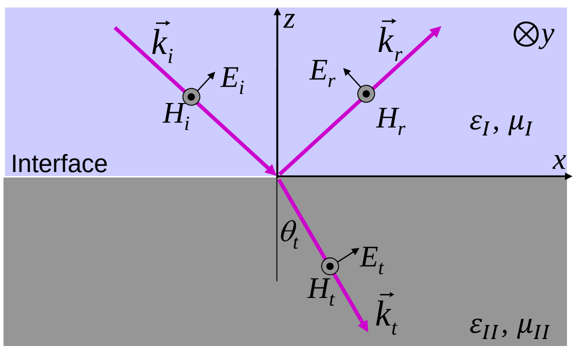
09/17/12, Lecture Note (Nick Fang)

A. Recap Fresnel coefficients, Impedance Approach

We now consider a problem of light beam incident obliquely at a flat interface (z=0) between two medium. Let ε_I, μ_I denote the material properties of the medium covering the z>0 half space, and $\varepsilon_{II}, \mu_{II}$ denote material properties of that z<0 half space accordingly. As we noted earlier, the light field could be decomposed of two distinct polarizations, and let's consider the cases involving these two polarizations separately.



Taking P-polarization as example, the H field of the input plane wave in frequency (ω)*domain* is:

$$\overrightarrow{H_i} = -\hat{y}H_i \exp(ik_{x,i}x - ik_{z,i}z),\ H_x = H_z = 0$$

From continuity of tangential fields at the boundary we obtain x and y components of E and H field should be continuous:

$$H_{y,i}(z=0) + H_{y,r}(z=0) = H_{y,t}(z=0)$$
$$E_{x,i}(z=0) + E_{x,r}(z=0) = E_{x,t}(z=0)$$

Using impedance to connect the E_x components to H_y at boundary:

$$E_{x,i} = -\frac{k_{z,i}}{\omega\varepsilon_0\varepsilon_I}H_{y,i} = -Z_I(k,\omega)H_{y,i}$$

$$E_{x,r} = +\frac{k_{z,r}}{\omega\varepsilon_0\varepsilon_I}H_{y,r} = Z_I(k,\omega)H_{y,r} \qquad \text{(note the +ive sign again!)}$$

$$E_{x,t} = -\frac{k_{z,t}}{\omega\varepsilon_0\varepsilon_{II}}H_{y,t} = -Z_{II}(k,\omega)H_{y,t}$$

So we arrive at:

$$Z_I H_i - Z_I H_r = Z_{II} H_t$$

Together with $$H_i + H_r = H_t$$

This helps to determine the ***Fresnel coefficients r, t:***

$$t_H = \frac{H_t}{H_i} = \frac{2Z_I}{Z_I + Z_{II}} = \frac{2\dfrac{k_{z,i}}{\omega\varepsilon_0\varepsilon_I}}{\dfrac{k_{z,i}}{\omega\varepsilon_0\varepsilon_I} + \dfrac{k_{z,t}}{\omega\varepsilon_0\varepsilon_{II}}}$$

$$r_H = \frac{H_r}{H_i} = \frac{Z_I - Z_{II}}{Z_I + Z_{II}} = \frac{\dfrac{k_{z,i}}{\omega\varepsilon_0\varepsilon_I} - \dfrac{k_{z,t}}{\omega\varepsilon_0\varepsilon_{II}}}{\dfrac{k_{z,i}}{\omega\varepsilon_0\varepsilon_I} + \dfrac{k_{z,t}}{\omega\varepsilon_0\varepsilon_{II}}}$$

Note that such impedance are polarization dependent too. In the case of TE waves, the impedance becomes:

$$Z(k,\omega) = \frac{\omega\mu\mu_0}{k_z}$$

B. Total Internal Reflection, Evanescent Waves

For arbitrary frequency of spatial variation k_x at the interface, we may define a complex wavenumber $k_z = \sqrt{\varepsilon\mu\left(\frac{\omega}{c_0}\right)^2 - k_x^2}$. When $k_x^2 > \varepsilon\mu\left(\frac{\omega}{c_0}\right)^2$, the wavenumber k_z becomes imaginary, therefore defining an 'evanescent' field that decays away from the interface.

e.g. $\overrightarrow{H_t} = -\hat{y}H_t\exp(ik_{x,t}x + \gamma z)$, (z<0)

$$\gamma = \pm\sqrt{k_x^2 - \varepsilon\mu\left(\frac{\omega}{c_0}\right)^2}$$

For a system that does not involve gain, we select the proper sign of γ to ensure the field at z=∞ is zero.

If both materials are loss-free, and only one k_z component is imaginary(say, $\varepsilon_I\mu_I\left(\frac{\omega}{c_0}\right)^2 > k_x^2 > \varepsilon_{II}\mu_{II}\left(\frac{\omega}{c_0}\right)^2$, then the corresponding impedance Z_{II} is imaginary while Z_I is real. We can see the reflection coefficient *r* become a complex number, while the amplitude |r|=1. This is referred as **total internal reflection**.

Using P-polarization as example, we see that when $\varepsilon_I \mu_I \left(\frac{\omega}{c_0}\right)^2 > k_x^2 > \varepsilon_{II} \mu_{II} \left(\frac{\omega}{c_0}\right)^2$:

$$r_H = \frac{\frac{k_{z,i}}{\varepsilon_I} - \frac{i\gamma}{\varepsilon_{II}}}{\frac{k_{z,i}}{\varepsilon_I} + \frac{i\gamma}{\varepsilon_{II}}} = \frac{\varepsilon_{II} k_{z,i} - i\varepsilon_I \gamma}{\varepsilon_{II} k_{z,i} + i\varepsilon_I \gamma} = \exp(-2i\delta)$$

$$\delta = \arctan\left(\frac{\varepsilon_I \gamma}{\varepsilon_{II} k_{z,i}}\right) = \arctan\left(\frac{Im(Z_{II})}{Re(Z_I)}\right).$$

The physical meaning of phase δ

Under total internal reflection we see that the reflected beam is retarded by a phase 2δ with respect to the incident field at the interface. Geometrically this is visualized as a lateral and vertical shift of the beam center (also known as Goos-Hanchen Effect, named after their seminal experiments in 1947, see homework 1). A refined experiment was reported by:

- **Lotsch, H. K. V.** 1970, Beam displacement at total reflection: the Goos–Hänchen effect, I–IV. Optik 32, 116–137.

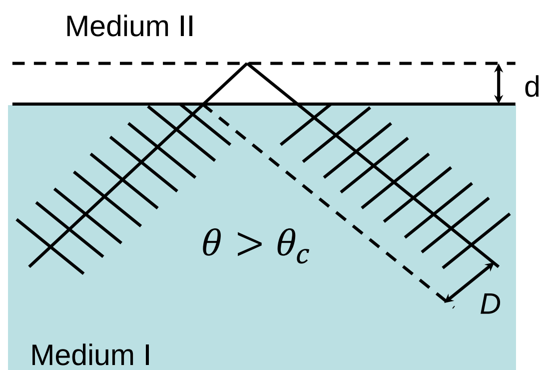
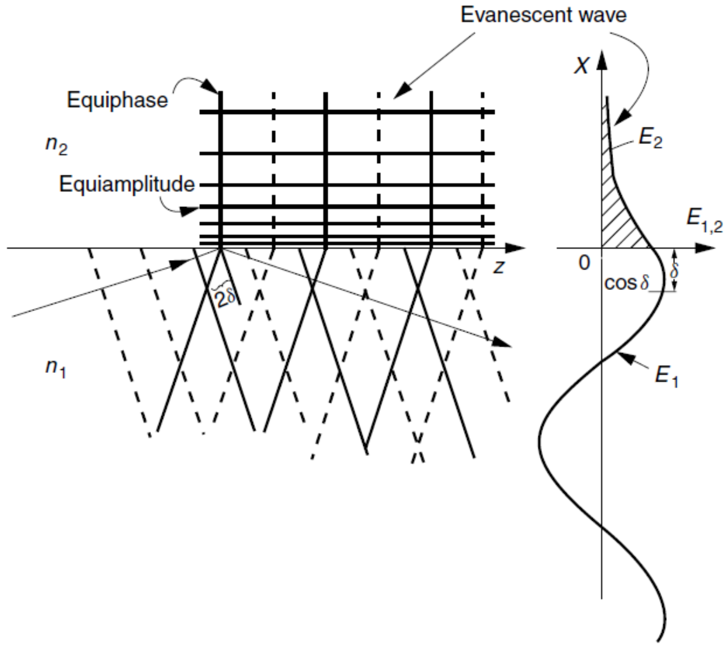


Illustration of Goos-Hanchen effect.

A related effect, transverse shift of circularly polarized light beams upon reflection, (Fedorov–Imbert shift) was reported by:

- **Costa de Beauregard, C. & Imbert, C.** 1972, Quantized longitudinal and transverse shifts associated with total internal reflection. Phys. Rev. Lett. 28, 1211–1213. (doi:10.1103/PhysRev Lett.28.1211)

Evanescent waves:



Like the case of reflection, we can show that the evanescent field

$$\mathrm{H_t} = \mathrm{t}_H \mathrm{H_i} \exp\left(ik_{x,t}x + \gamma z\right) = |\mathrm{t}_H| \mathrm{H_i} \exp\left(ik_{x,t}x - i\delta + \gamma z\right)$$

This is an indication that the presence of the field at the boundary caused the reflected wave to lag behind. Indeed, it is impressive to see such pronounced delay of light field, although the evanescent field may only penetrate in the sparse medium by a fraction of the wavelength. You may make an analogy to "no-slip" boundary conditions in fluid mechanics, in which a bullet shaped laminar flow profile can be developed. However when the boundary is allowed to slip, we no longer have zero flow velocity at the interface and the velocity profile is closer to a flat hat shape.

Graphical methods for Evanescent waves:

Consider the fact that the lateral momentum k_x is conserved, we may focus on the difference of k_z across the interface.

$$k_{z,I}{}^2 + k_x{}^2 = n_I{}^2 \left(\frac{\omega}{c}\right)^2$$

$$k_x{}^2 - \gamma^2 = n_{II}{}^2 \left(\frac{\omega}{c}\right)^2$$

Therefore,

$$k_{z,I}{}^2 + \gamma^2 = (n_I^2 - n_{II}^2)\left(\frac{\omega}{c}\right)^2$$

From the above we can start to draw a circle that connects k_z and γ. Given any incident angle from the prism, we first use Descartes sphere (red circle) to determine k_z, and then project onto the small blue circle to find γ. The larger γ, the faster the field decays away from the interface. You may also observe that, for small k_x (thus the incident angle), the k_z goes outside the blue circle, indicating there is no evanescent field present.