fftw_export_wisdom_to_filename( “wise-dft.wis” );
  ```

- Can then restore the wisdom next time with:

  ```
  fftw_import_wisdom_from_filename( “wise-dft.wis” );
  ```

- While wisdom accumulates over time, one can discard it with:

  ```
  fftw_forget_wisdom();
  ```

Example: Beam Propagation

- Starting from the Helmholtz equation:

$$-\nabla^2\psi = \left(\frac{n\omega}{c}\right)^2 \psi$$

- One can assume a solution of the form:

$$\psi = \phi e^{-j\beta z}$$

- Where ϕ is slowly varying, which gives rise to:

$$-\nabla^2\phi + 2j\beta\hat{z}\cdot\nabla\phi = k_\perp^2\phi$$

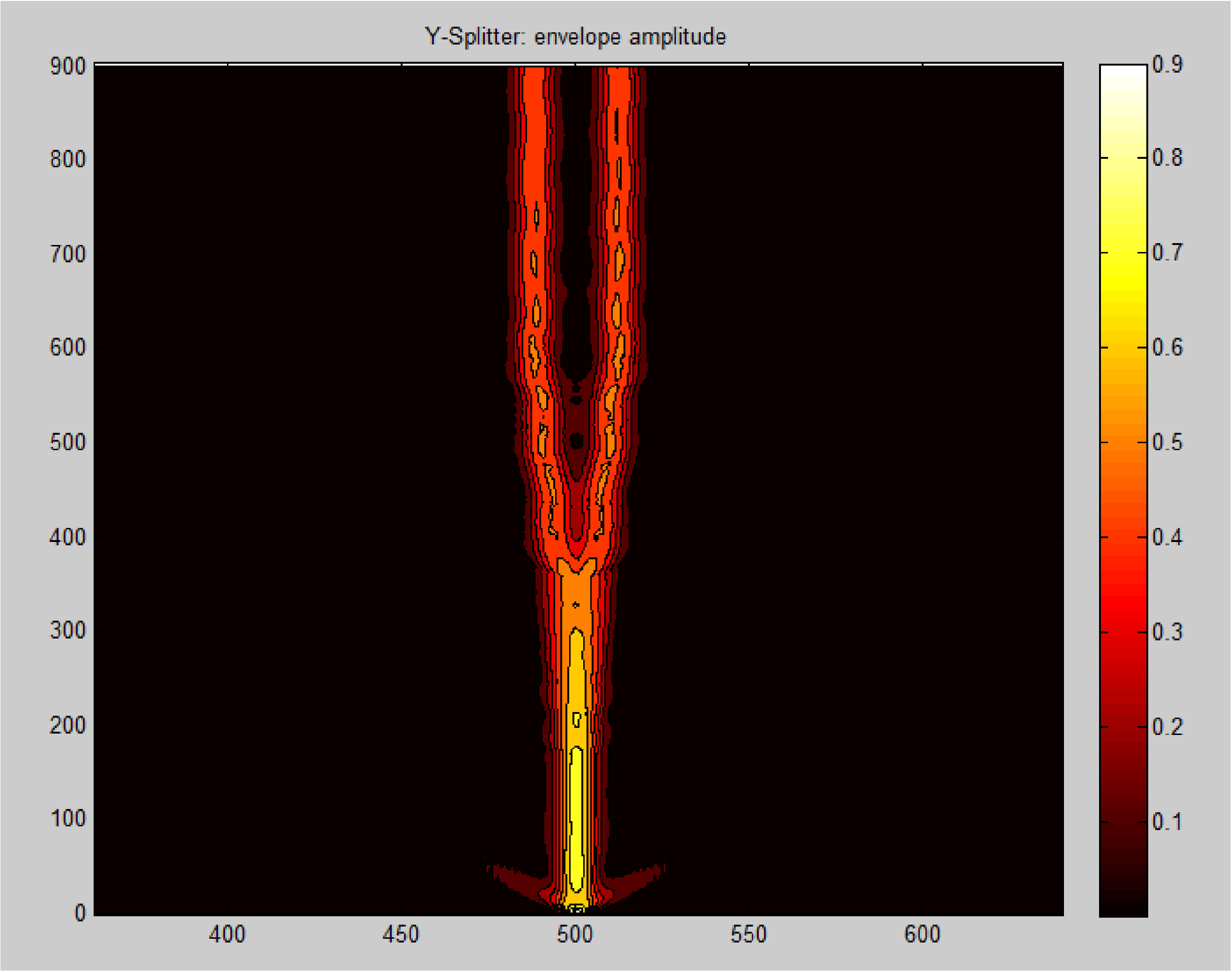
Example: Beam Propagation

- BPM closely resembles the nonlinear Schrodinger equation, which describes a broad class of problems
- For now, we'll focus on direct applications in optics
- Can solve in real-space or Fourier-space

Example: Beam Propagation

```
[xx,yy] = meshgrid([xa:del:xb-del],[1:1:zmax]);
mode = A*exp(-((x+x0)/W0).^2);   % Gaussian pulse
dftmode = fix(fft(mode));         % DFT of Gaussian pulse
zz = imread('ybranch.bmp','BMP'); %Upload image with the profile
…
phase1 = exp((i*deltaz*kx.^2)./(nbar*k0 + sqrt(max(0,nbar^2*k0*2
- kx.^2))));
for k = 1:zmax,
    phase2 = exp(-(od + i*(n(k,:) - nbar)*k0)*deltaz);
    mode = ifft((fft(mode).*phase1)).*phase2;
    zz(k,:) = abs(mode);
end
```

Example: Beam Propagation



Next Class

- Is on Monday, Feb. 11
- Will discuss beam propagation method
- Recommended reading: Obayya, Sections 2.2-2.6