ECE 595, Section 10
Numerical Simulations
Lecture 21: 3D Bandstructures

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February 27, 2013

Outline

- Recap from Monday
- Bandstructure Symmetries
- 2D Photonic Bandstructures
- Periodic Dielectric Waveguides
- Photonic Crystal Slabs

Recap from Monday

- Bandstructure Problem Formulation
- Bloch’s Theorem
- Reciprocal Lattice Space
- Numerical Solutions
  - 1D crystal
  - 2D triangular lattice

Recap from Monday

- In the case of photonic bandstructures:

$$\nabla \times [\epsilon^{-1}(\nabla \times H)] = \left(\frac{\omega}{c}\right)^2 H$$

- We can obtain:

$$-(\boldsymbol{k} + \boldsymbol{G}) \times \left[\epsilon^{-1}_{\boldsymbol{G}\boldsymbol{G}'}(\boldsymbol{k} + \boldsymbol{G}') \times \boldsymbol{h}_{G-G'}\right] = \left(\frac{\omega}{c}\right)^2 \boldsymbol{h}_G$$

- Implemented numerically in MIT Photonic Bands (MPB): http://jdj.mit.edu/mpb/

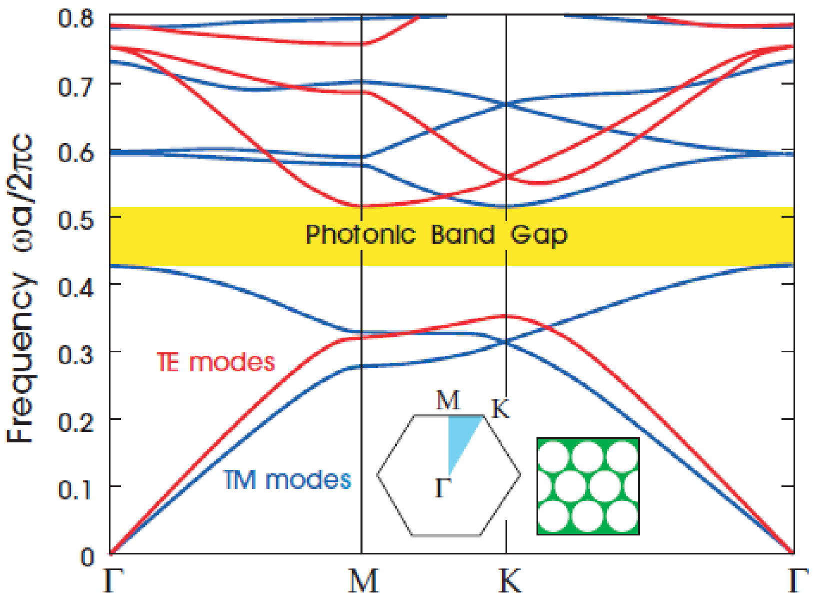
Bandstructure Symmetries

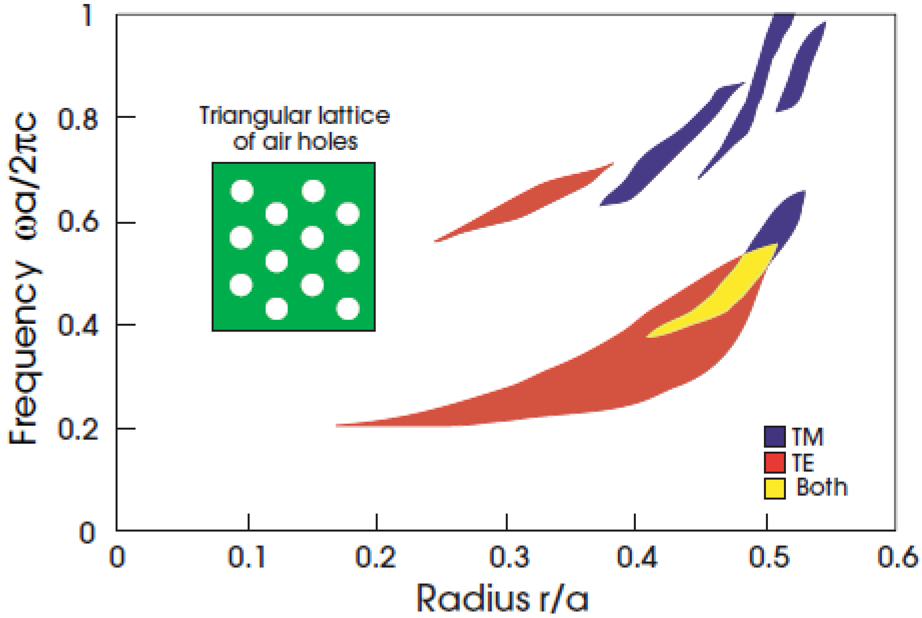
- Can be formally defined as operators that commute with eigenproblem operator
- Periodicity gives rise to k vectors and Brillouin zone
- Time-reversal invariance:
  - True for all Hermitian operators
  - Implies $\omega_n(k) = \omega_n(-k)$

Bandstructure Symmetries

- Mirror-plane symmetries
  - Mirror reflection defined s.t. $\widehat{M}H = \pm H$
  - In 2D, z-reflection gives rise to TE and TM polarizations
- Rotational symmetries
  - Defined s.t. $\omega_n(k) = \omega_n(\mathcal{R}k)$
  - $\mathcal{R}$ depends on crystallographic point group
  - In 2D, 3-fold, 4-fold, and 6-fold symmetries
  - Other symmetries give rise to quasicrystals

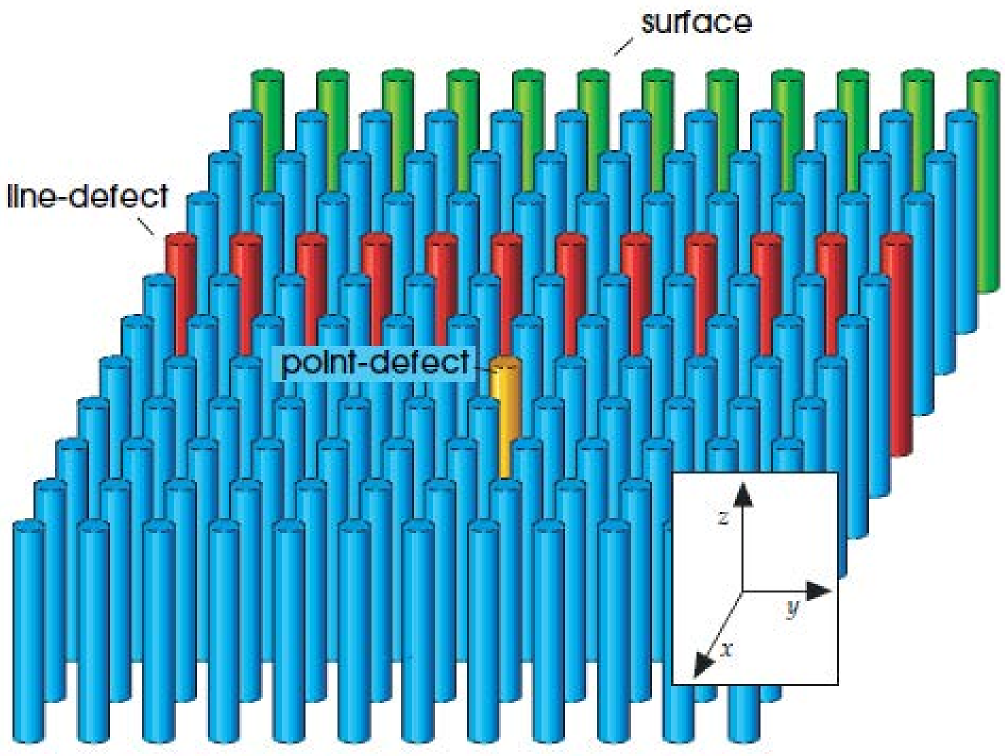
2D Photonic Crystals





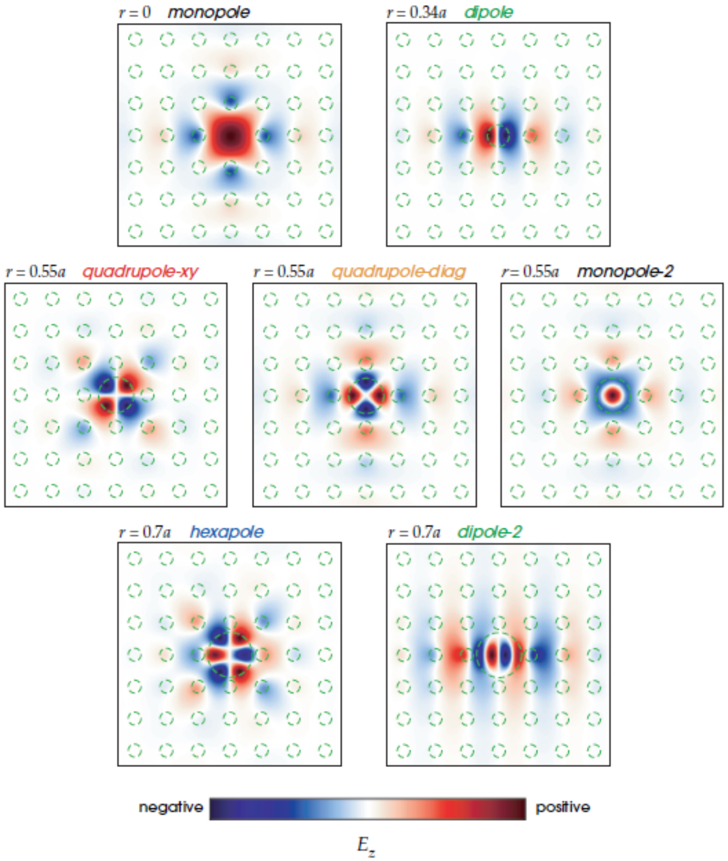
- 2D triangular lattice can give rise to band gap for all polarizations for certain radii

2D Photonic Crystals



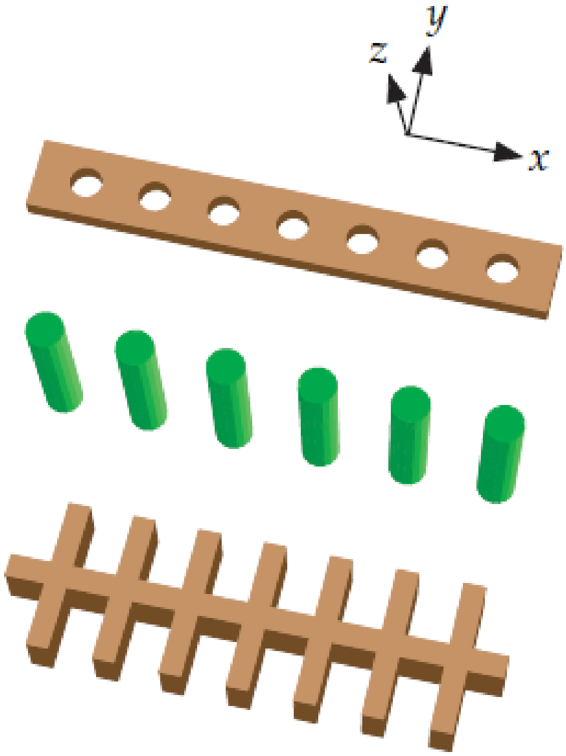
- Introducing defects can give rise to states in the bandgap

2D Photonic Crystals



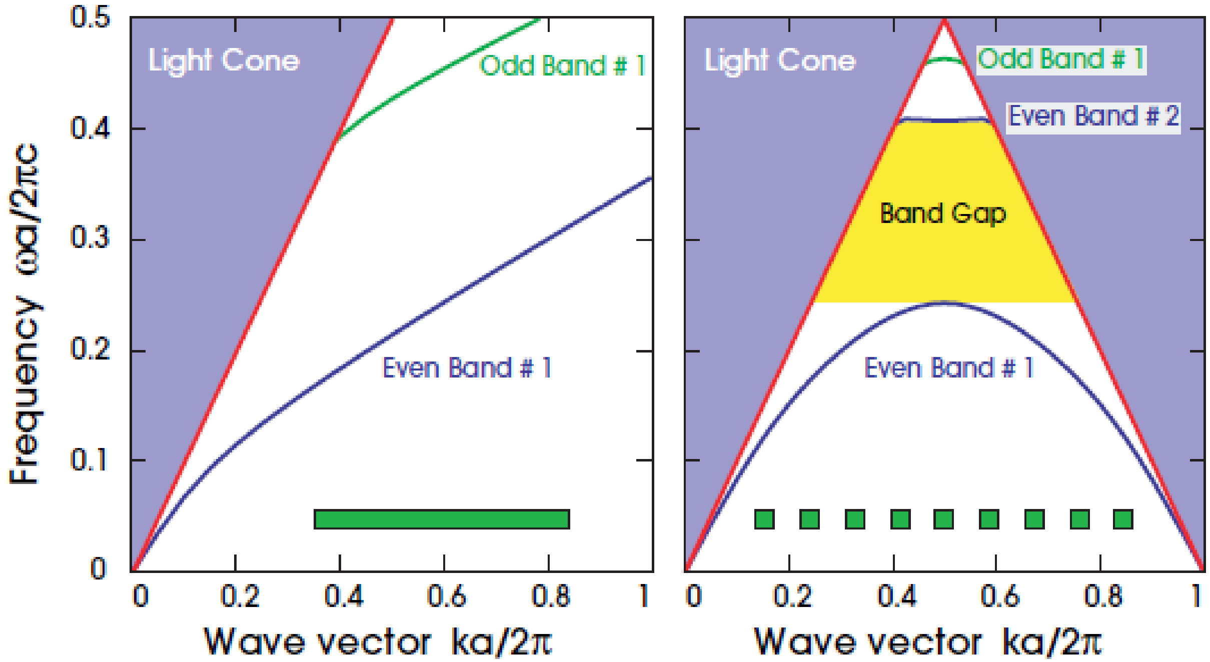
- Various localized modes observed front a point defect in a square lattice of rods

Periodic Dielectric Waveguides



- To confine light to a small volume, can combine a 1D photonic crystal with index guiding in other 2 dimensions

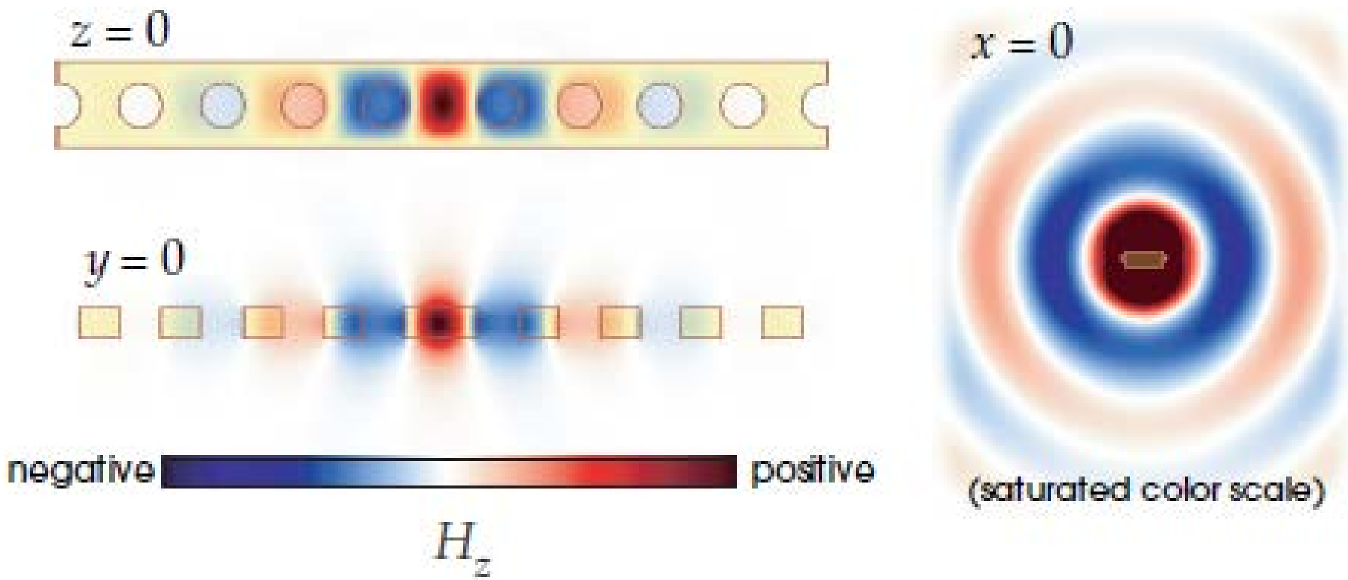
Periodic Dielectric Waveguides



Uniform index waveguide   Periodic graded waveguide

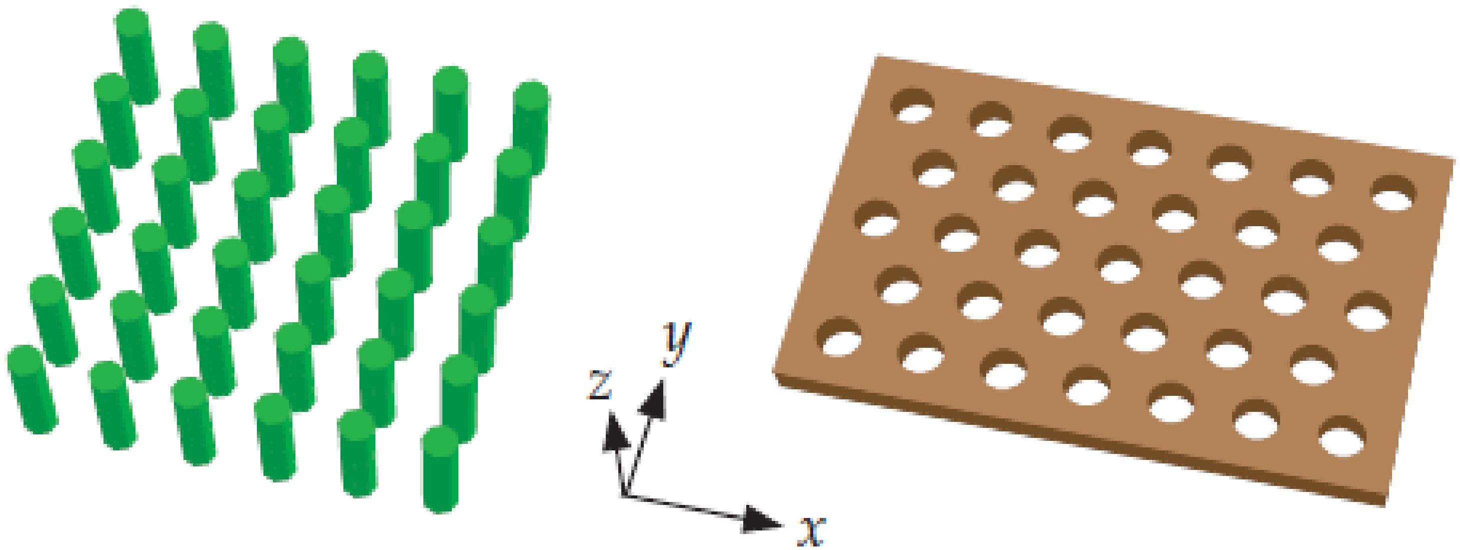
- Bandstructures for index-guided waveguides
- Introducing periodicity restricts Brillouin zone

Periodic Dielectric Waveguides



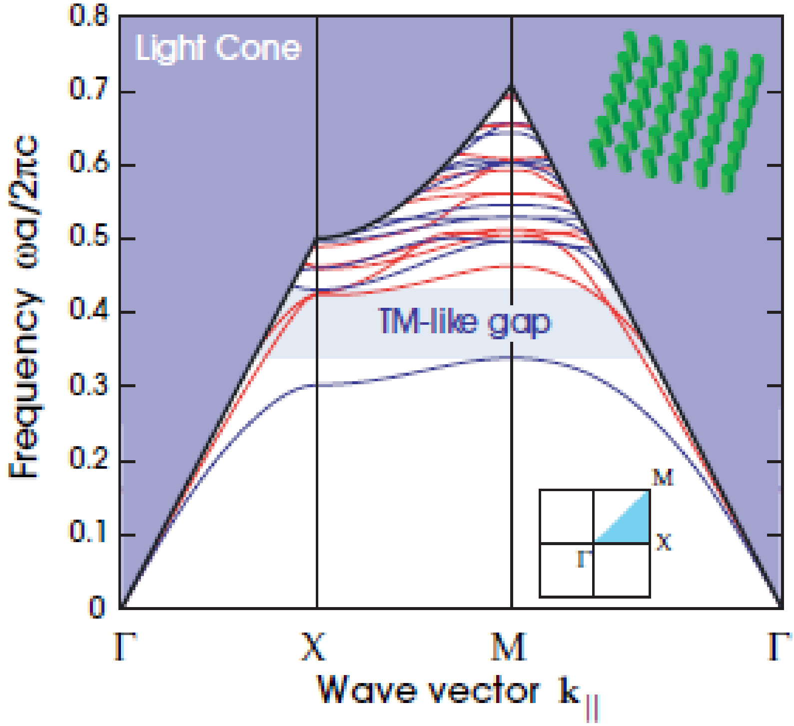
- Introducing a pointlike defect creates 3D confinement at one or more bandgap frequencies

Photonic Crystal Slabs

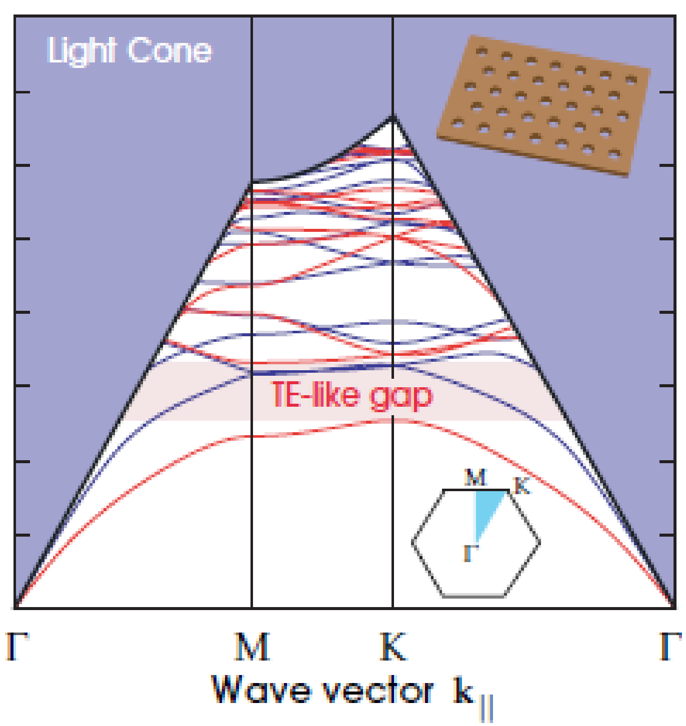


- To confine light in 3D, use bandgap in plane and index confinement out of plane

Photonic Crystal Slabs



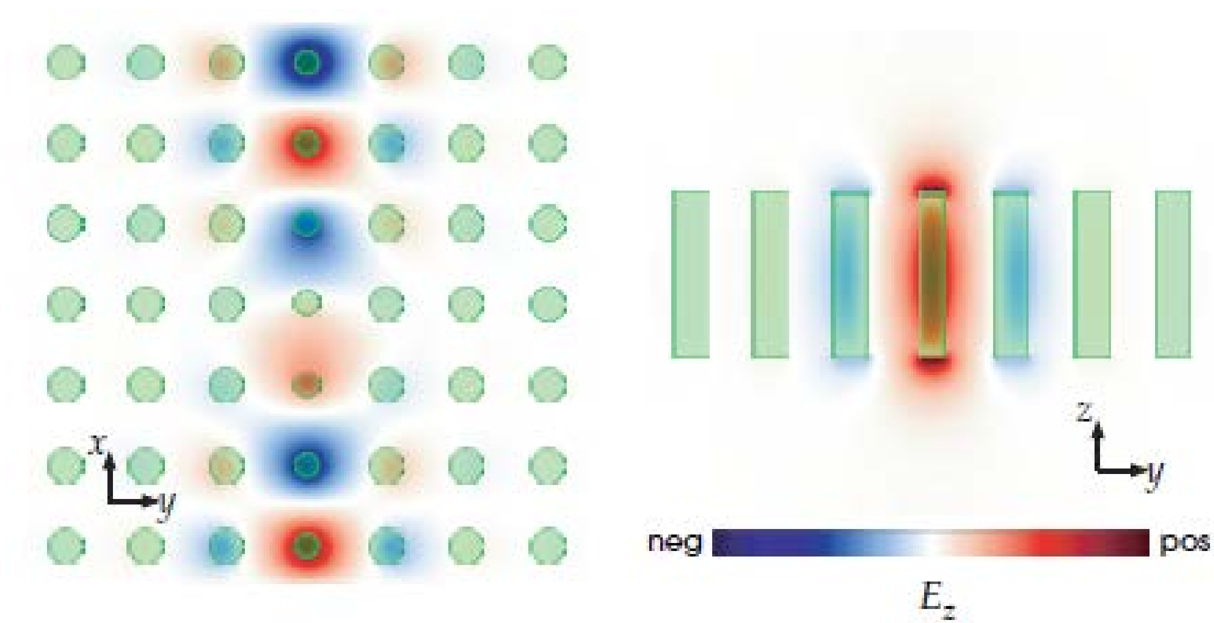
Square lattice of rods



Triangular lattice of holes

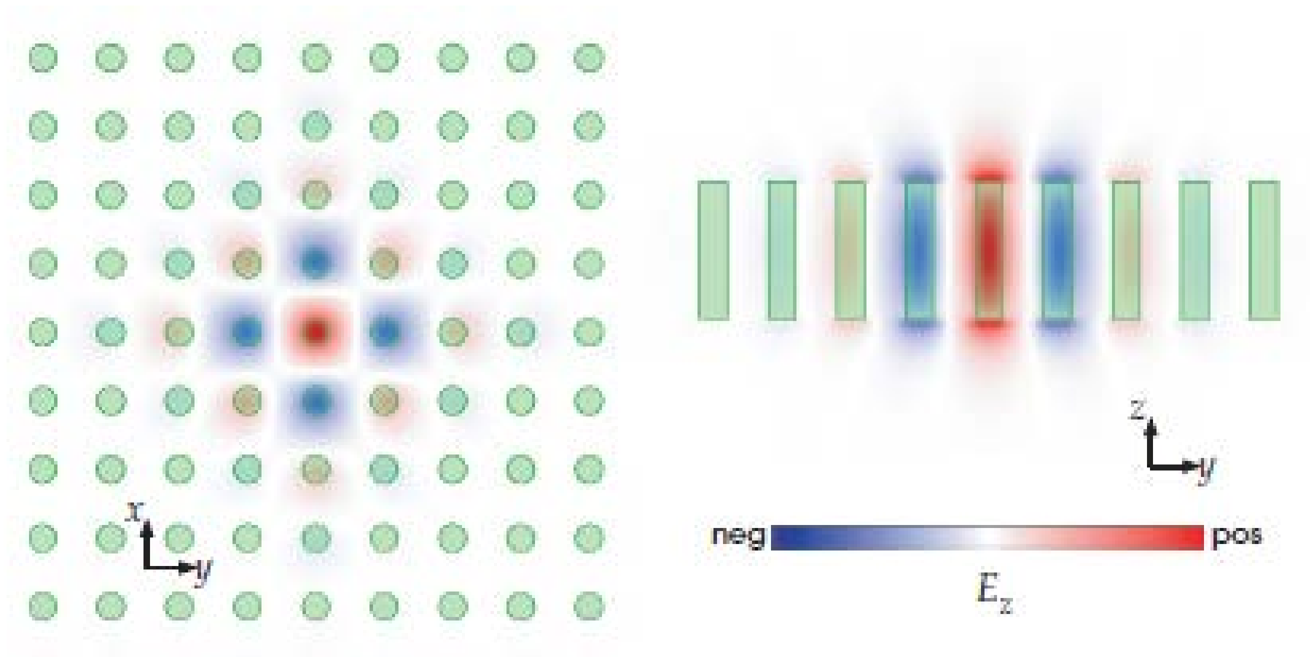
Photonic bandstructures for 2D slabs

Photonic Crystal Slabs



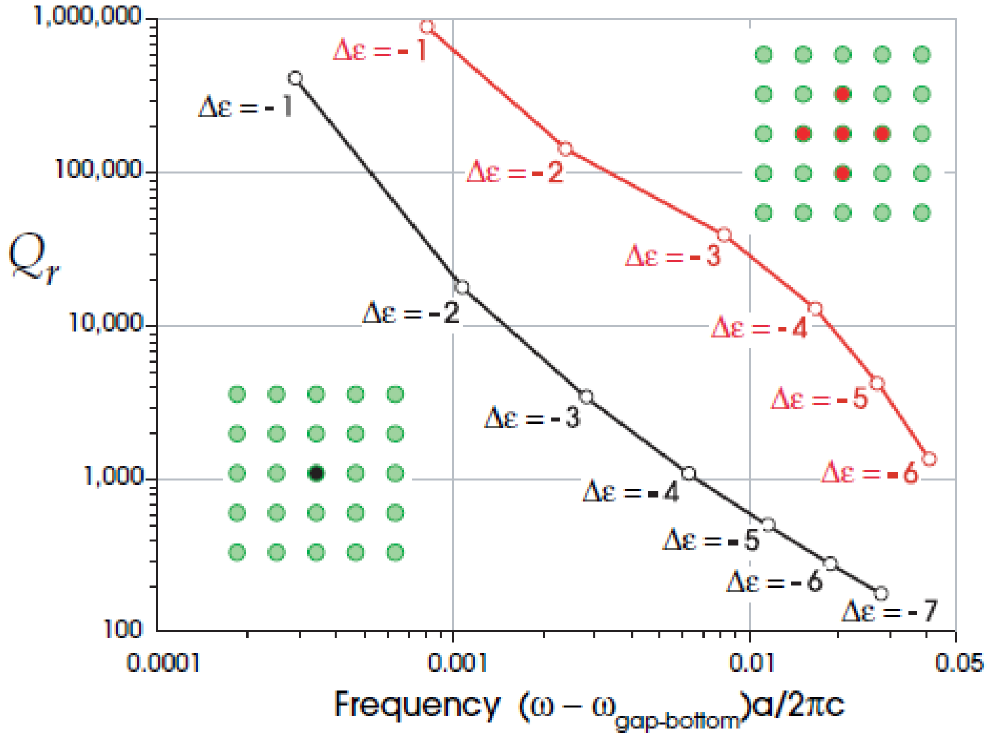
- Line defects create a low-loss waveguide; $\frac{dP}{dz} = \frac{\alpha}{v_g^2} + \frac{\beta}{v_g}$

Photonic Crystal Slabs



- Pointlike defects create a high quality-factor localized mode

Photonic Crystal Slabs



- Quality factor of pointlike defects varies strongly with frequency and index contrast

Next Class

- Is on Friday, March 1
- Will discuss applications for bandstructures
- Recommended reading: Joannopoulos, Chapter 8