ECE 595, Section 10
Numerical Simulations
Lecture 18: FEM for Thermal Transport

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Outline

- Recap from Monday
- Thermal transfer overview
  – Convection
  – Conduction
  – Radiative transfer

Recap from Monday

- Applications of Beam Propagation Method
  – Tunable Photonic Crystal Fibers
  – Electro-Optic Modulator
  – Electro-Optic Switch

Thermal Transport Mechanisms

- Convection: heat transfer by surface contact with gas or fluid molecules
- Conduction: volumetric heat transfer by propagation of phonons
- Radiative thermal transfer: emission of thermal photons from source to receiver

Thermal Transport: Convection

- Heat transfer by gas or fluid molecules
- Transfer rate per unit area given by:

$$Q = h(T_1 - T_2)$$

- Heat transfer constant *h* determined by many factors, including material choice, microstructures, fluid flow environment, etc.

Thermal Transport: Conduction

- Volumetric heat transfer through phonon transfer
- Heat transfer rate quantified by Fourier's law:

$$Q = -k\nabla T$$

- Conservation of energy yields:

$$\frac{\partial u}{\partial t} = L - \nabla \cdot Q$$

$$\frac{\partial u}{\partial t} - L = k\nabla^2 T = \frac{k}{\rho c_v}\nabla^2 u \equiv \alpha \nabla^2 u$$

Thermal Transport: Radiative Transfer

- Heat transfer via photon emission
- For a blackbody, total emission follows Stefan-Boltmann law:

$$P = \sigma T^4$$

- Net thermal transfer between two infinite surfaces becomes:

$$Q = \sigma(T_1^4 - T_2^4)$$

Thermal Transport: Radiative Transfer

- Emission for real materials depends on emissivity
- In thermal equilibrium, Kirchoff's law states emissivity=absorptivity at each wavelength
- Emission spectrum is given by:

$$\frac{dQ}{d\lambda} = \frac{2\pi hc^2\,\epsilon(\lambda)}{\lambda^5[e^{hc/\lambda kT} - 1]}$$

- Blackbody result recovered by setting $\epsilon(\lambda) = 1$ and integrating

Thermal Transport: Modeling

- Convection amounts to a boundary condition in most problems
- Will thus be first combined with conduction
- Strategy:
  - Create FEM grid for thermal conduction
  - Impose BC's from convection
  - (Optionally) include radiative transfer from disconnected bodies

Thermal Transport FEM

- Employ Galerkin method to reduce to linear algebra problem as before (see Petr Krysl's step-by-step introduction, Chapter 6):

$$C\dot{T} + (K + H)T = \sum_i L_i$$

- Where:

$$C_{ji} = \int_{S_c} N_j c_V N_i \, \mathrm{d}S \qquad L_{Q,j} = \int_{S_c} N_j Q \, \mathrm{d}S$$

$$K_{ij} = \int_{S_c} (\mathrm{grad} N_j) \, \boldsymbol{\kappa} (\mathrm{grad} N_i)^T \, \mathrm{d}S \qquad L_{q2,j} = -\int_{C_{c,2}} N_j \, \overline{q}_n \, \mathrm{d}C$$

$$H_{ji} = \int_{C_{c,3}} N_j \, h N_i \, \mathrm{d}C \qquad L_{q3,j} = \int_{C_{c,3}} N_j \, h T_a \, \mathrm{d}C$$

SOFEA: MATLAB FEM Toolbox

- 1D Meshing routine:

```
for j= 1:n+1
    fens=[fens fenode(struct ('id',j,'xyz',[x]));];
    x = x+(L/n);
end
gcells = [];
for j= 1:n
    gcells = [gcells gcell_l2(struct('id',j,'conn',[j j+1]))];
end
```

- Construct finite element block:

```
feb = feblock_defor_taut_wire(struct ('mater',mater_defor,...
    'gcells',gcells,...
    'integration_rule',simpson_1_3_rule,...
    'P',P));
geom = field(struct ('name',['geom'], 'dim', 1, 'fens',fens));
w   = 0*clone(geom,'w');
```

SOFEA: MATLAB FEM Toolbox

- Apply boundary conditions:

```
fenids=[1]; prescribed=[1]; component=[1]; val=0;
w   = set_ebc(w, fenids, prescribed, component, val);
w   = apply_ebc (w);
w   = numbereqns (w);
```

- Assemble and solve equations:

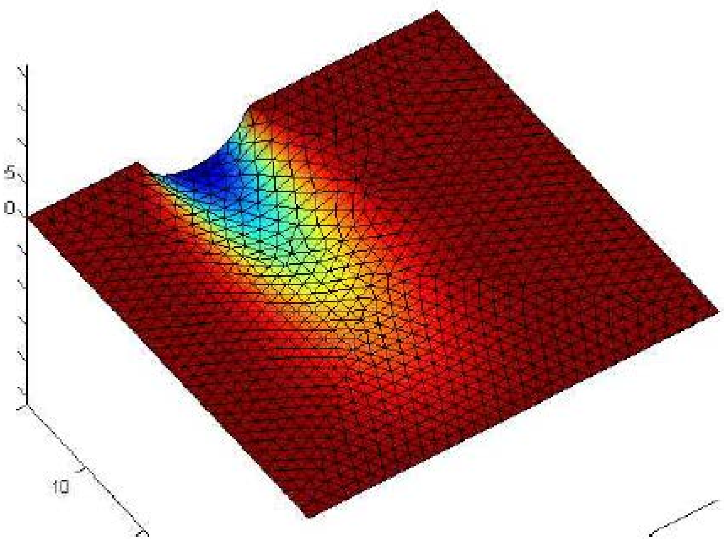
```
K = start (dense_sysmat, get(w, 'neqns'));
K = assemble (K, stiffness(feb, geom, w));
bl = body_load(struct ('magn',inline(num2str(q))));
F = start (sysvec, get(w, 'neqns'));
F = assemble (F, body_loads(feb, geom, w, bl));
w = scatter_sysvec(w, get(K,'mat')\get(F,'vec'));
```

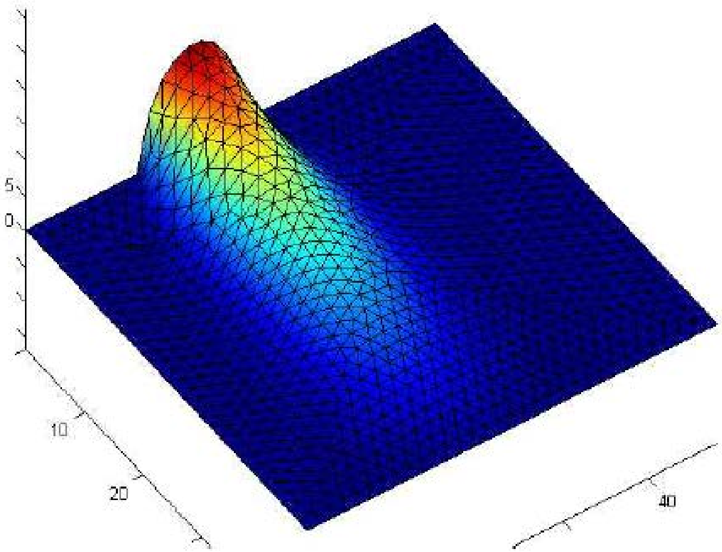
SOFEA: MATLAB FEM Toolbox

- Note: latest version is now called FAESOR:

http://hogwarts.ucsd.edu/~pkrysl/faesor/faesor_publish.html

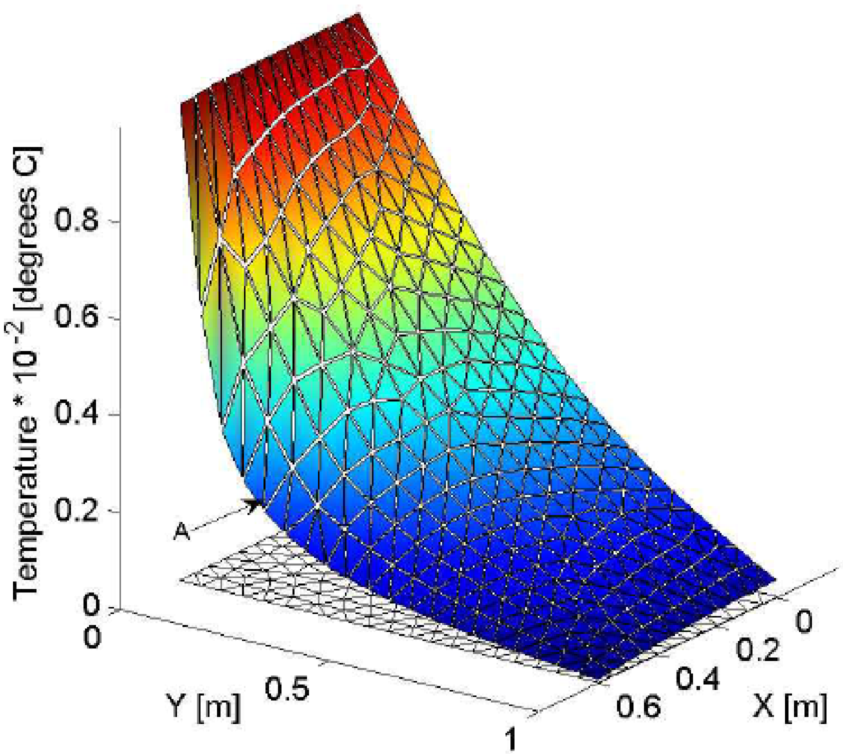
Results





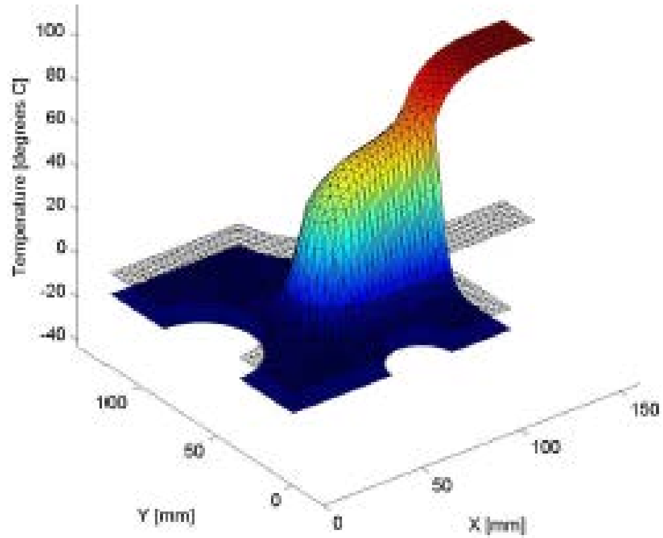
- Steady-state solution for a thermally insulating medium, with a variable temperature placed along one surface

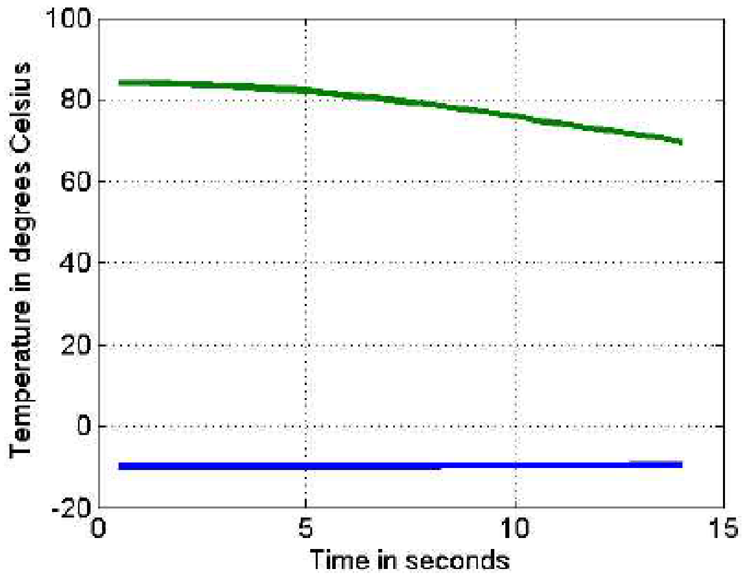
Thermal Conduction: Results



- Steady-state thermal gradient between two adjoining walls with different temperatures

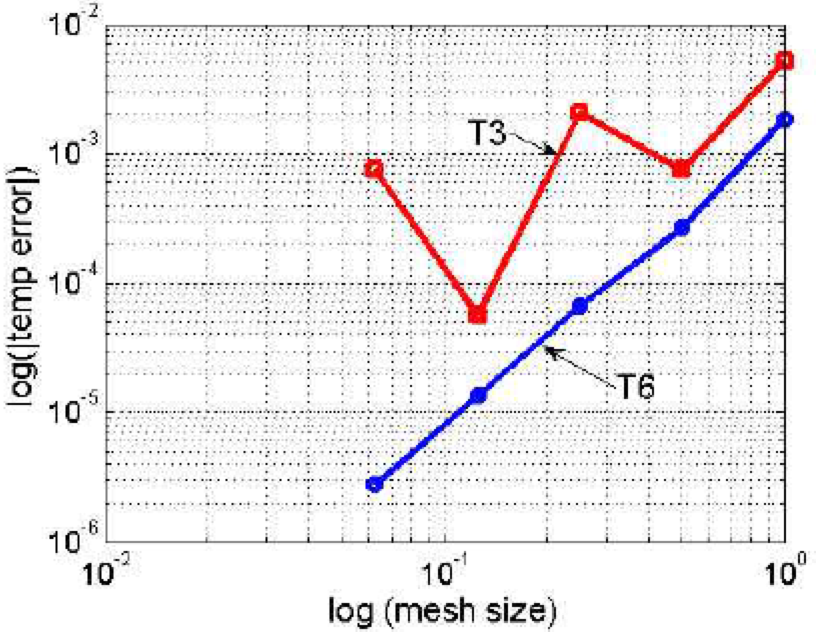
Thermal Conduction: Results





- Transient cooling of a shrink-fitted assembly:
  - In red: highest temperature vs. cooling time
  - In blue: lowest temperature vs. cooling time

Error Evaluation



- Error for linear elements T3 higher overall than quadratic elements T6; both decrease almost quadratically with mesh size

Next Class

- Is on Friday, Feb. 22
- Next time, we will cover other FEM applications in electronic transport