

EE-612:

Lecture 10

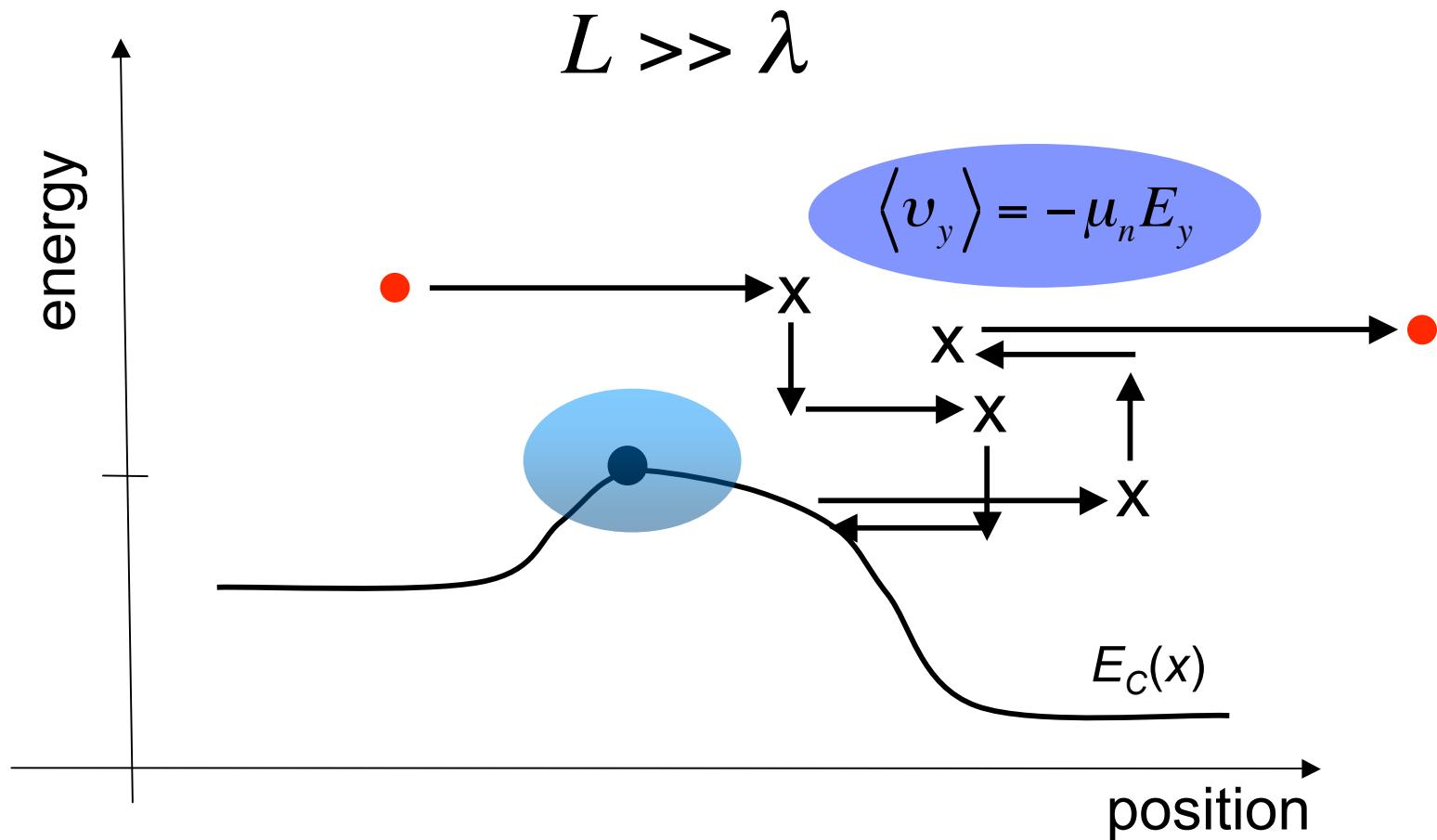
The Ballistic MOSFET

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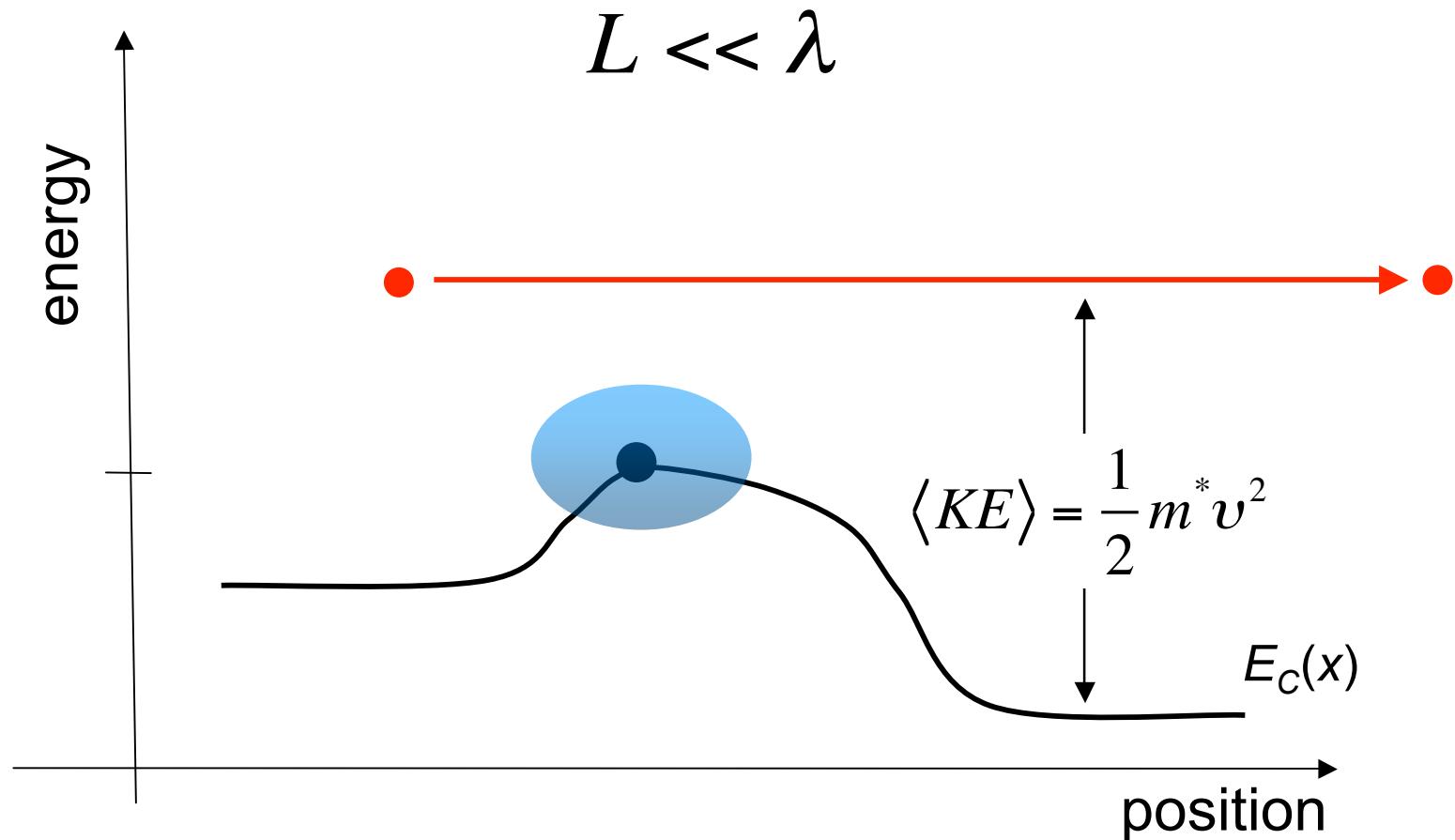
outline

- 1) Introduction
- 2) Physics of the ballistic MOSFET
- 3) Mathematical theory

what is diffusive transport?

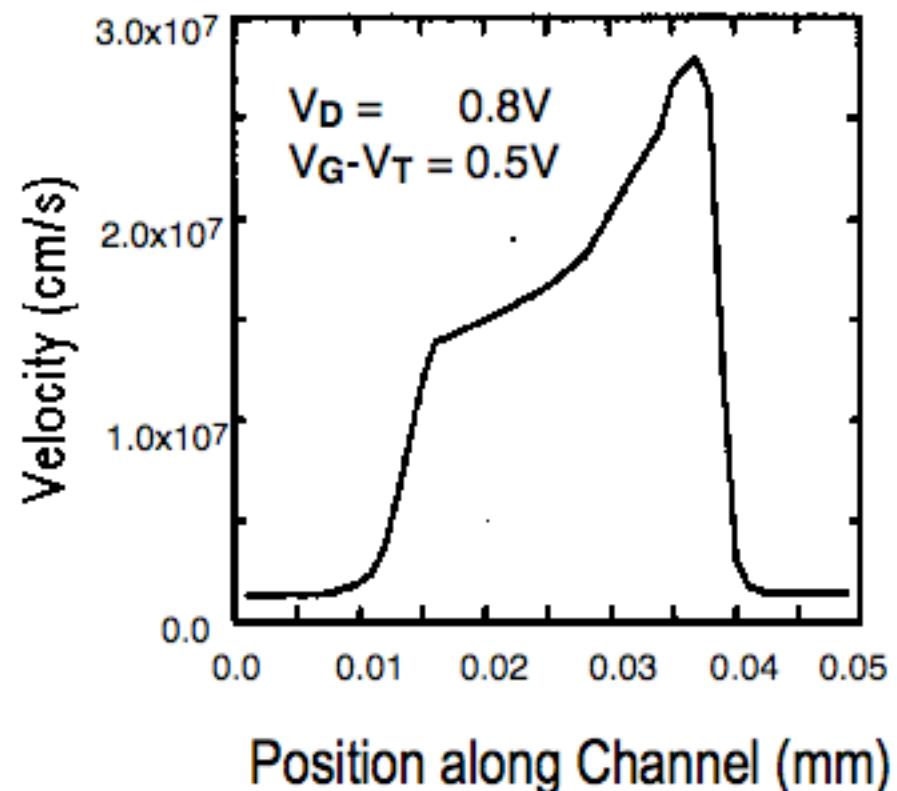
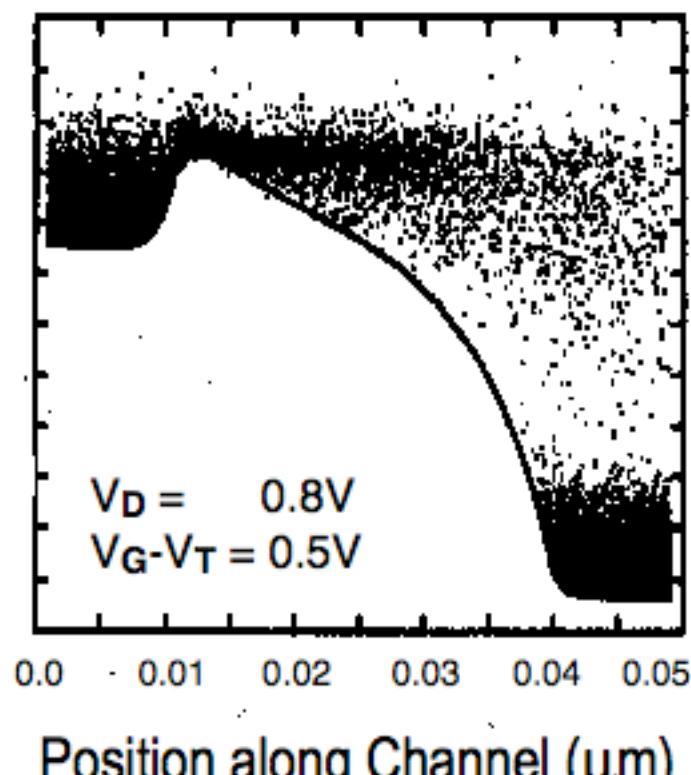


what is ballistic transport?



quasi-ballistic transport

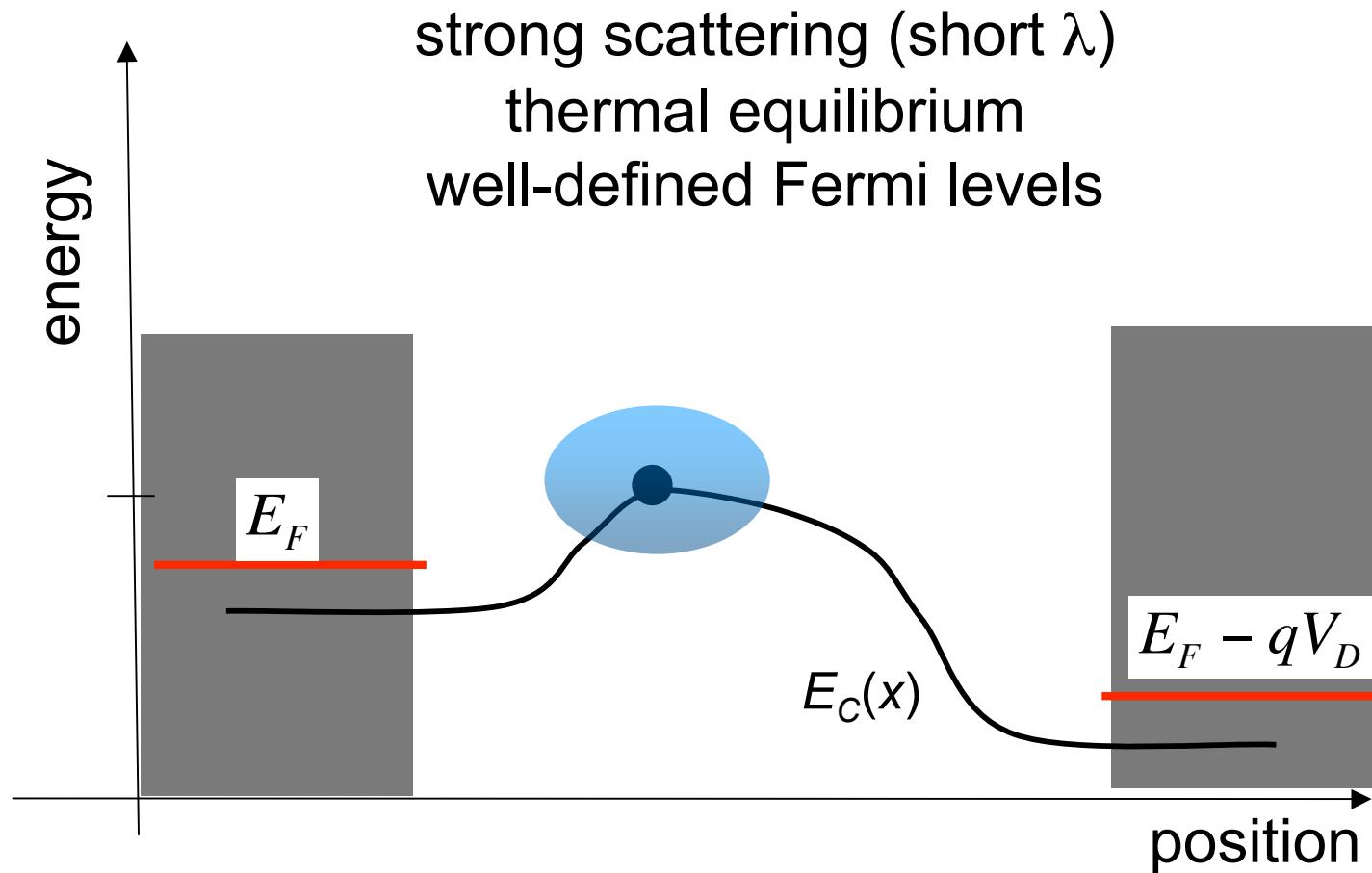
$$L \sim \lambda$$



Frank, Laux, and Fischetti, IEDM Tech. Dig., p. 553, 1992

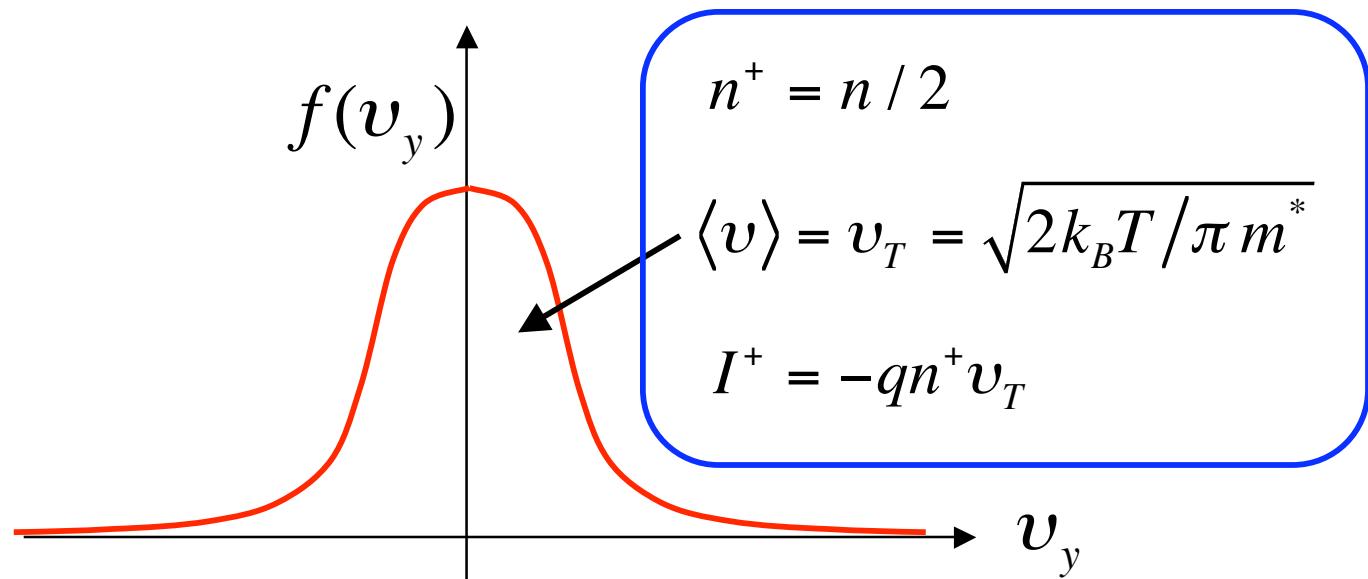
Lundstrom EE-612 F06

contacts



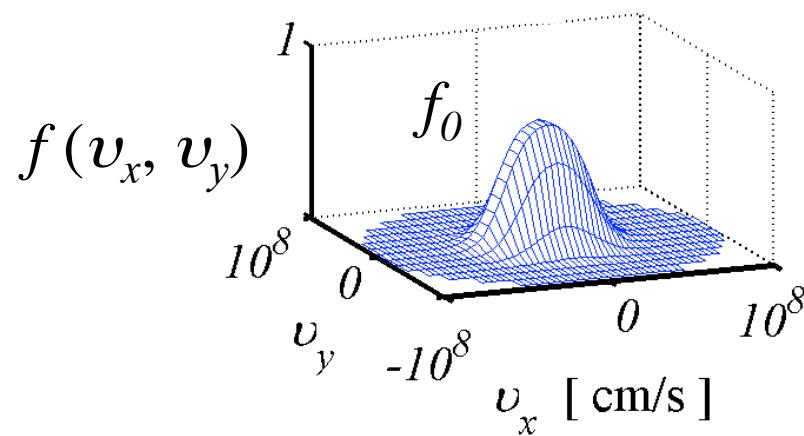
carrier distributions

$$f(E) = \frac{1}{1 + e^{(E-E_F)/k_B T}} \approx e^{(E_F - E)/k_B T} = e^{(E_F - E_C)/k_B T} \times e^{-m^* v^2 / 2 k_B T}$$



2D distributions

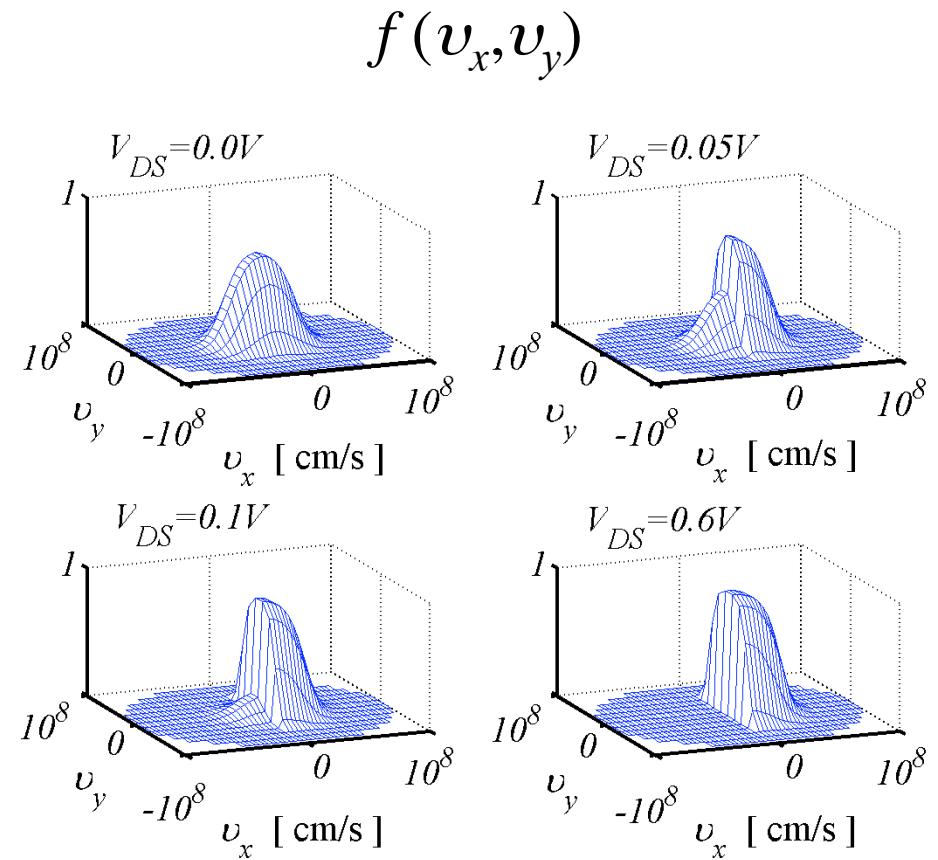
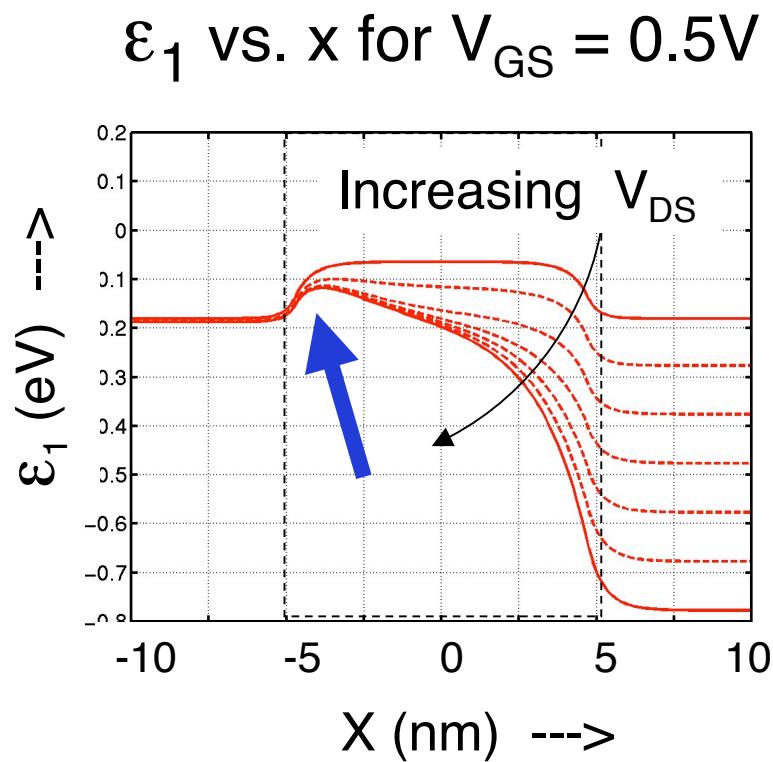
$$f(v_x, v_y) \propto e^{-\frac{1}{2}m^*(v_x^2 + v_y^2)/k_B T}$$



outline

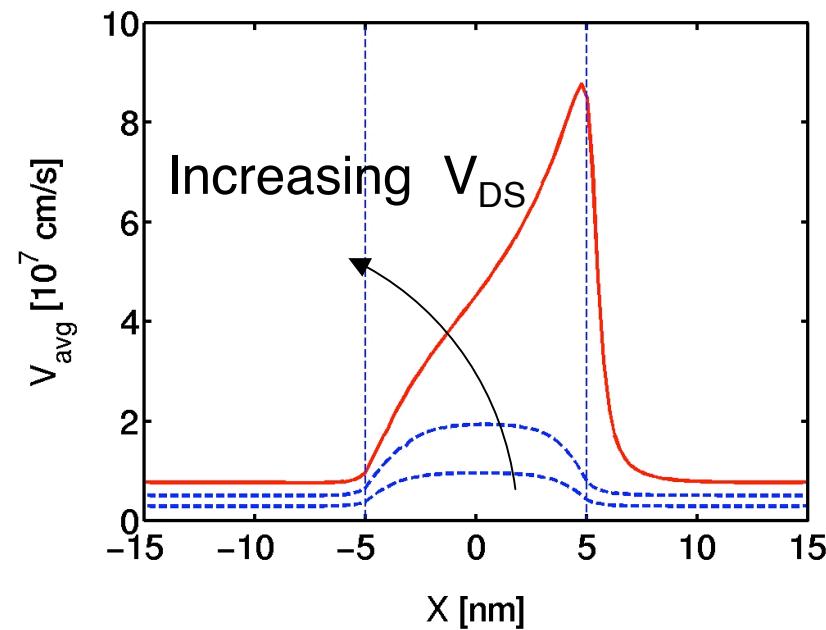
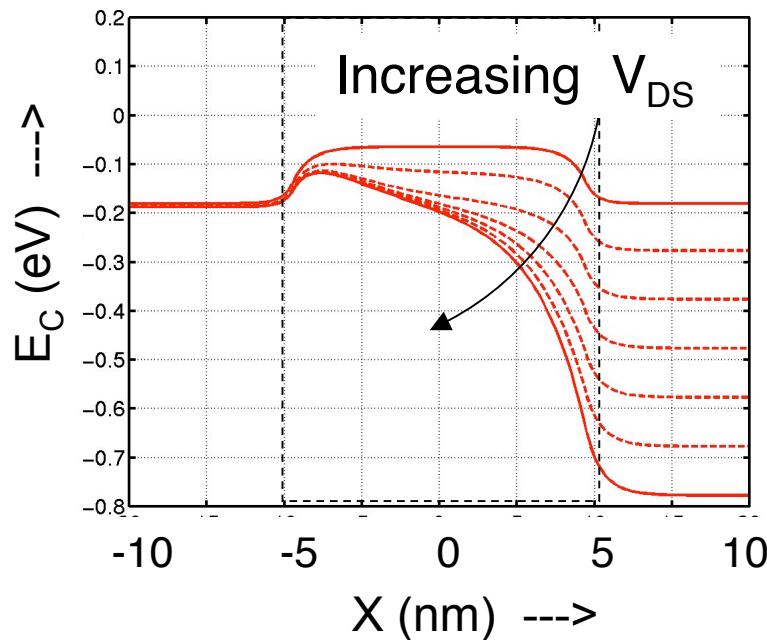
- 1) Introduction
- 2) Physics of the ballistic MOSFET**
- 3) Mathematical theory

$f(v_x, v_y)$ at the top-of-the-barrier



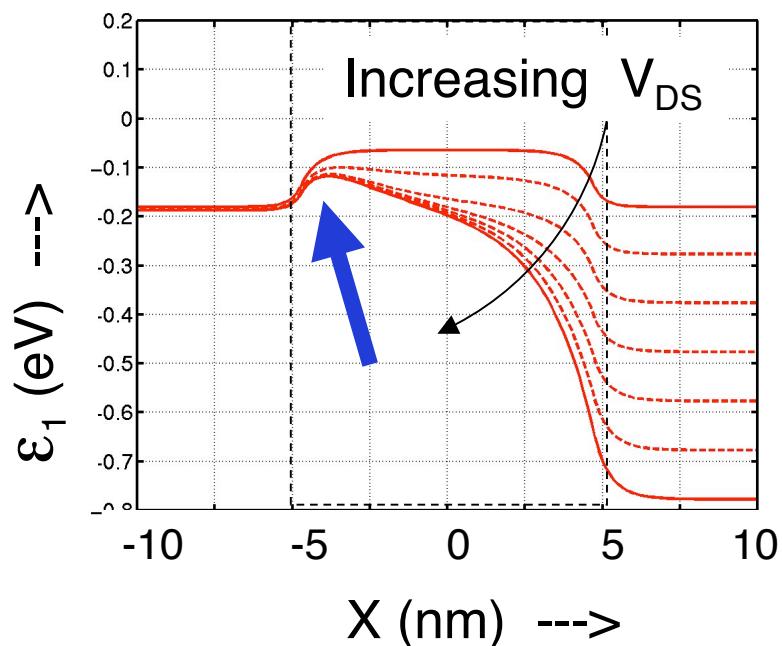
carrier velocity in a ballistic MOSFET

ϵ_1 vs. x for $V_{GS} = 0.5V$

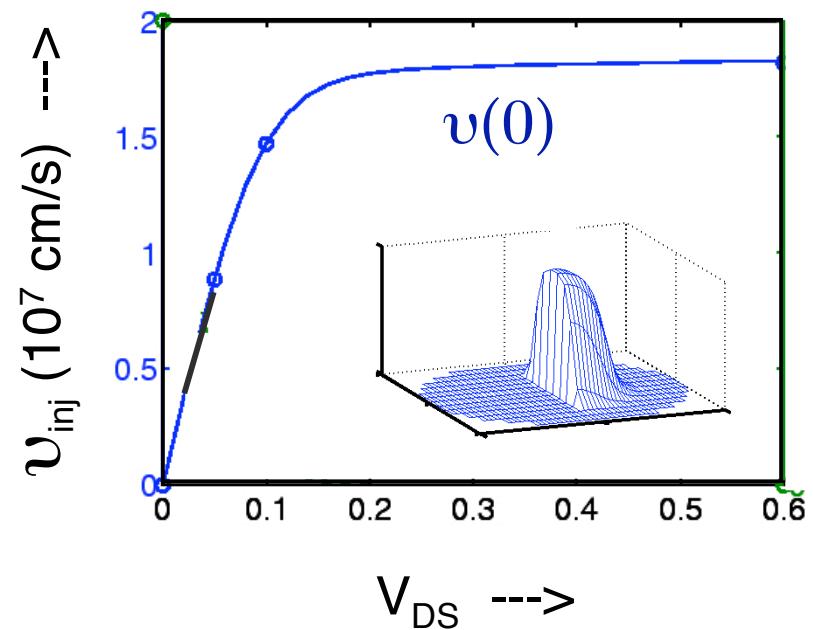


velocity saturation in a ballistic MOSFET

ϵ_1 vs. x for $V_{GS} = 0.5V$



$v(0) \rightarrow \tilde{v}_T$



"ballistic injection velocity" $\tilde{v}_T = \sqrt{\frac{2k_B T}{\pi m^*}} \left(\frac{\mathcal{F}_{1/2}(\eta_{F1})}{\mathcal{F}_0(\eta_{F1})} \right)^S$

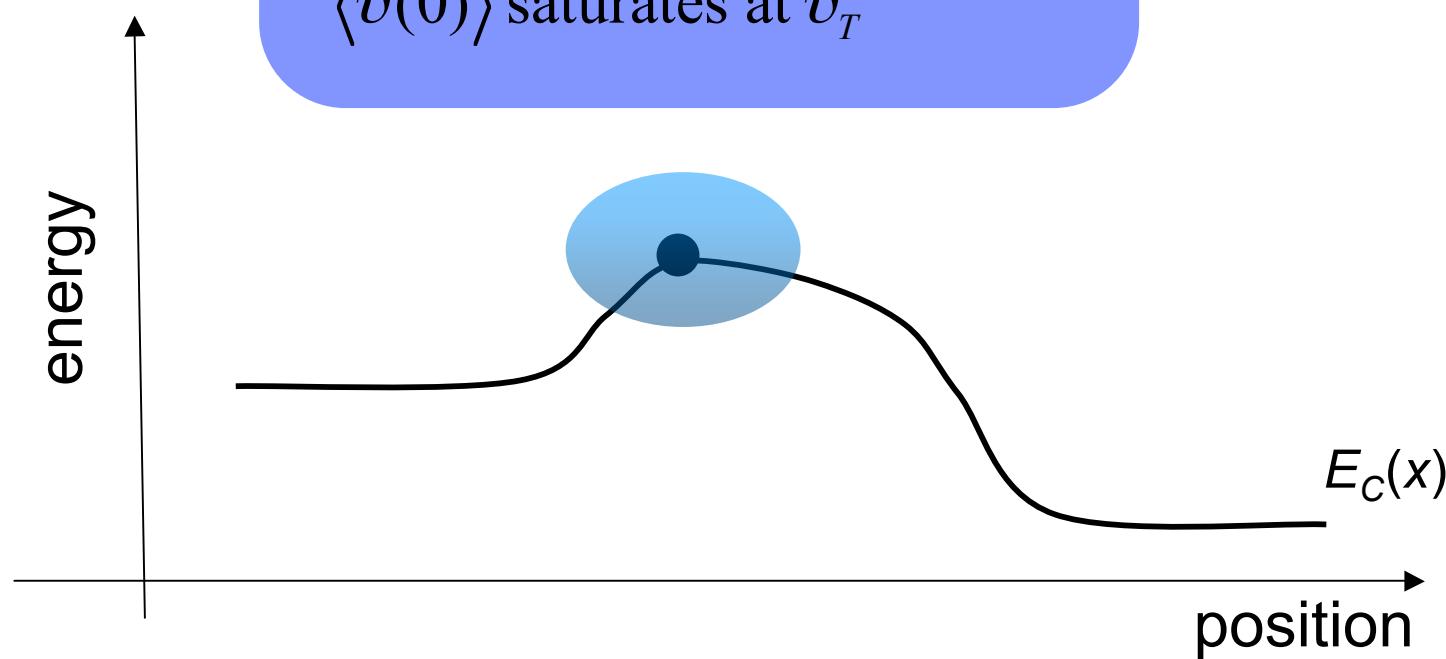
the ballistic MOSFET

$$Q_i(0) \approx -C_G(V_{GS} - V_T)$$

$Q_i^+(0), Q_i^-(0)$ depend on V_{DS}

$\langle v(0) \rangle$ depends on V_{DS}

$\langle v(0) \rangle$ saturates at \tilde{v}_T



outline

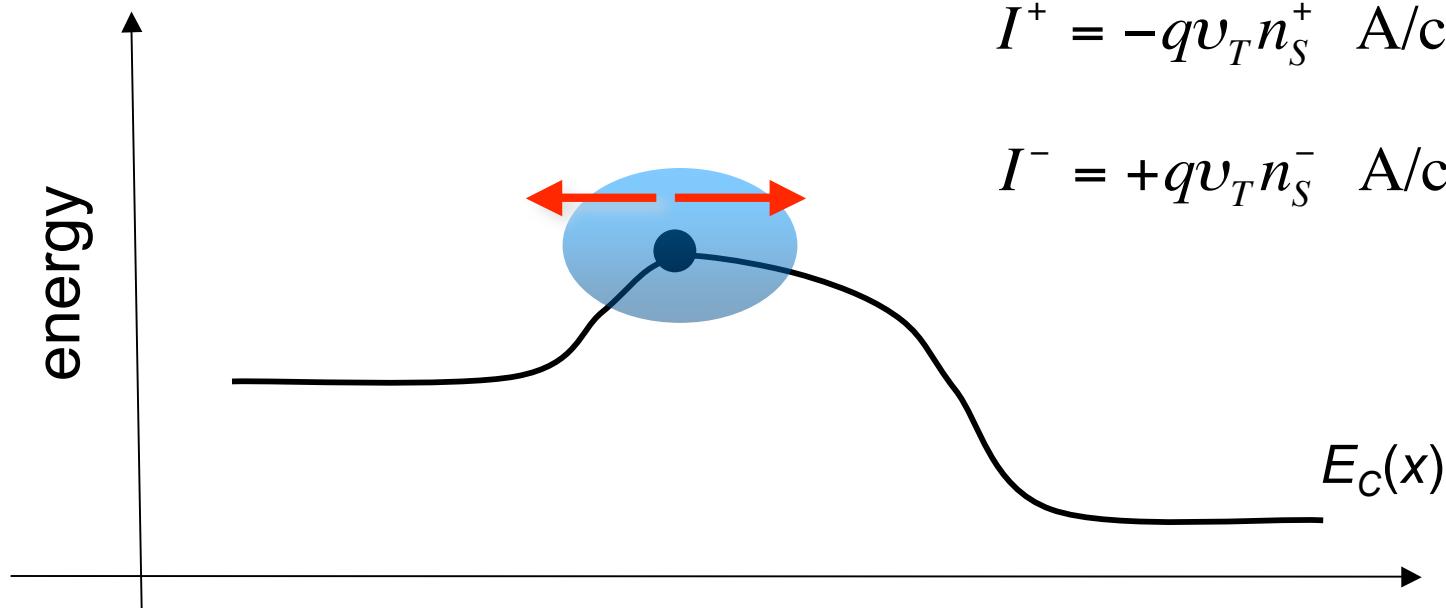
- 1) Introduction
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the ballistic MOSFET

$$n_S = n_S^+ + n_S^- \text{ cm}^{-2}$$

$$I^+ = -qv_T n_S^+ \text{ A/cm}$$

$$I^- = +qv_T n_S^- \text{ A/cm}$$



$$I_D = -W(I^+ - I^-) \text{ A}$$

the ballistic MOSFET (ii)

$$I_D = -W(I^+ - I^-) \text{ A}$$

$$I_D = Wq\upsilon_T(n_S^+ - n_S^-)$$

$$I_D = Wq\upsilon_T n_S^+ \left(1 - n_S^- / n_S^+\right) \quad (1)$$

$$\begin{cases} n_S = n_S^+ + n_S^- \text{ cm}^{-2} \\ I^+ = -q\upsilon_T n_S^+ \text{ A/cm} \\ I^- = +q\upsilon_T n_S^- \text{ A/cm} \end{cases}$$

$$qn_S = C_G(V_{GS} - V_T) = q(n_S^+ + n_S^-)$$

$$n_S^+ = \frac{C_G(V_{GS} - V_T)/q}{\left(1 + n_S^- / n_S^+\right)} \quad (2)$$

$$I_D = WC_G\upsilon_T(V_{GS} - V_T) \frac{\left(1 - n_S^- / n_S^+\right)}{\left(1 + n_S^- / n_S^+\right)}$$

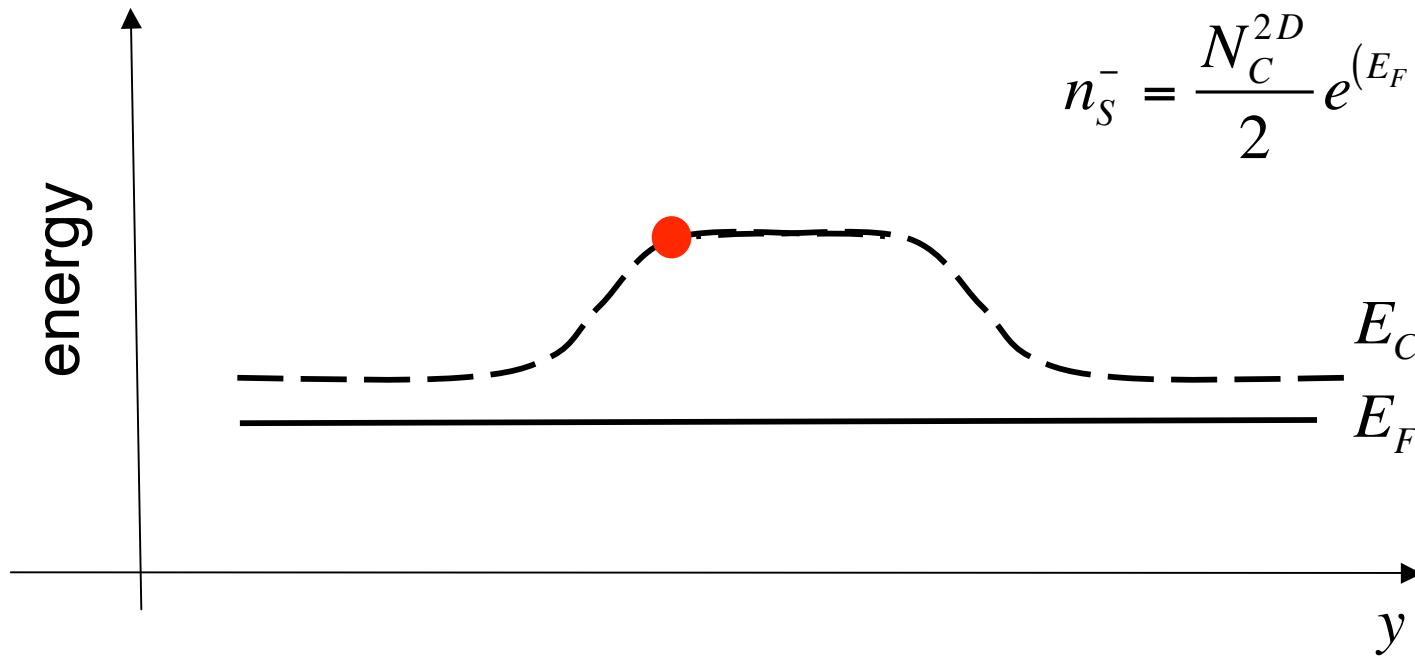
the ballistic MOSFET (iii)

Boltzmann statistics

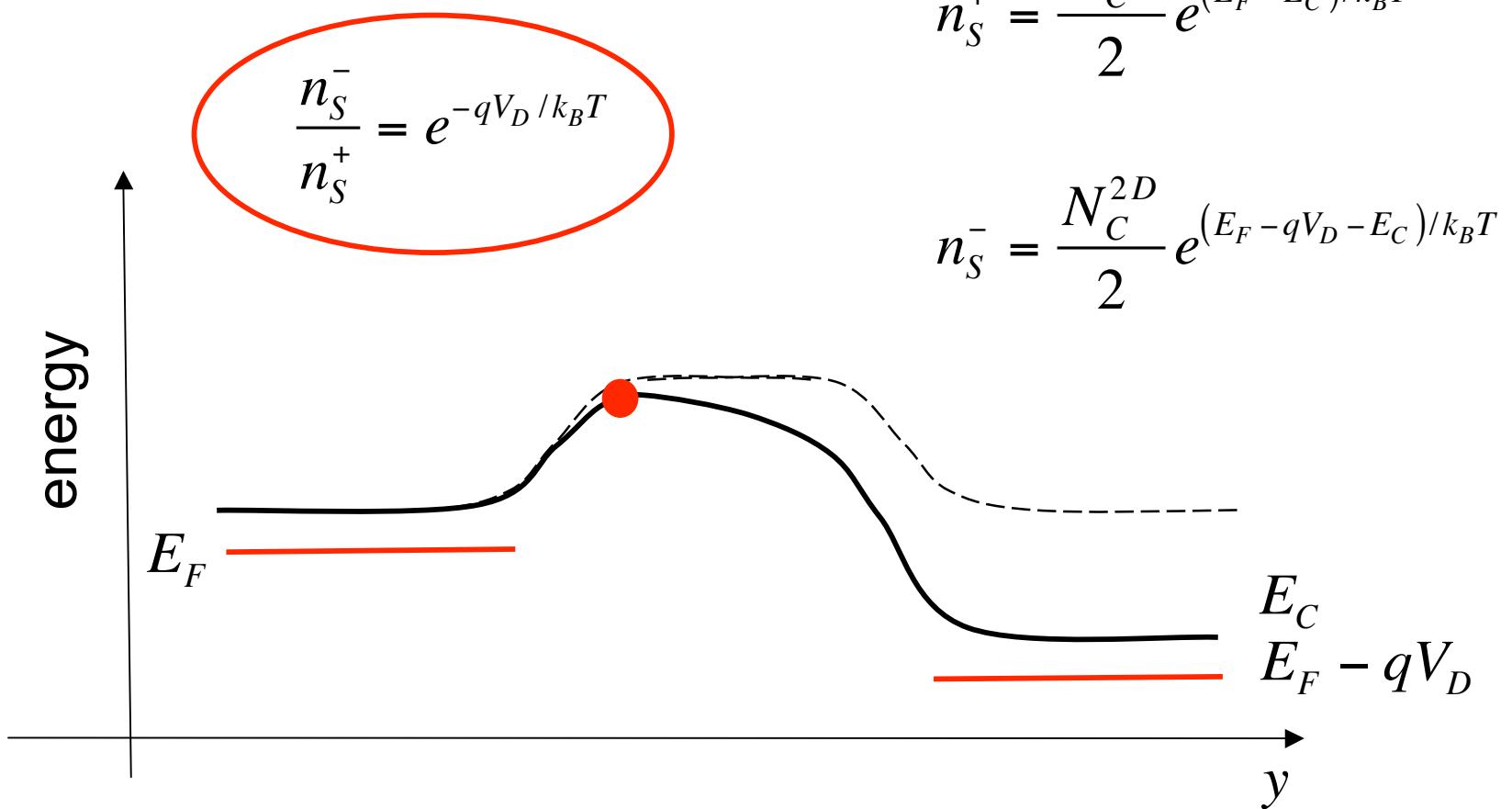
$$n_S = N_C^{2D} e^{(E_F - E_C)/k_B T}$$

$$n_S^+ = \frac{N_C^{2D}}{2} e^{(E_F - E_C)/k_B T}$$

$$n_S^- = \frac{N_C^{2D}}{2} e^{(E_F - E_C)/k_B T}$$



the ballistic MOSFET (iii)



the ballistic MOSFET (iv)

$$I_D = Wq\upsilon_T \left(n_S^+ - n_S^- \right)$$

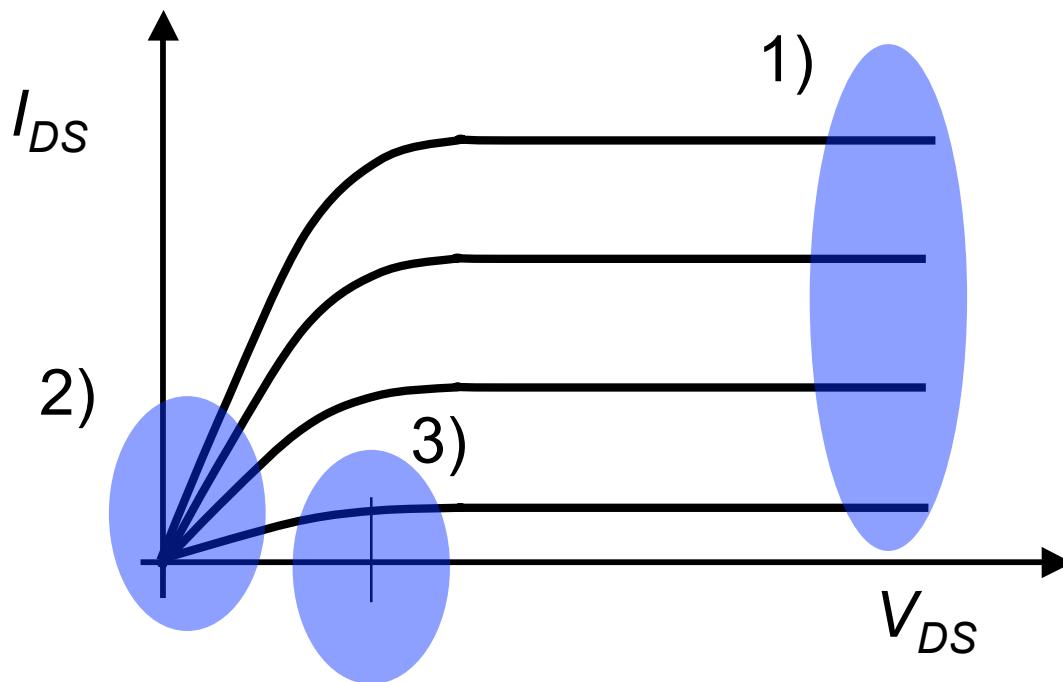
$$C_G(V_{GS} - V_T) = q \left(n_S^+ + n_S^- \right)$$

$$I_D = WC_G\upsilon_T \left(V_{GS} - V_T \right) \frac{\left(1 - n_S^- / n_S^+ \right)}{\left(1 + n_S^- / n_S^+ \right)}$$

$$\frac{n_S^-}{n_S^+} = e^{-qV_D/k_B T}$$

$$I_D = WC_G\upsilon_T \left(V_{GS} - V_T \right) \frac{\left(1 - e^{-qV_{DS}/k_B T} \right)}{\left(1 + e^{-qV_{DS}/k_B T} \right)}$$

the ballistic MOSFET IV



$$I_D = WC_G v_T \left(V_{GS} - V_T \right) \frac{\left(1 - e^{-qV_{DS}/k_B T} \right)}{\left(1 + e^{-qV_{DS}/k_B T} \right)}$$

1) high V_{DS}

$$I_D = WC_G v_T \left(V_{GS} - V_T \right) \frac{\left(1 - e^{-qV_{DS}/k_B T} \right)}{\left(1 + e^{-qV_{DS}/k_B T} \right)}$$

$$V_{DS} \gg k_B T / q$$

$$I_{ON} = WC_G v_T \left(V_{GS} - V_T \right)$$

$$v_T = \sqrt{2k_B T / \pi m^*} \approx 1.2 \times 10^7 \text{ cm/s}$$

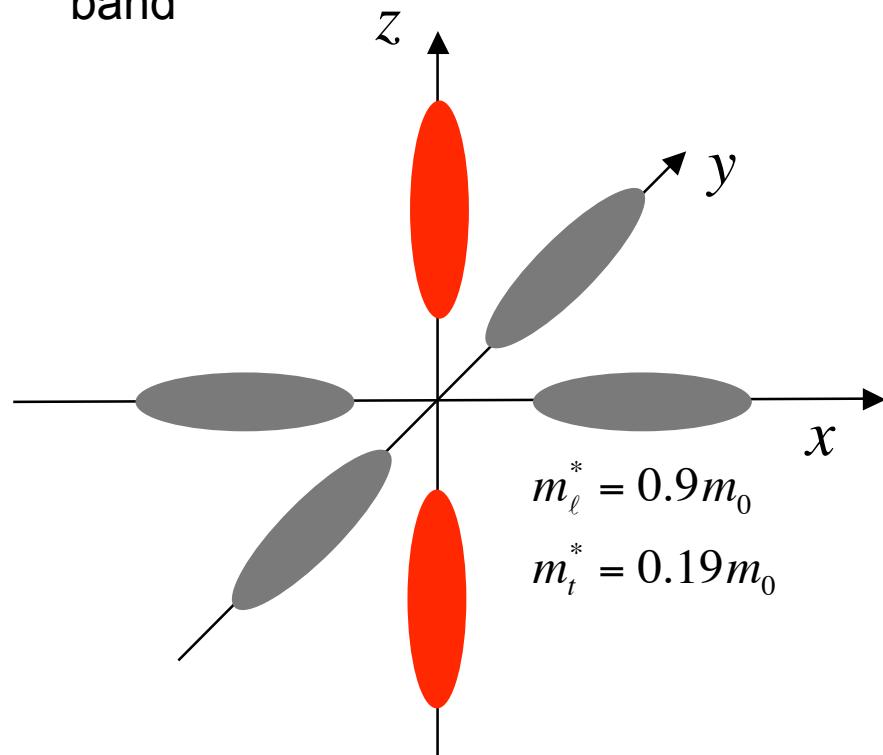
$$I_{ON} = WC_G v_{sat} \left(V_{GS} - V_T \right)$$

$$v_{sat} \approx 1 \times 10^7 \text{ cm/s}$$

(complete velocity saturation)

a comment on effective mass

Si conduction
band



for confinement in z -direction:

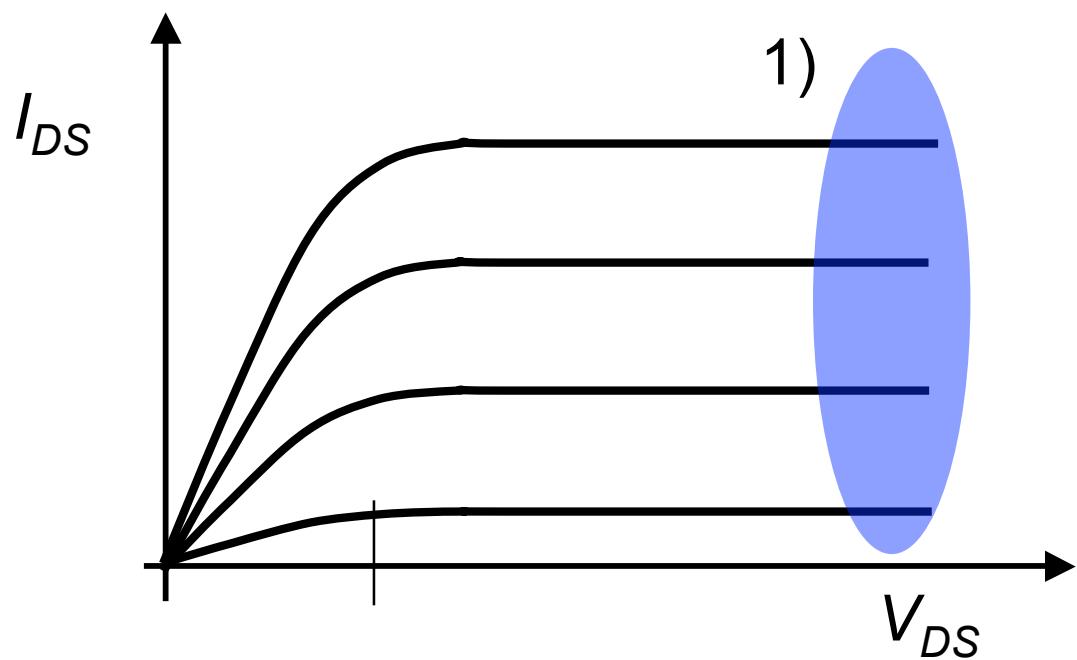
m_{ℓ}^* determines ϵ_1

transport in x - y plane with

m_t^*

$$v_T = \sqrt{2k_B T / \pi m_t^*}$$
$$\approx 1.2 \times 10^7 \text{ cm/s}$$

1) high V_{DS}



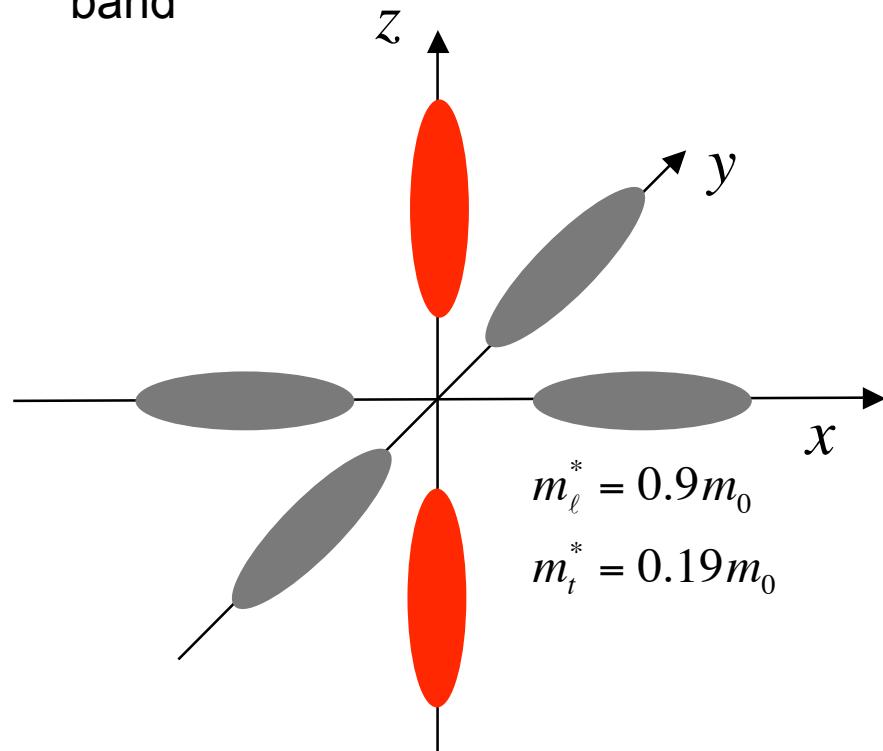
$$I_{ON} = WC_G v_T (V_{GS} - V_T)$$

$$I_{ON} \sim (V_{GS} - V_T)^\alpha$$

$\alpha = 1$ for ballistic transport
(nondegenerate)

a comment on effective mass

Si conduction
band



for confinement in z -direction:

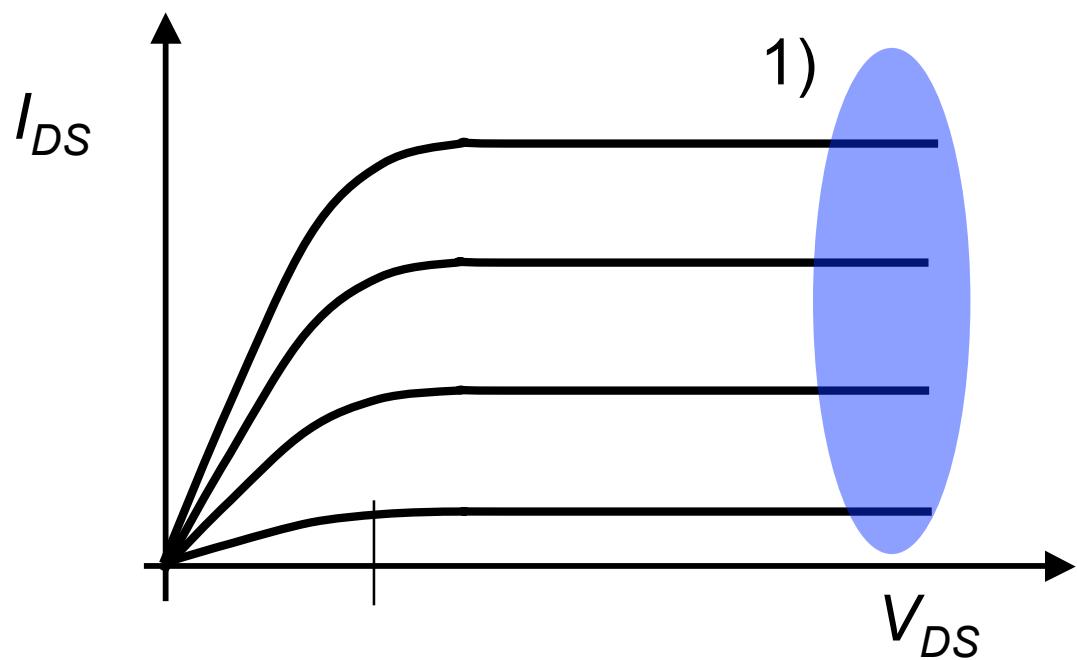
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$$v_T = \sqrt{2k_B T / \pi m_t^*}$$
$$\approx 1.2 \times 10^7 \text{ cm/s}$$

1) high V_{DS}



$$I_{ON} = WC_G v_T (V_{GS} - V_T)$$

$$I_{ON} \sim (V_{GS} - V_T)^\alpha$$

$\alpha = 1$ for ballistic transport
(nondegenerate)

2) low V_{DS}

$$I_D = WC_G v_T \left(V_{GS} - V_T \right) \frac{\left(1 - e^{-qV_{DS}/k_B T} \right)}{\left(1 + e^{-qV_{DS}/k_B T} \right)}$$

$$V_{DS} \ll k_B T / q$$

$$I_D = WC_G \frac{v_T}{2k_B T / q} \left(V_{GS} - V_T \right) V_{DS} = V_{DS} / R_{CH}^{ball}$$

$$R_{CH}^{ball} = \left(2k_B T / q \right) / WC_G v_T \left(V_{GS} - V_T \right)$$

$$R_{Ch}^{ball} = \left(2k_B T / q \right) / I_{ON}$$

independent of L!

2) low V_{DS}

drift-diffusion:

$$I_D = \frac{W}{L} C_G \mu_{eff} (V_{GS} - V_T) V_{DS} \quad R_{CH} = L / [WC_G \mu_{eff} (V_{GS} - V_T)]$$

ballistic:

$$I_D = WC_G \frac{v_T}{2k_B T / q} (V_{GS} - V_T) V_{DS}$$

$$I_D = \frac{W}{L} C_G \mu_B (V_{GS} - V_T) V_{DS}$$

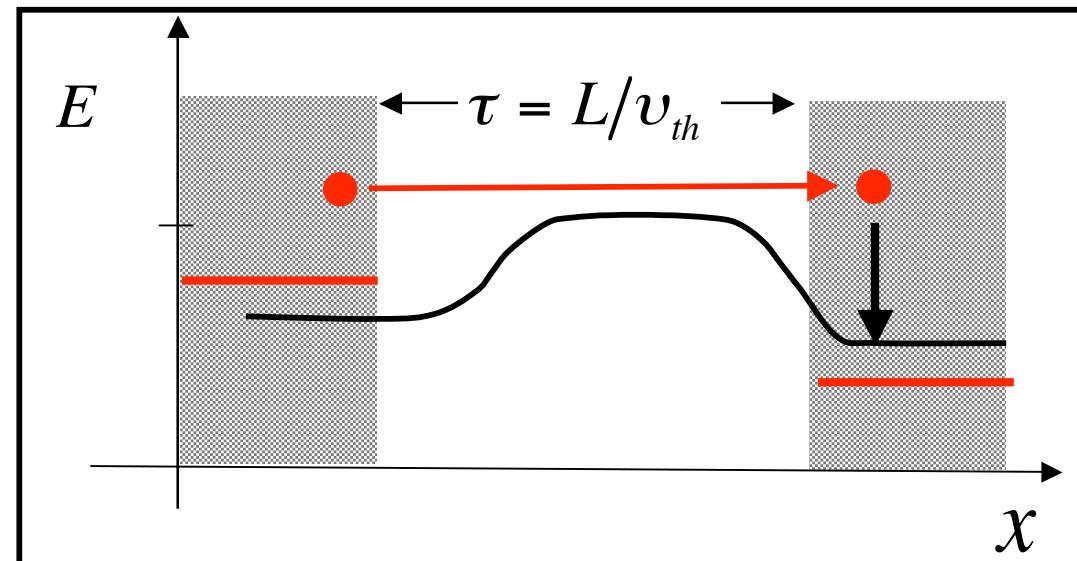
$$\mu_B = \left(\frac{v_T L}{2k_B T / q} \right)$$

2) low V_{DS}

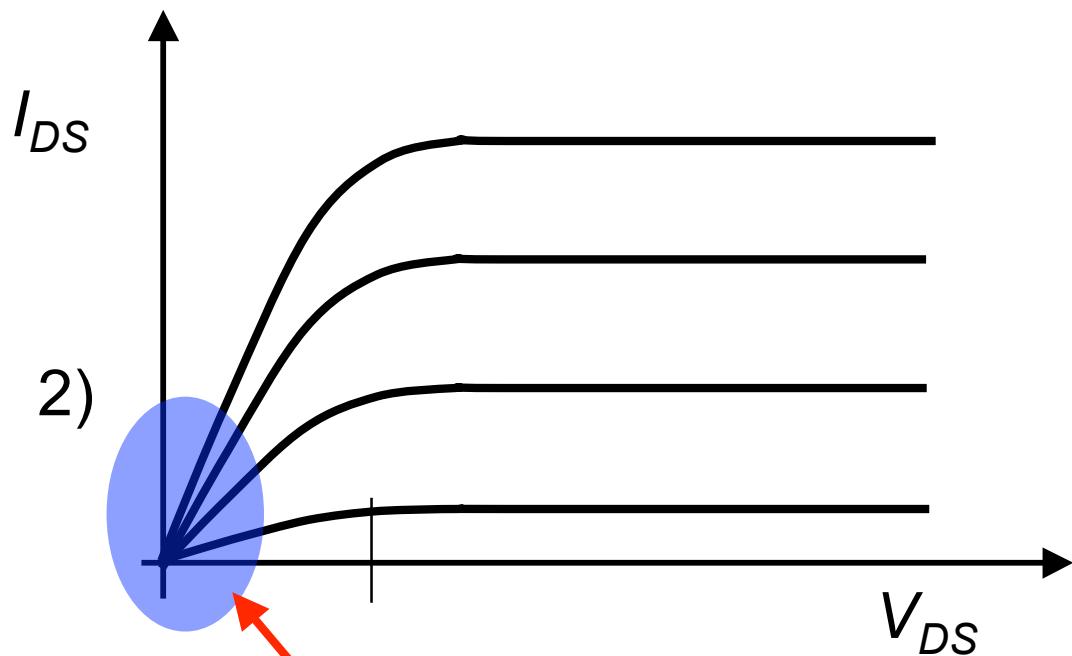
$$\mu_B \equiv \left(\frac{v_T L}{2k_B T / q} \right) \quad v_T = \sqrt{2k_B T / \pi m^*}$$

$$\mu_B = \left(\frac{q(L/v_{th})}{m^*} \right) \quad v_{th} = \sqrt{\pi k_B T / 2m^*}$$

$$\mu \equiv \left(\frac{q \tau}{m^*} \right)$$



2) low V_{DS}



$$I_D = WC_G \frac{v_T}{2k_B T / q} (V_{GS} - V_T) V_{DS}$$

$$R_{Ch}^{ball} = (2k_B T / q) / I_{ON} \approx 50 \Omega - \mu m$$

3) V_{DSAT}

$$I_D = W Q_i(0) \langle v(0) \rangle$$

$$I_D = W C_G (V_{GS} - V_T) v_T \frac{\left(1 - e^{-qV_{DS}/k_B T}\right)}{\left(1 + e^{-qV_{DS}/k_B T}\right)}$$



$$Q_i(0) \quad \langle v(0) \rangle$$

$$\langle v(0) \rangle = v_T \frac{\left(1 - e^{-qV_{DS}/k_B T}\right)}{\left(1 + e^{-qV_{DS}/k_B T}\right)}$$

I_D saturates because $\langle v(0) \rangle$ saturates
 $V_{DSAT} \approx 2k_B T / q$

velocity vs. “field” for a ballistic MOSFET

