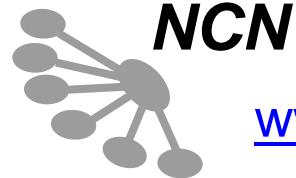


EE-612:

Lecture 3

1D MOS Electrostatics

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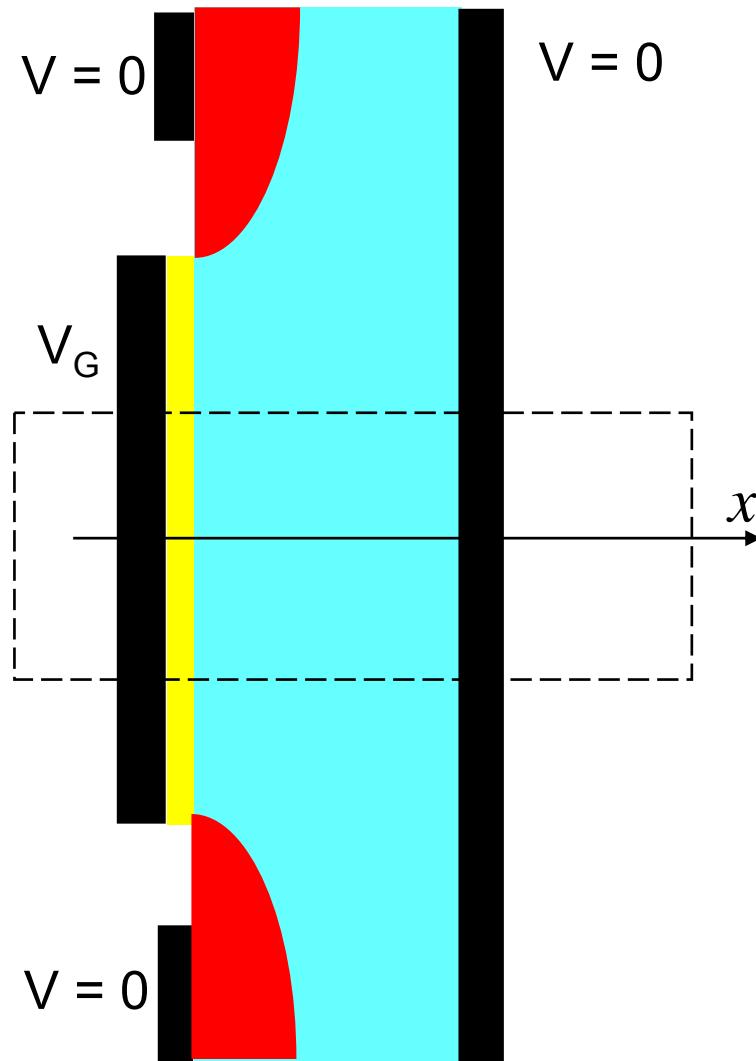
Lundstrom EE-612 F06

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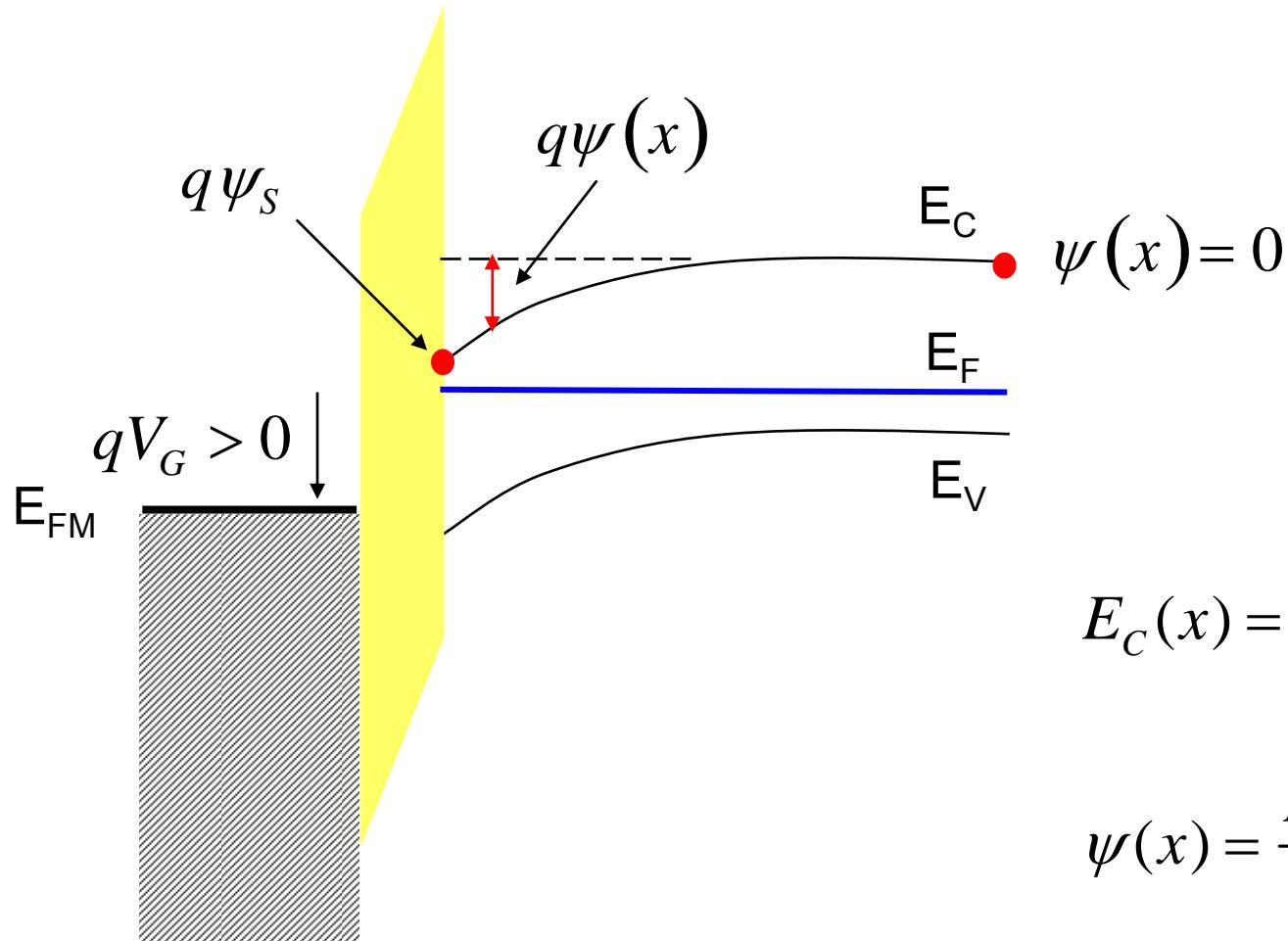
outline

- 1) Introduction
- 2) **'Exact' solution (bulk)**
- 3) Approximate solution (bulk)
- 4) Approximate solution (ultra-thin body)
- 5) Summary

1D MOS Electrostatics ($L \gg T_{ox}$)



electrostatic potential



$$E_C(x) = \text{constant} - q\psi(x)$$

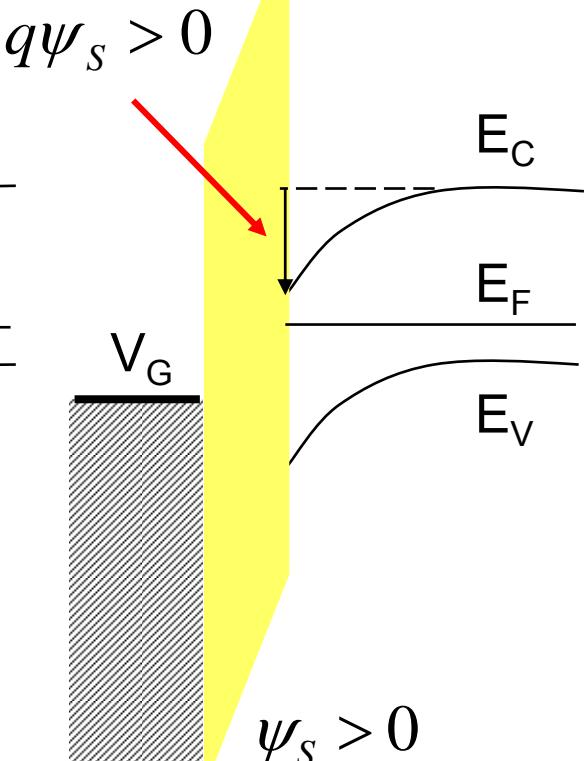
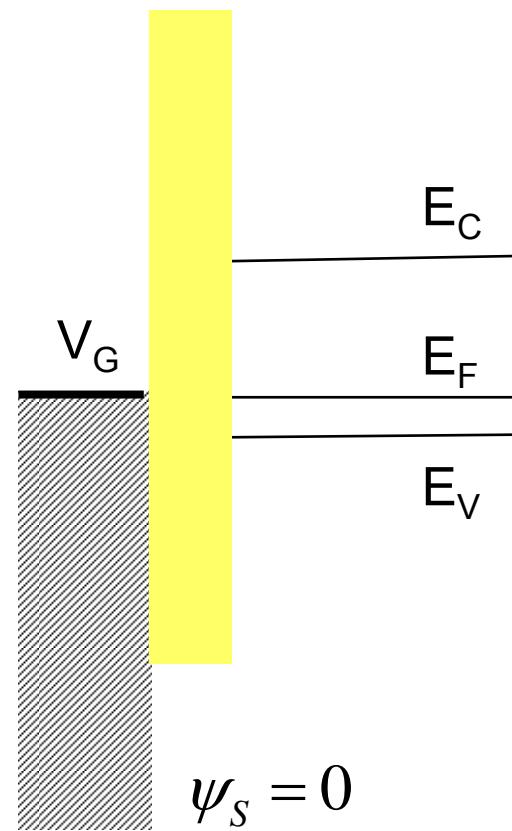
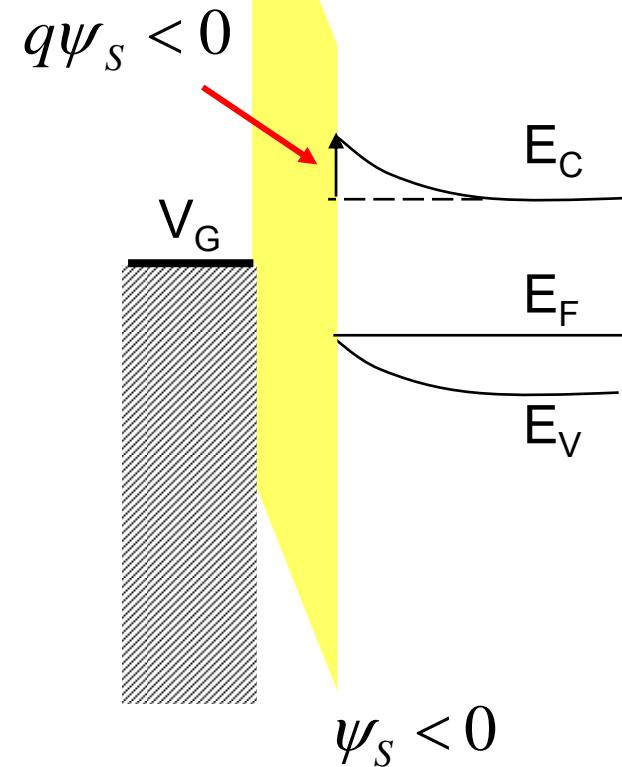
$$\psi(x) = \frac{E_C(\infty) - E_C(x)}{q}$$

MOS Electrostatics

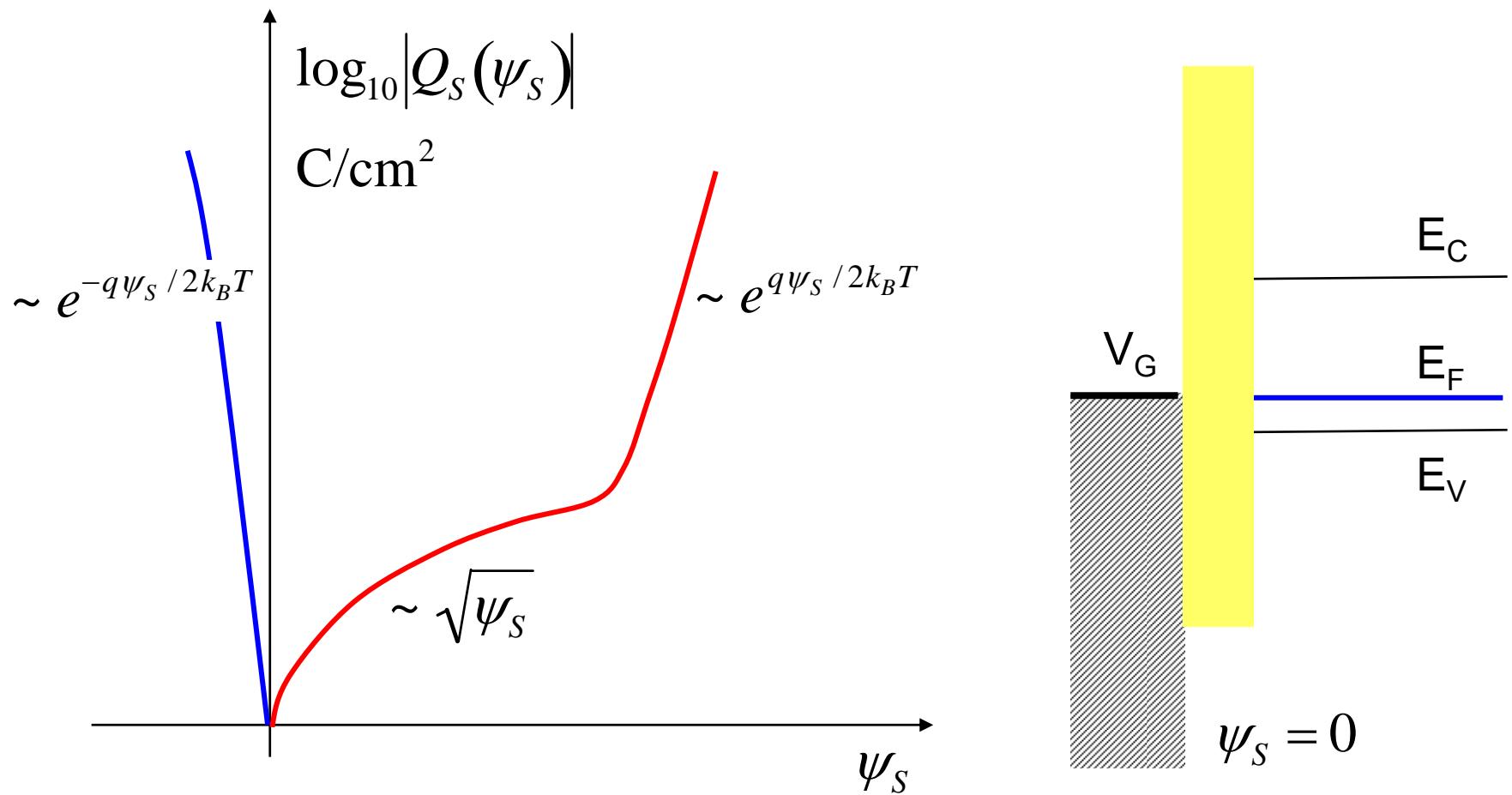
accumulation

flat band

depletion/
inversion



MOS Electrostatics



'Exact' solution of $Q_S(\psi_S)$

$$\nabla \bullet \vec{D} = \rho$$

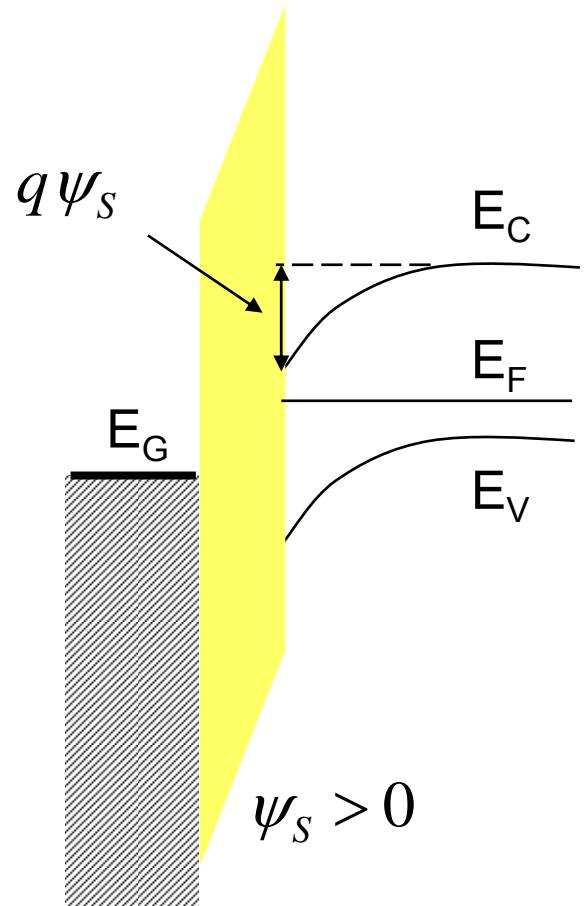
$$\nabla \bullet (\vec{J}_n / -q) = (G - R)$$

$$\nabla \bullet (\vec{J}_p / q) = (G - R)$$

equilibrium



$$\frac{d^2\psi}{dx^2} = \frac{-q}{\epsilon_{Si}} [p_0(x) - n_0(x) + N_D^+ - N_A^-]$$



Poisson-Boltzmann Equation

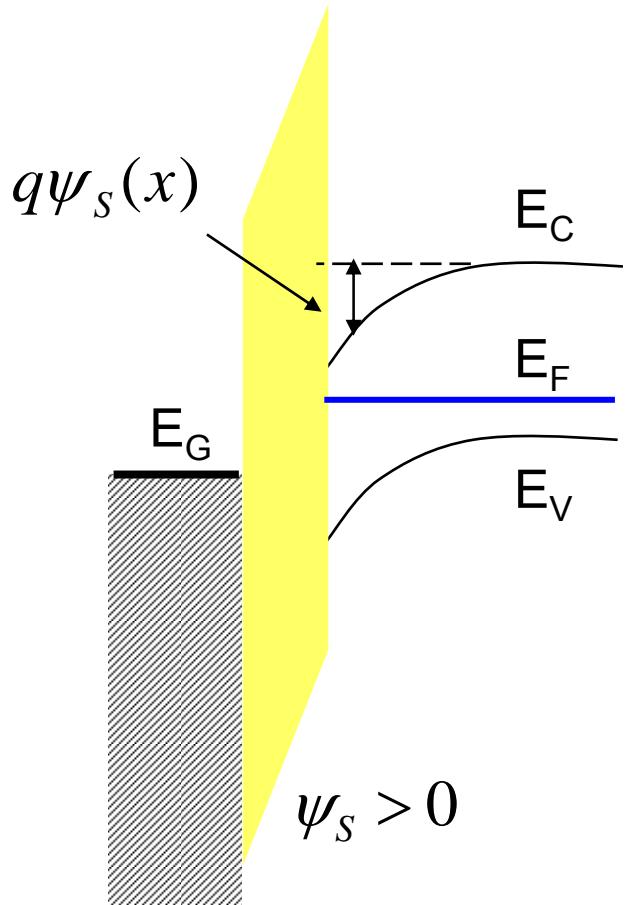
$$n_0(x) = N_C e^{[E_F - E_C(x)]/k_B T}$$

$$p_0(x) = N_V e^{[E_V(x) - E_F]/k_B T}$$

$$E_C(x) = \text{constant} - q\psi(x)$$

$$n_0(x) = n_B e^{q\psi(x)/k_B T}$$

$$p_0(x) = p_B e^{-q\psi(x)/k_B T}$$



Poisson-Boltzmann Equation

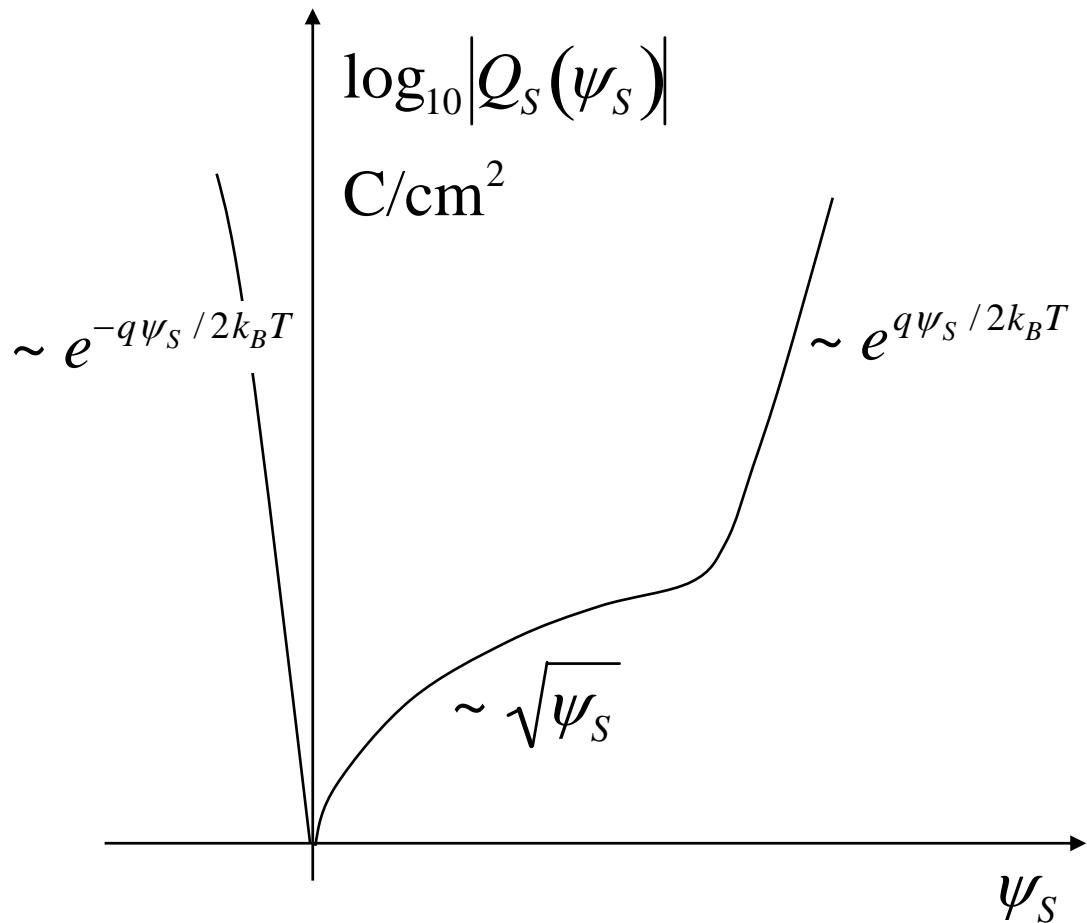
$$\left. \frac{d^2\psi}{dx^2} \right|_{x \rightarrow \infty} = 0 = \frac{-q}{\epsilon} \left[p_B - n_B + N_D^+ - N_A^- \right]$$

$$(N_D^+ - N_A^-) = -p_B + n_B$$

$$\frac{d^2\psi}{dx^2} = \frac{-q}{\epsilon} \left[(p_0(x) - p_B) - (n_0(x) - n_B) \right]$$

$$\frac{d^2\psi}{dx^2} = \frac{-q}{\epsilon} \left[N_A \left(e^{-q\psi/k_B T} - 1 \right) - \frac{n_i^2}{N_A} \left(e^{q\psi/k_B T} - 1 \right) \right]$$

Poisson-Boltzmann Equation



see:

Taur and Ning, pp. 63-65

and

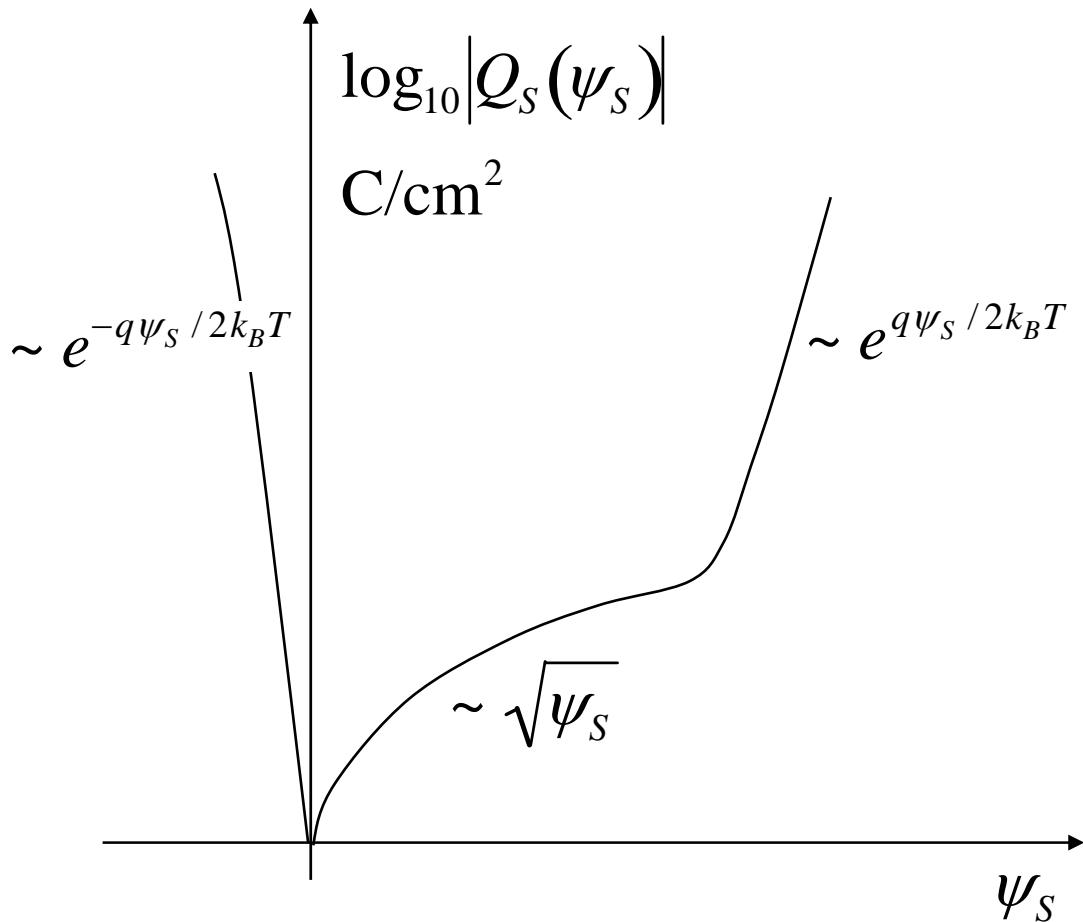
Lundstrom's notes on the
Poisson-Boltzmann equation.

$$\frac{d^2\psi}{dx^2} = \frac{-q}{\epsilon} \left[N_A \left(e^{-q\psi/k_B T} - 1 \right) - \frac{n_i^2}{N_A} \left(e^{q\psi/k_B T} - 1 \right) \right]$$

outline

- 1) Introduction
- 2) ‘Exact’ solution (bulk)
- 3) Approximate solution (bulk)**
- 4) Approximate solution (ultra-thin body)
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1D MOS electrostatics



need:

$Q_s(\psi_s)$ for capacitance

$Q_i(\psi_s)$ for current

Can we understand the essential features of the PB solution simply?

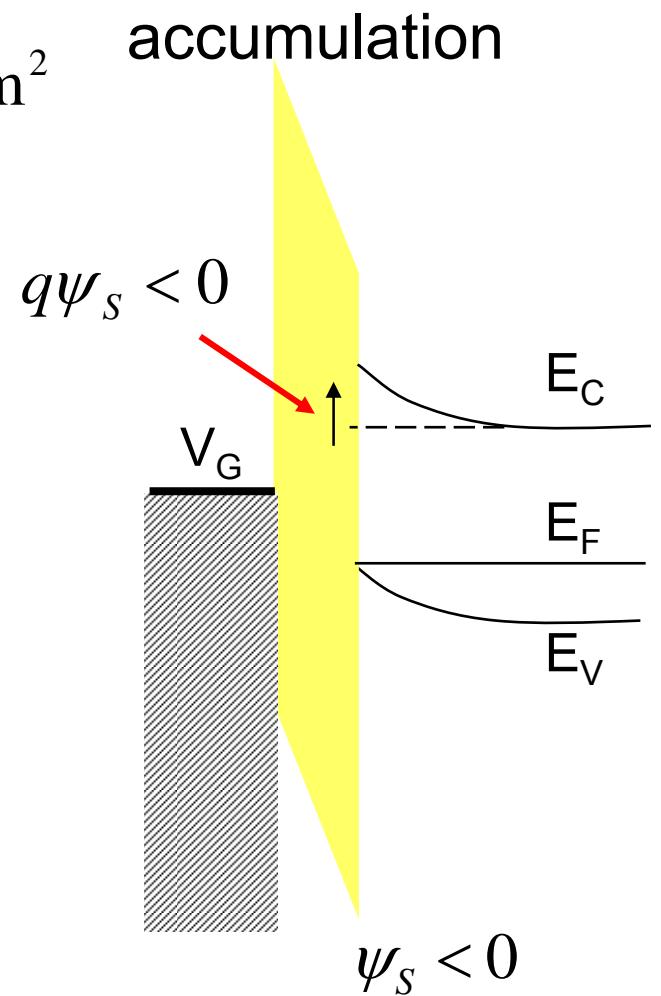
i) accumulation

$$Q_s = \int_0^\infty [(p(x) - p_B) - (n(x) - n_B)] dx \text{ C/cm}^2$$

$$\approx \int_0^\infty \Delta p(x) dx$$

$$\Delta p(x) = (p(x) - p_B) = p_B (e^{-q\psi/k_B T} - 1)$$

$$Q_s = q \int_0^\infty \Delta p(x) dx ; \quad q p_B \int_0^\infty e^{-q\psi/k_B T} dx$$



i) accumulation

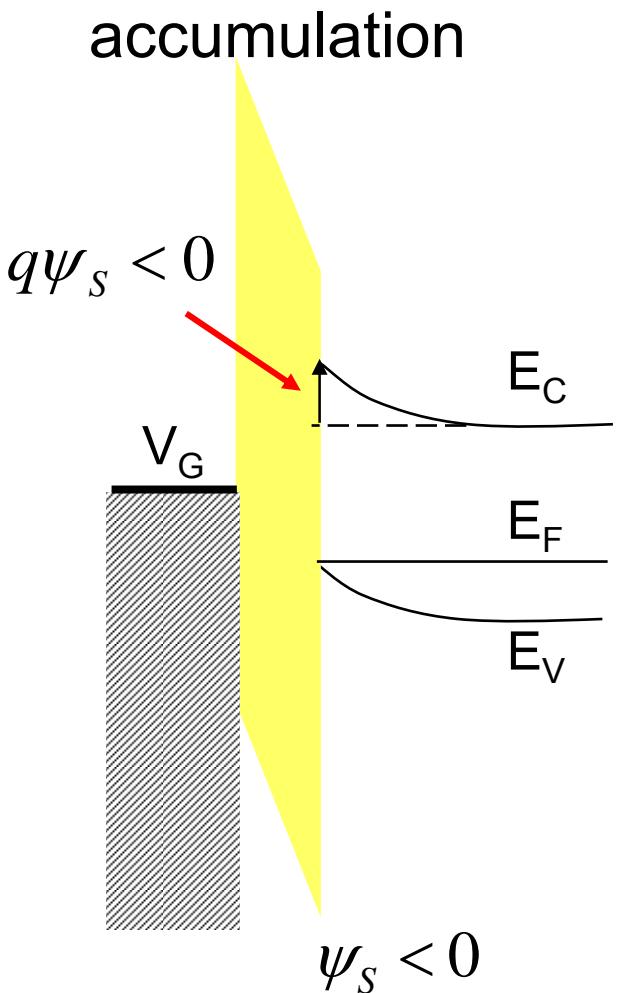
$$Q_S = qp_B \int_0^{\infty} e^{-q\psi/k_B T} dx$$

$$Q_S \square qp_B \int_{\psi_S}^0 \frac{e^{-q\psi/k_B T}}{d\psi/dx} d\psi$$

$$\square \frac{qp_B}{E_S} \int_0^{\psi_S} e^{-q\psi/k_B T} d\psi$$

$$Q_S \square \frac{qp_B}{-E_S} e^{-q\psi_S/k_B T} \frac{k_B T}{q}$$

$$\square qp(0) \left(\frac{k_B T / q}{|E_S|} \right) = \text{circled term}$$

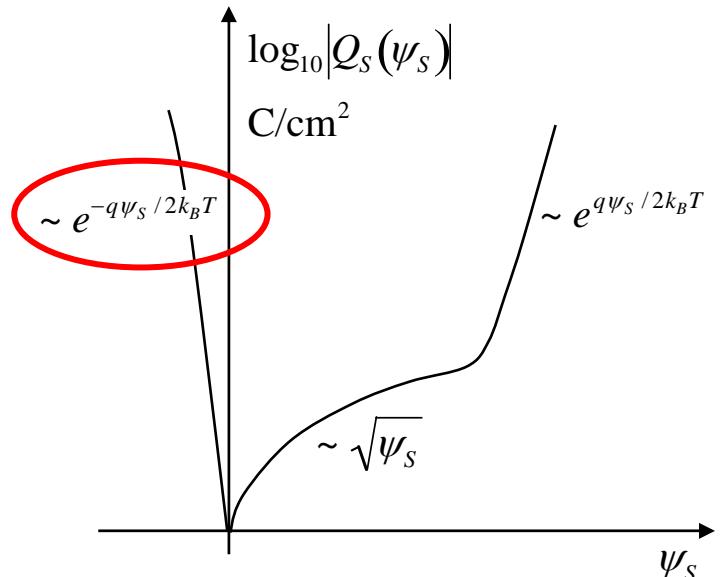


i) accumulation

$$Q_S \square \frac{qp_B}{E_S} e^{-q\psi_S/k_B T} \frac{k_B T}{q} \quad (1)$$

$$D_S = \varepsilon_S E_S = -Q_S$$

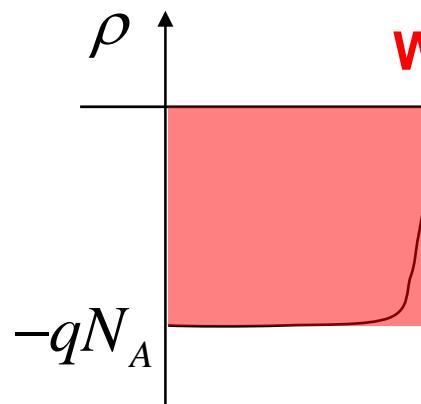
$$Q_S \square \frac{q\varepsilon_{Si} p_B}{Q_S} e^{-q\psi_S/k_B T} \frac{k_B T}{q}$$



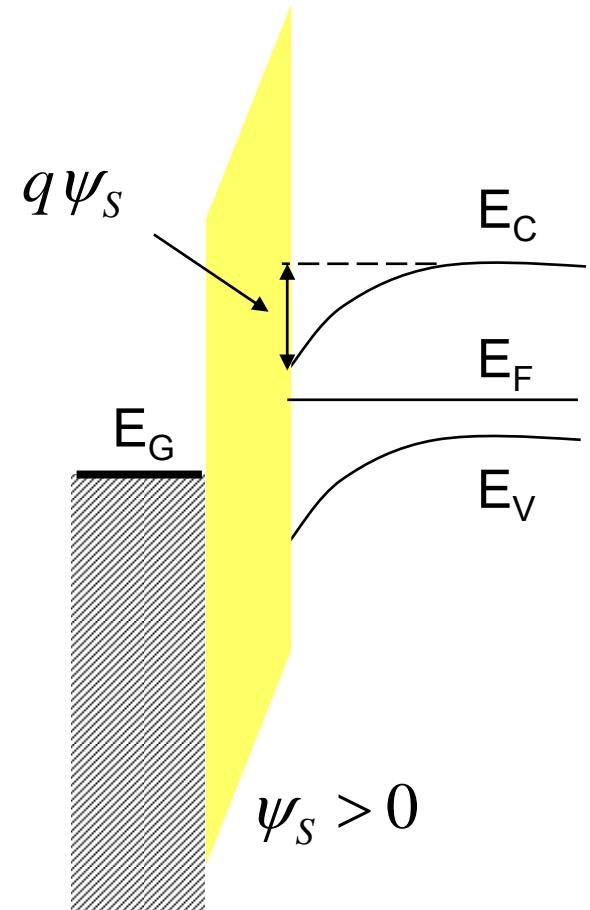
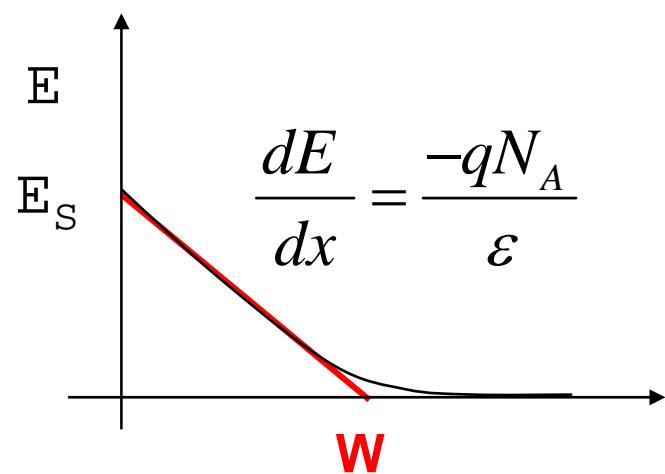
$$Q_S \square \sqrt{\varepsilon_{Si} p_B k_B T} e^{-q\psi_S/2k_B T} = qN_A L_D e^{-q\psi_S/2k_B T}$$

$$L_D = \sqrt{\frac{\varepsilon_{Si} k_B T}{q^2 N_A}} \quad \text{'extrinsic Debye Length'}$$

ii) depletion

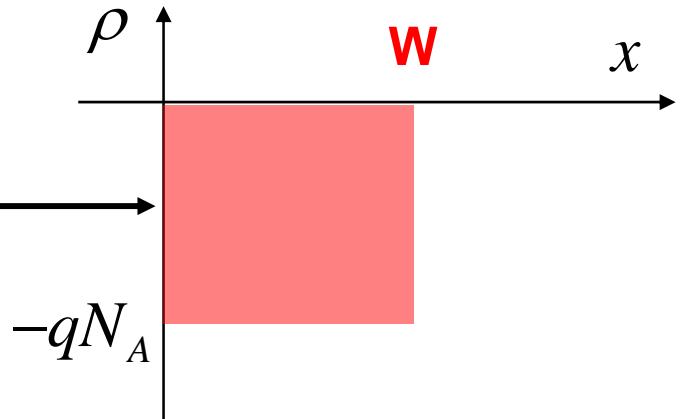


depletion approximation



ii) depletion

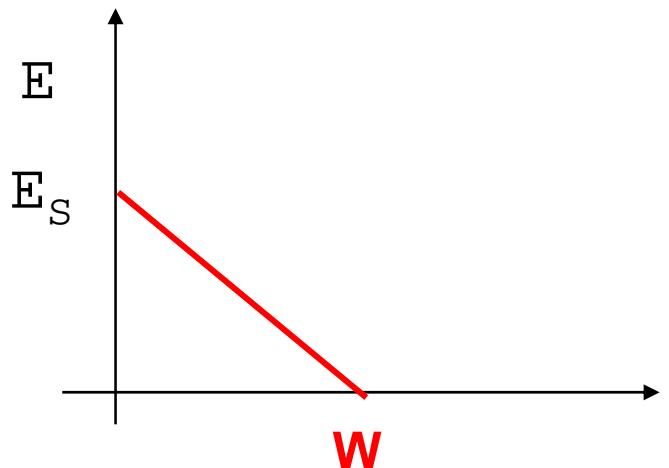
$$D_S = \varepsilon_{st} E_S = -Q_S = qN_A W \quad D_S$$



$$\psi_S = \frac{1}{2} E_S W$$

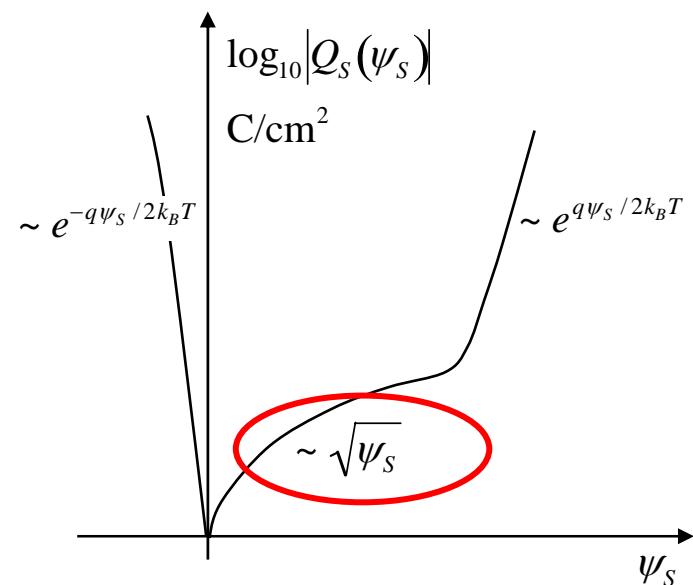
$$W = \sqrt{\frac{2\varepsilon_{Si}\psi_S}{qN_A}}$$

$$Q_S(\psi_S) = -qN_A W = \sqrt{2qN_A \varepsilon_{Si} \psi_S}$$



ii) depletion

$$Q_s(\psi_s) = -qN_A W = \sqrt{2qN_A \epsilon_{Si} \psi_s}$$



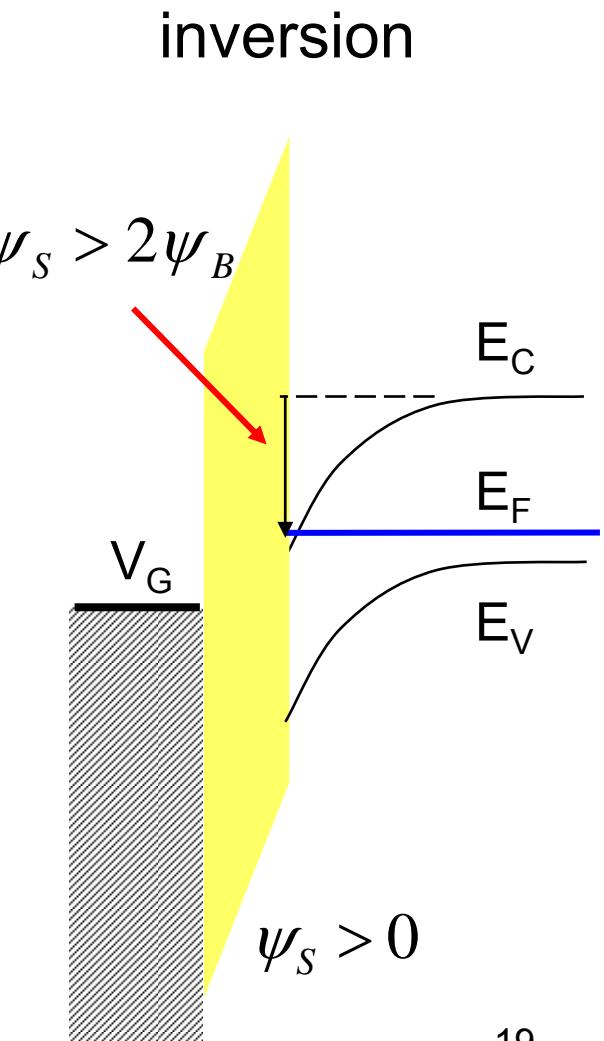
iii) inversion

$$Q_s(\psi_s) = Q_d(\psi_s) + Q_i(\psi_s)$$

$$Q_d(\psi_s) = \sqrt{2qN_A\epsilon_{Si}\psi_s}$$

$$Q_i(\psi_s) = -qn(0)\frac{k_B T / q}{E_s} = -qn(0)W_{inv}$$

$$\left. \begin{aligned} n(0) &= n_B e^{q\psi_s/k_B T} \\ W_{inv} &= \frac{k_B T / q}{E_s} \end{aligned} \right\}$$



weak inversion (sub-threshold)

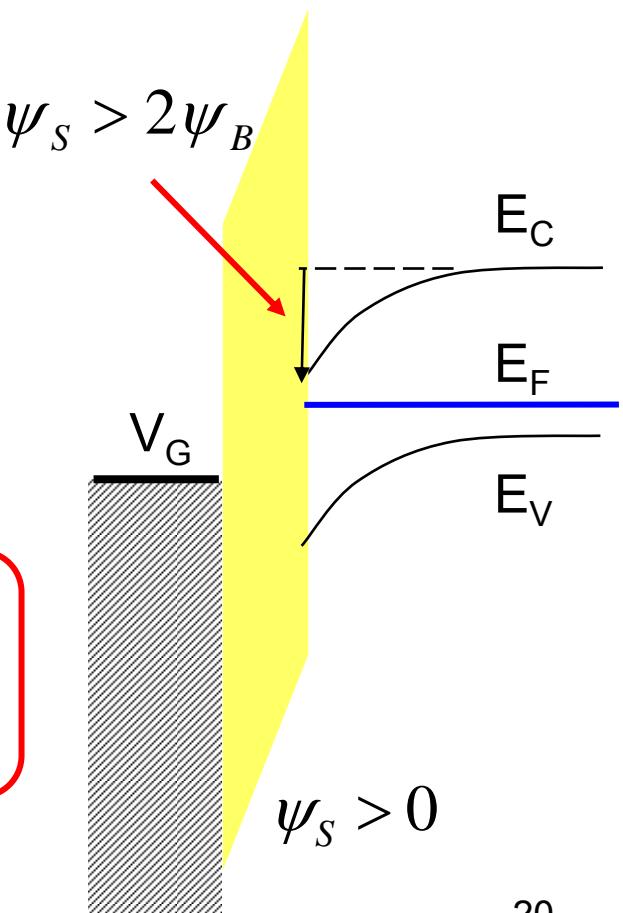
$$Q_s(\psi_s) \approx Q_d(\psi_s)$$

$$Q_i(\psi_s) = -qn(0) \frac{k_B T / q}{E_s}$$

$$E_s(\psi_s) = \sqrt{2qN_A\epsilon_{Si}\psi_s} / \epsilon_{Si}$$

$$Q_i(\psi_s) = -qn_B e^{q\psi_s/k_B T} \frac{k_B T / q}{\sqrt{2q\epsilon_{Si}N_A\psi_s} / \epsilon_{Si}}$$

weak inversion



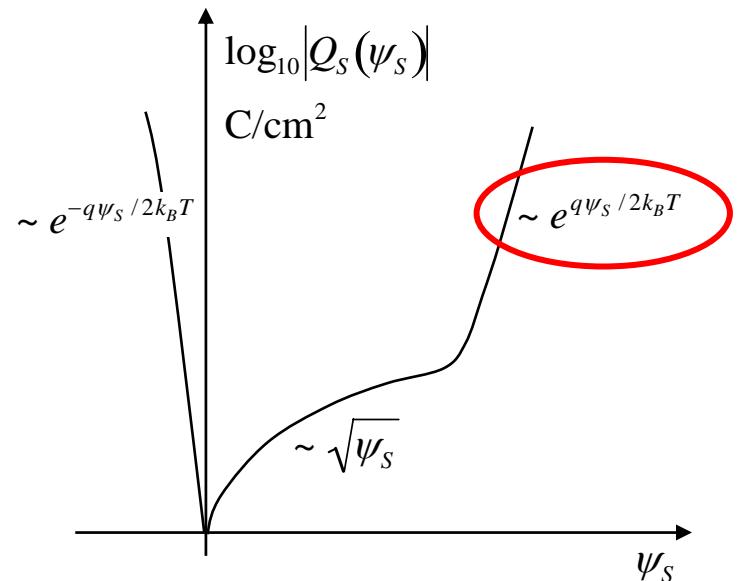
strong inversion (above threshold)

$$Q_s(\psi_s) \approx Q_i(\psi_s)$$

$$Q_i(\psi_s) = -qn(0) \frac{k_B T / q}{E_s}$$

$$E_s(\psi_s) = Q_i / \varepsilon_{Si}$$

$$Q_i(\psi_s) = -\sqrt{\varepsilon_{Si} k_B T n_B} e^{q\psi_s / 2k_B T}$$



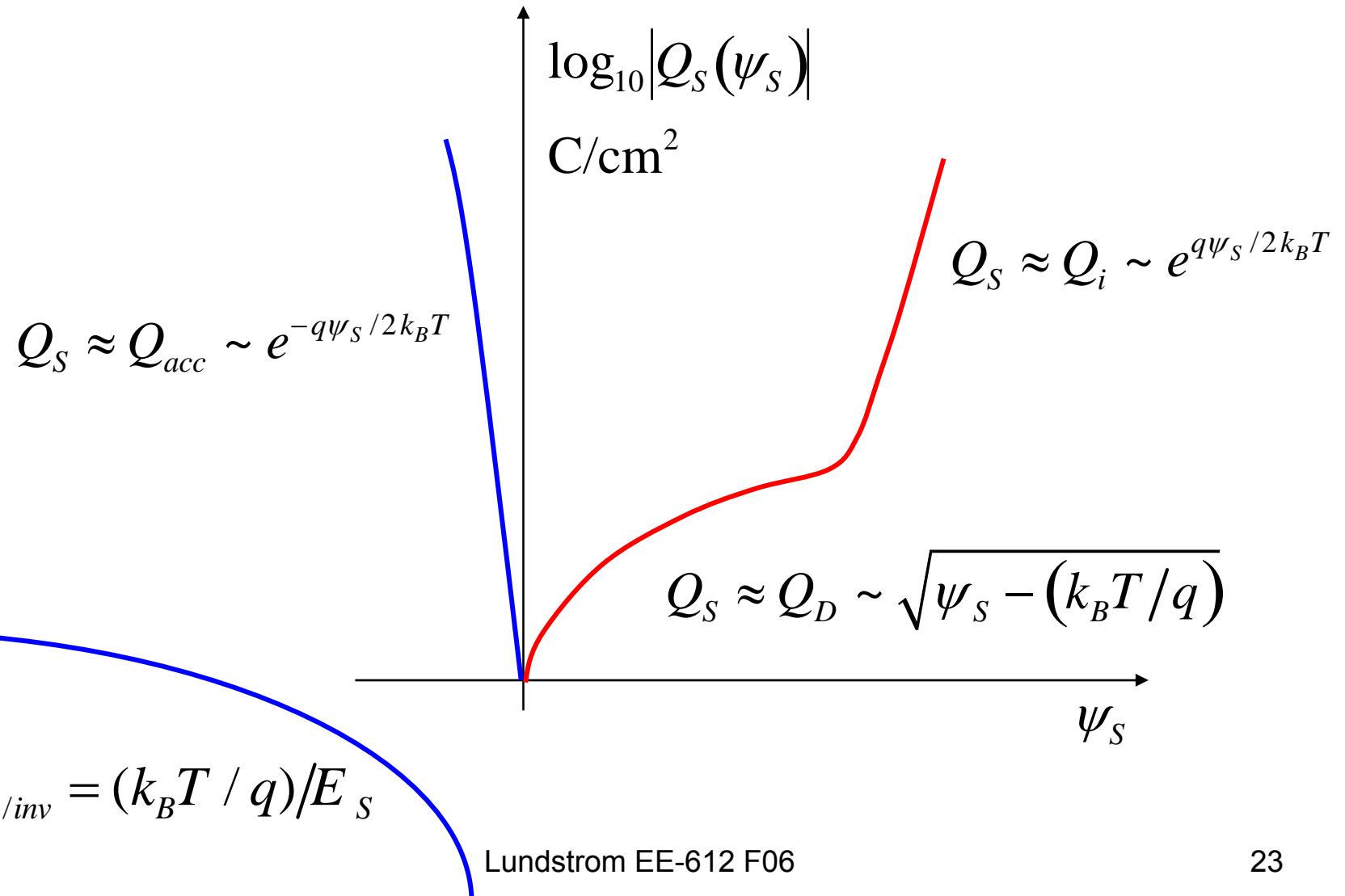
strong inversion criterion

$$n(0) \approx p_B$$

$$n_B e^{q\psi_S/k_B T} = \frac{n_i^2}{N_A} e^{q\psi_S/k_B T} \approx p_B = N_A$$

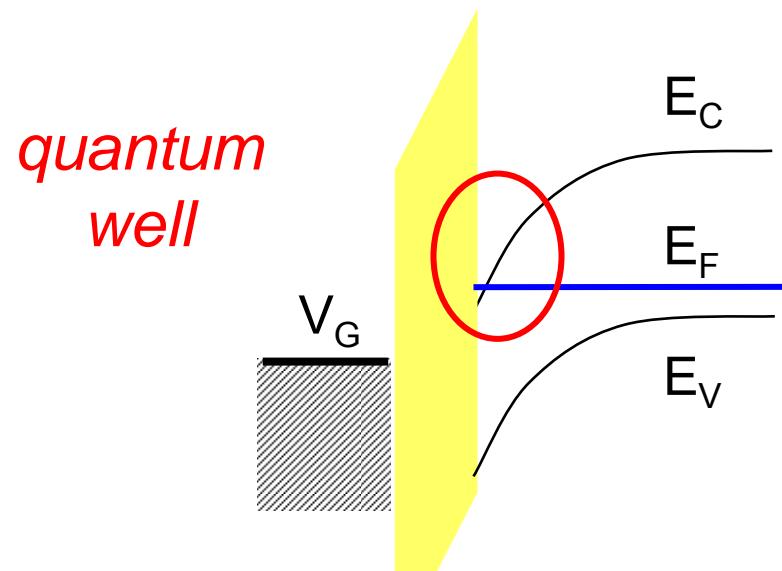
$$\psi_S = 2 \frac{k_B T}{q} \ln \left(\frac{N_A}{n_i} \right) = 2 \psi_B$$

summary



assumptions

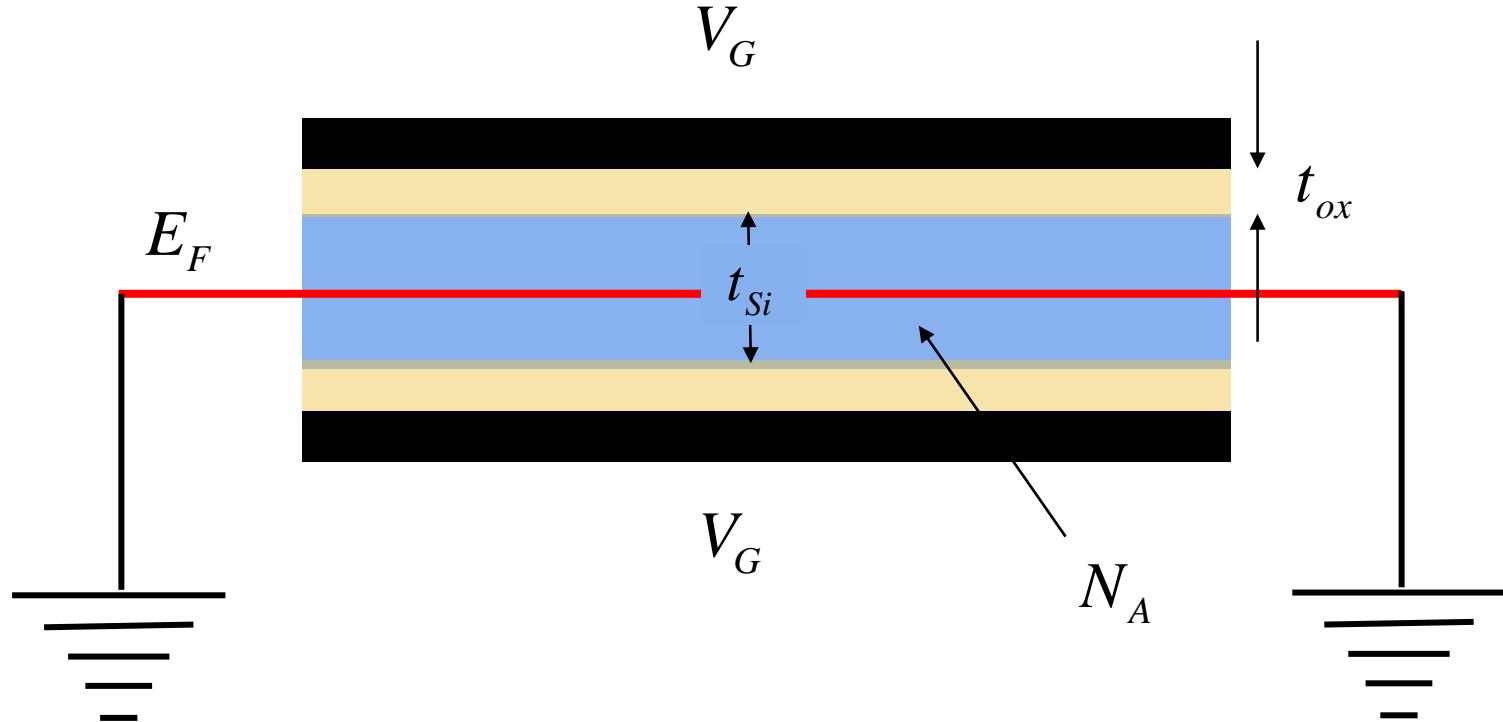
- 1) Boltzmann statistics (not valid above threshold)
- 2) Uniform doping (not valid in practice)
- 3) No quantum confinement (not valid above threshold)



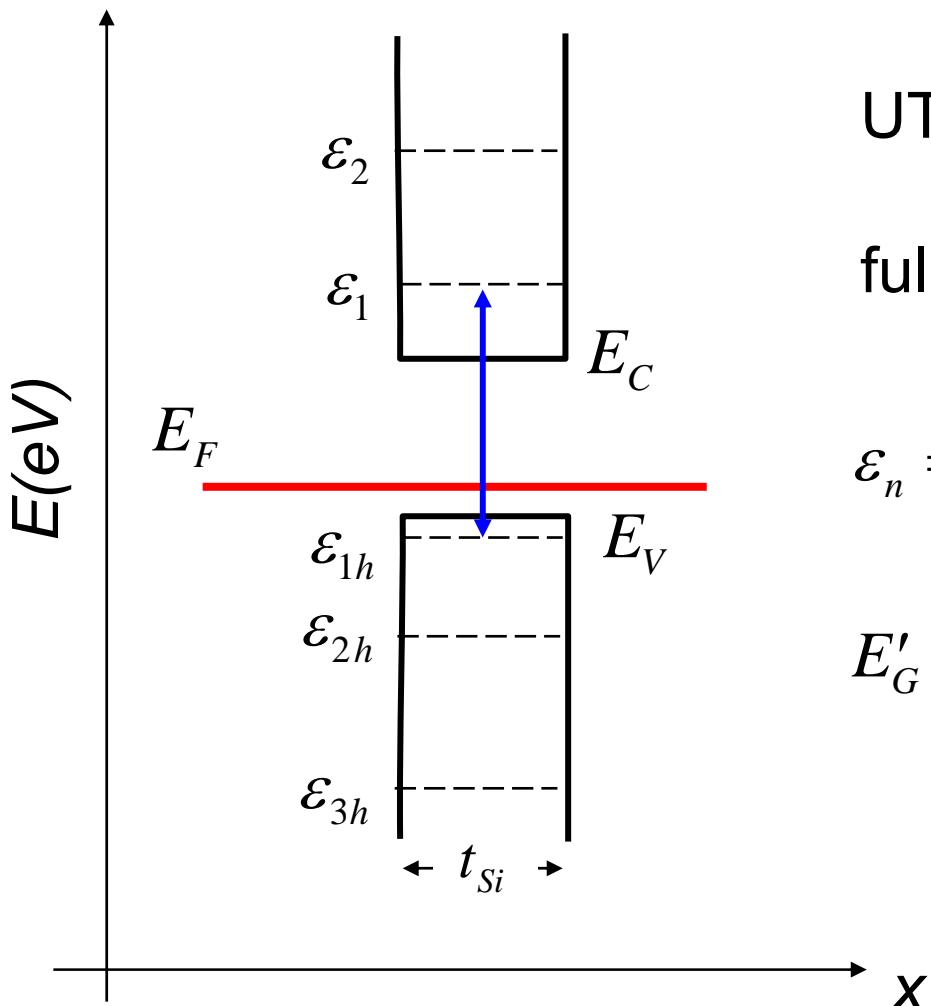
outline

- 1) Introduction
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ultra-thin body double gate MOSFET



UTB energy band diagram



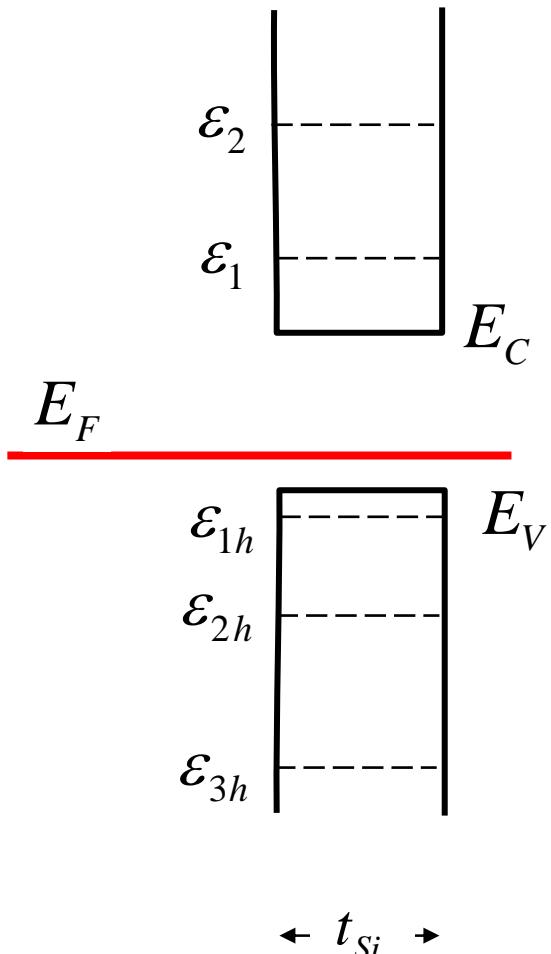
UTB (neglect band bending)

fully depleted (for $\psi > 0$)

$$\epsilon_n = \frac{\hbar^2 n^2 \pi^2}{2m^* t_{Si}^2}$$

$$E'_G = E_G + \epsilon_1 + \epsilon_{1h}$$

2D carrier densities



$$n_s = N_c^{2D} F_0(\eta_c) = N_c^{2D} (1 + e^{\eta_c}) \text{cm}^{-2}$$

$$p_s = N_v^{2D} F_0(\eta_v) = N_v^{2D} (1 + e^{\eta_v}) \text{cm}^{-2}$$

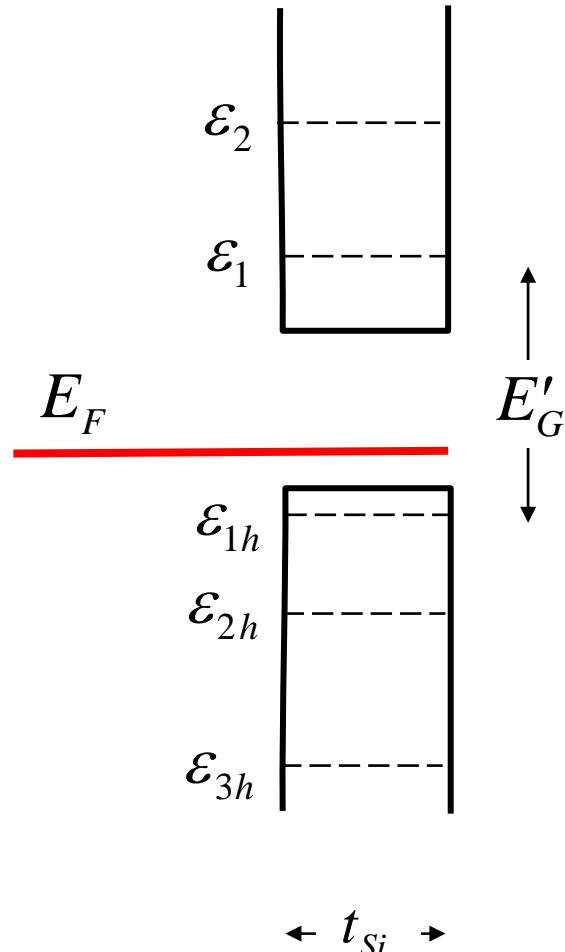
$$N_c^{2D} = \frac{m_n^* k_B T}{\pi \hbar^2} \quad N_v^{2D} = \frac{m_p^* k_B T}{\pi \hbar^2}$$

$$\eta_c = (E_F - E_C - \epsilon_1) / k_B T$$

$$\eta_v = (E_V - \epsilon_{1h} - E_F) / k_B T$$

(these eqns. assume that only 1 subband is occupied)

2D carrier densities (Boltzmann statistics)



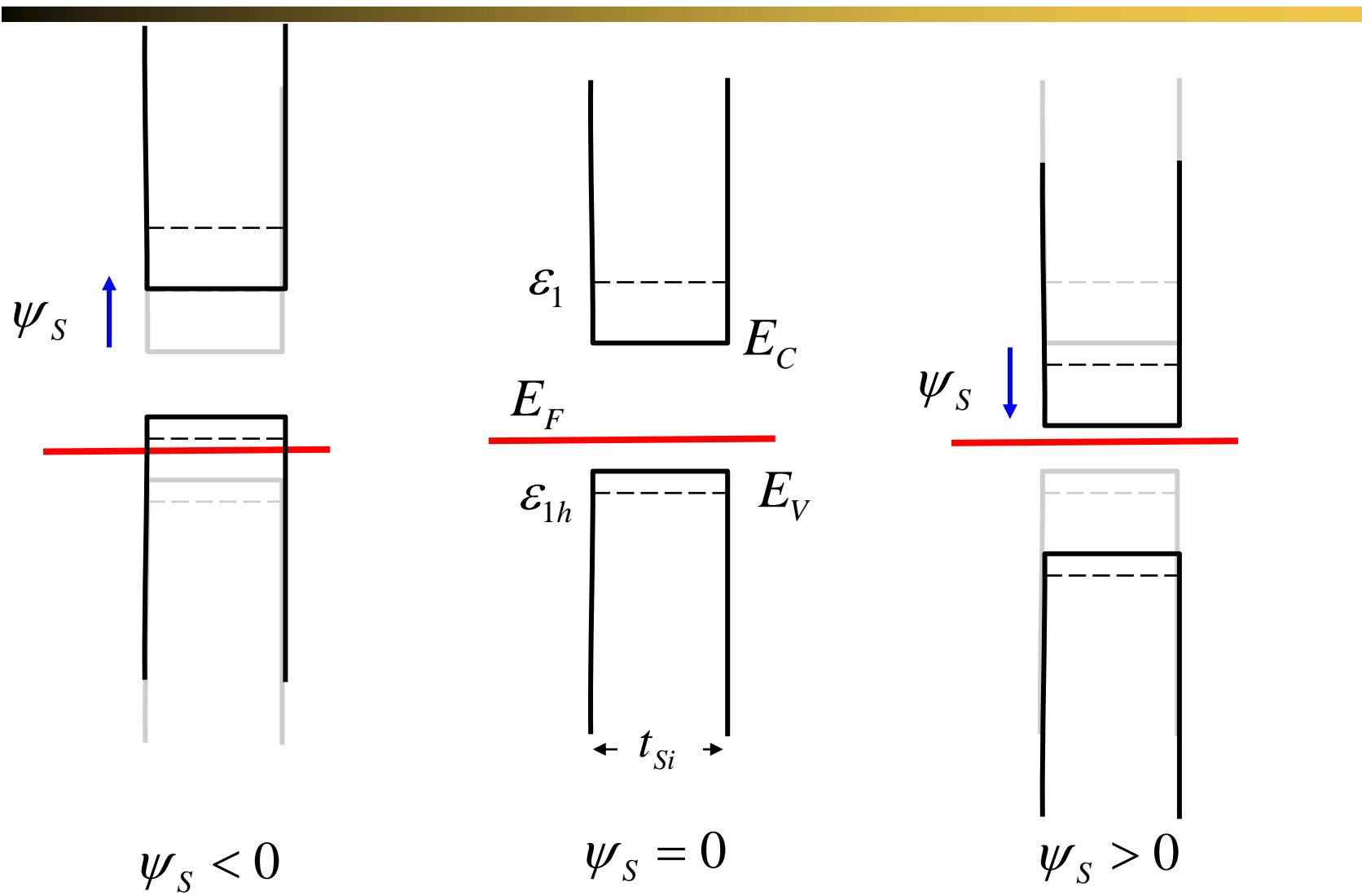
$$n_S \approx N_C^{2D} e^{\eta_c} \text{ cm}^{-2} \quad p_S = N_V^{2D} e^{\eta_v} \text{ cm}^{-2}$$

$$n_S p_S = (n_i^{2D})^2$$

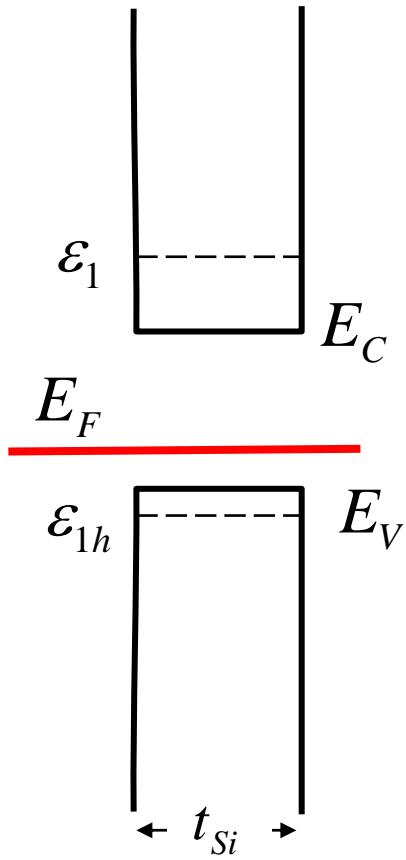
$$n_i^{2D} = \sqrt{N_C^{2D} N_V^{2D}} e^{-E'_G / 2k_B T} \text{ cm}^{-2}$$

$$E'_G = E_G + \epsilon_1 + \epsilon_{1h}$$

UTB ('surface' potential)



UTB (carrier densities and ψ_S)



$\psi_S = 0$ is analogous to the bulk in a Si wafer

$$p_{SB} \approx N_A t_{Si} \text{ cm}^{-2}$$

for $\psi_S \neq 0$

$$p_S = p_{SB} e^{-q\psi_S/k_B T} \text{ cm}^{-2}$$

$$n_S = n_{SB} e^{q\psi_S/k_B T} \text{ cm}^{-2}$$

$$\left(n_{SB} = \left(n_i^{2D} \right)^2 / p_{SB} \text{ cm}^{-2} \right)$$

$$\psi_S = 0$$

$Q_S(\psi_S)$ for UTB MOS

$$Q_S = q \left[(p_S(\psi_S) - p_{SB}) - (n_S(\psi_S) - n_{SB}) \right] \text{ C/cm}^2$$

$$Q_S = q \left[p_{SB} (e^{-q\psi_s/k_B T} - 1) - n_{SB} (e^{q\psi_s/k_B T} - 1) \right] \quad (2)$$

$$\left. \begin{aligned} p_{SB} &\approx N_A t_{Si} \text{ cm}^{-2} \\ n_{SB} &= (n_i^{2D})^2 / p_{SB} \text{ cm}^{-2} \end{aligned} \right\}$$

From equation (2), we can readily plot $Q_S(\psi_S)$

$Q_S(\psi_S)$ for UTB MOS

$$Q_S = q \left[p_{SB} (e^{-q\psi_s/k_B T} - 1) - n_{SB} (e^{q\psi_s/k_B T} - 1) \right] \quad (2)$$

1) strong accumulation ($\psi_S \ll 0$)

$$Q_S = q p_{SB} e^{-q\psi_s/k_B T}$$

2) depletion ($\psi_S > 0$)

$$Q_S = -q p_{SB} = -q N_A t_{Si}$$

Note: We are not resolving band bending within the Si body.
We assume that the film is fully depleted when $\psi_S > 0$).

3) inversion ($\psi_S \gg 0$)

$$Q_S = -q n_{SB} e^{q\psi_s/k_B T}$$

UTB MOS: inversion criterion

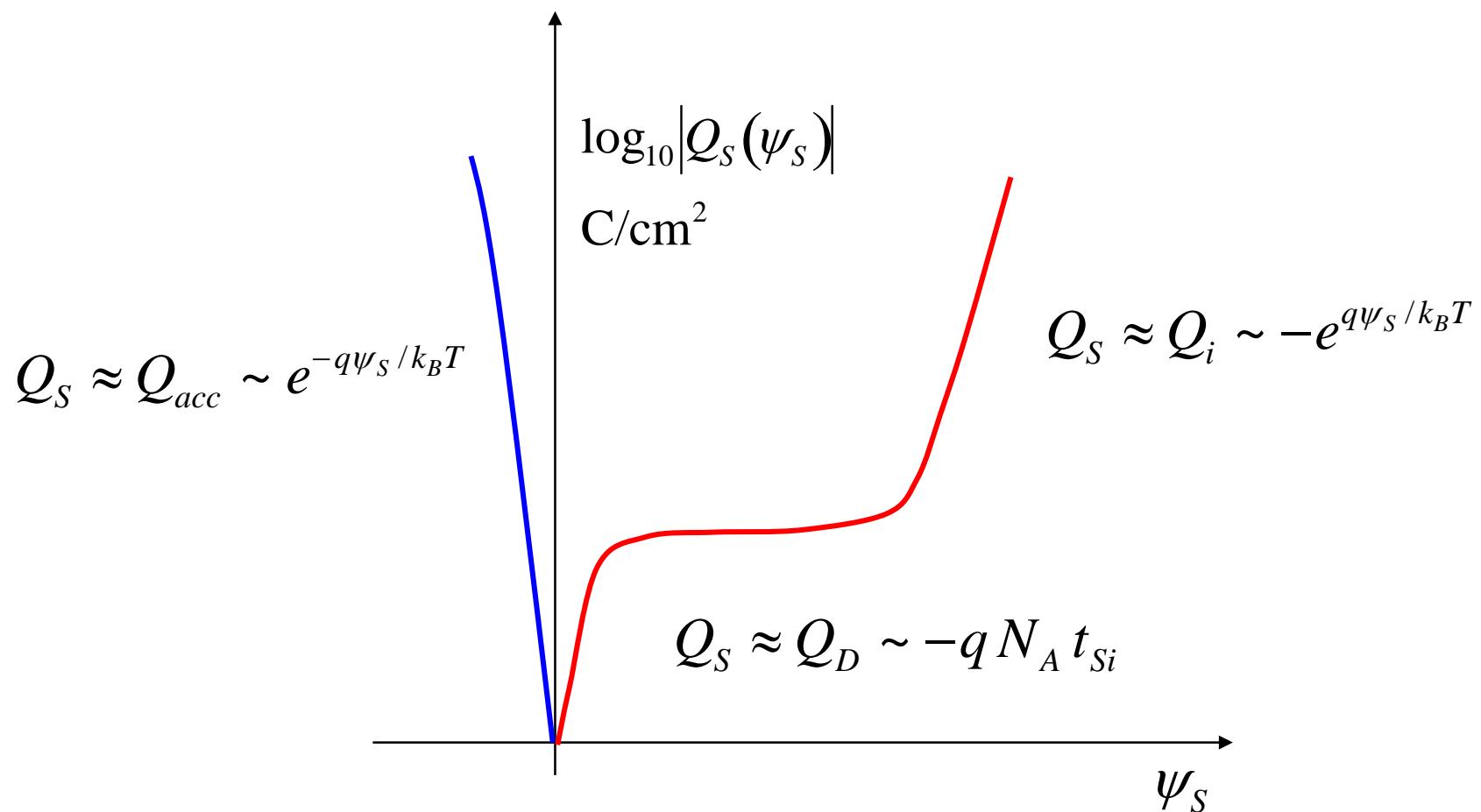
$$n_S \approx p_{SB}$$

$$n_{SB} e^{q\psi_S / k_b T} = \frac{\left(n_i^{2D}\right)^2}{N_A t_{Si}} e^{q\psi_S / k_b T} \approx p_{SB} = N_A t_{Si}$$

$$\psi_S = 2 \frac{k_B T}{q} \ln \left(\frac{N_A t_{Si}}{n_i^{2D}} \right) = 2\psi_B$$

$Q_S(\psi_S)$ for UTB MOS

$$Q_S = q \left[p_{SB} (e^{-q\psi_s/k_B T} - 1) - n_{SB} (e^{q\psi_s/k_B T} - 1) \right] \quad (2)$$



$Q_S(\psi_S)$ for UTB MOS: summary

- 1) The UTB case is easier to solve than the bulk Si case.
- 2) Results are qualitatively similar - except for the depletion charge and acc and inv layers that vary as $\exp(\psi_S/k_B T)$ rather than $\exp(\psi_S/2k_B T)$.

Can you explain why this difference occurs?

- 3) We have included quantum mechanics (without self-consistent electrostatics inside the silicon film)
but not Fermi-Dirac statistics.

Fermi-Dirac statistics are important above threshold.

$Q_S(\psi_S)$ for UTB MOS: exercise

- 1) Repeat the derivation, but include Fermi-Dirac statistics.
(This can be done analytically for the UTB.)

- 2) Plot Q_S vs. ψ_S from accumulation to inversion for both Boltzmann and Fermi-Dirac statistics and compare the results.

Summary

- 1) Understanding $Q_S(\psi_S)$ and $Q_i(\psi_S)$ are essential for understanding MOS C-V and MOSFETs.
- 2) Both $Q_S(\psi_S)$ and $Q_i(\psi_S)$ are readily computed for simple, model structures.
- 3) The general features of the $Q_S(\psi_S)$ and $Q_i(\psi_S)$ vs. ψ_S are readily understood.
- 4) Before we proceed to MOS-C's and MOSFETs, we need to relate ψ_S to the V_G that produced it.