What happened at the previous lectures?

Light interaction with small objects ($d < \lambda$)

- Insulators (Rayleigh Scattering, blue sky..)
- Semiconductors (Size dependent absorption, fluorescence..)
- Metals…Resonant absorption at $\omega_{sp}$

Microparticles

- Particles with $d \approx \lambda$ ($\lambda$-independent scattering, white clouds)
- Particles with $d >> \lambda$ (Intuitive ray-picture useful)

Dielectric photonic crystal

- Molding the flow of light
Metal Optics: An Introduction

Majority of optical components based on dielectrics

• High speed, high bandwidth ($\omega$), but…
• Does not scale well  $\Rightarrow$ Needed for large scale integration

Problems

Bending losses

Diffraction Limit

$\frac{\lambda_0}{2n_{\text{CORE}}}$

Optical mode in waveguide > $\frac{\lambda_0}{2n_{\text{CORE}}}$

Solutions?

Some fundamental problems!

Photonic functionality based on metals?!

Some:

What is a plasmon?

- Compare electron gas in a metal and real gas of molecules
- Metals are expected to allow for electron density waves: plasmons

Bulk plasmon

- Metals allow for EM wave propagation above the plasma frequency
  They become transparent!

Surface plasmon

- Sometimes called a surface plasmon-polariton (strong coupling to EM field)
Local Field Intensity Depends on Wavelength

Characteristics plasmon-polariton
- Strong localization of the EM field
- High local field intensities easy to obtain

Applications:
- Guiding of light below the diffraction limit (near-field optics)
- Non-linear optics
- Sensitive optical studies of surfaces and interfaces
- Bio-sensors
- Study film growth
- ……
Laser excitation \( \lambda = 532 \text{ nm} \)

8.1 \( \mu \text{m} \) Au rod

Light at the other end

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Plasmon-Polariton Excitation using a Launch Pad

• Array of 50 nanometer diameter Au particles spaced by 75 nanometer
• Guides electromagnetic energy at optical frequency below the diffraction limit
• Enables communication between nanoscale devices
• Information transport at speeds and densities exceeding current electronics

S.A. Maier et al., Advanced materials 13, 1501 (2001)
Purdue Near-Field Optical Microscope

- Nanonics MultiView 2000
- NSOM / AFM
- Tuning Fork Feedback Control
  - Normal or Shear Force
- Aperture tips down to 50 nm
- AFM tips down to 30 nm
- Radiation Source
  - 532 nm
Enhanced Transmission through Sub-\(\lambda\) Apertures

- Ag film with a 440 nm diameter hole surrounded by circular grooves
- Transmission enhancement of 10 x compared to a bare hole
- 3x more light than directly impinging on hole!
- Reason: Excitation of plasmon-polaritons

Optical Properties of an Electron Gas (Metal)

Dielectric constant of a free electron gas (no interband transitions)

- Consider a time varying field:
  \[ E(t) = \text{Re}\left\{ E(\omega) \exp(-i\omega t) \right\} \]

- Equation of motion electron (no damping)
  \[ m \frac{d^2 r}{dt^2} = -eE \]
  \[ p(t) = -er(t) \]

- Dipole moment electron
  \[ \frac{d^2 p}{dt^2} = e^2 E \]

- Harmonic time dependence
  \[ p(t) = \text{Re}\left\{ p(\omega) \exp(-i\omega t) \right\} \]

- Substitution \( p \) into Eq. of motion:
  \[ -m\omega^2 p(\omega) = e^2 E(\omega) \]

- This can be manipulated into:
  \[ p(\omega) = -\frac{e^2}{m\omega^2} \frac{1}{\varepsilon_0} E(\omega) \]

- The dielectric constant is:
  \[ \varepsilon_r = 1 + \kappa = \frac{Np(\omega)}{\varepsilon_0 E(\omega)} = 1 - \frac{Ne^2}{\varepsilon_0 m\omega^2} = 1 - \frac{\omega_p^2}{\omega^2} \]
Dispersion Relation for EM Waves in Electron Gas

Determination of dispersion relation for bulk plasmons

- The wave equation is given by:
  \[
  \frac{\varepsilon_r}{c^2} \frac{\partial^2 E(r,t)}{\partial t^2} = \nabla^2 E(r,t)
  \]

- Investigate solutions of the form:
  \[
  E(r,t) = \text{Re} \left\{ E(r,\omega) \exp(ik \cdot r - i\omega t) \right\}
  \]

  \[\implies \omega^2 \varepsilon_r = c^2 k^2\]

- Dielectric constant:
  \[
  \varepsilon_r = 1 - \frac{\omega_p^2}{\omega^2}
  \]

  \[\implies \omega^2 \left(1 - \frac{\omega_p^2}{\omega^2}\right) = \omega^2 - \omega_p^2 = c^2 k^2\]

- Dispersion relation:
  \[
  \omega = \sqrt{\omega_p^2 + c^2 k^2}
  \]

  No allowed propagating modes
  (imaginary k)

Note1: Solutions lie above light line

Note2: Metals: \( \hbar \omega_p \approx 10 \text{ eV} \); Semiconductors \( \hbar \omega_p < 0.5 \text{ eV} \) (depending on dopant conc.)
Dispersion Relation Surface-Plasmon Polaritons

Solve Maxwell’s equations with boundary conditions

• We are looking for solutions that look like:

\[ \mathbf{E}_{\text{d}} = (0, H_{yd}, 0) \exp i \left( k_{xd} x + k_{zd} z - \omega t \right) \]
\[ \mathbf{E}_{\text{m}} = (E_{xm}, 0, E_{zm}) \exp i \left( k_{xm} x + k_{zm} z - \omega t \right) \]
\[ \mathbf{H}_{\text{d}} = (0, H_{ym}, 0) \exp i \left( k_{ym} x + k_{zm} z - \omega t \right) \]
\[ \mathbf{H}_{\text{m}} = (E_{xm}, 0, E_{zm}) \exp i \left( k_{xm} x + k_{zm} z - \omega t \right) \]

• Mathematically:

\[ \nabla \cdot \varepsilon_i \mathbf{E} = 0 \quad \nabla \cdot \mathbf{H} = 0 \quad \nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t} \quad \nabla \times \mathbf{H} = \varepsilon_i \frac{\partial \mathbf{E}}{\partial t} \]

• At the boundary:

\[ E_{x,m} = E_{x,d} \quad \varepsilon_m E_{zm} = \varepsilon_d E_{zm} \quad H_{ym} = H_{yd} \]
Dispersion Relation Surface-Plasmon Polaritons

- Start with curl equation for $\mathbf{H}$ in medium i (as we did for EM waves in vacuum)

$$\nabla \times \mathbf{H}_i = \varepsilon_i \frac{\partial \mathbf{E}_i}{\partial t}$$

where

$$\mathbf{H}_i = (0, H_{yi}, 0) \exp i(k_{xi}x + k_{zi}z - \omega t)$$

$$\mathbf{E}_i = (E_{xi}, 0, E_{zi}) \exp i(k_{xi}x + k_{zi}z - \omega t)$$

$$\left( \frac{\partial H_{zi}}{\partial y} - \frac{\partial H_{yi}}{\partial z}, \frac{\partial H_{xi}}{\partial z} - \frac{\partial H_{zi}}{\partial x}, \frac{\partial H_{yi}}{\partial x} - \frac{\partial H_{xi}}{\partial y} \right) = (ik_{zi}H_{yi}, 0, ik_{xi}H_{yi}) = (-i\omega \varepsilon_i E_{xi}, 0, i\omega \varepsilon_i E_{zi})$$

- We will use that: $k_{zi}H_{yi} = -\omega \varepsilon_i E_{xi} \Rightarrow \begin{cases} k_{zm}H_{yi} = -\omega \varepsilon_m E_{xm} \\ k_{zd}H_{yd} = -\omega \varepsilon_d E_{xd} \end{cases}$

$$\frac{k_{zm}}{\varepsilon_m} H_{ym} = \frac{k_{zd}}{\varepsilon_d} H_{yd}$$

- $E_{\parallel}$ across boundary is continuous:

$$E_{x,m} = E_{x,d}$$

- $H_{\parallel}$ across boundary is continuous:

$$H_{ym} = H_{yd} \Rightarrow \begin{cases} \frac{k_{zm}}{\varepsilon_m} H_{ym} = \frac{k_{zd}}{\varepsilon_d} H_{yd} \end{cases}$$

Combine with:

$$\frac{k_{zm}}{\varepsilon_m} = \frac{k_{zd}}{\varepsilon_d}$$
Dispersion Relation Surface-Plasmon Polaritons

Relations between k vectors

- Condition for SP’s to exist:
  \[ \frac{k_{zm}}{\varepsilon_m} = \frac{k_{zd}}{\varepsilon_d} \]

- Relation for \( k_x \) (Continuity \( E_{//}, H_{//} \)):
  \[ k_{xm} = k_{xd} \]

- For any EM wave:
  \[ k_x^2 + k_{zi}^2 = \varepsilon_i \left( \frac{\omega}{c} \right)^2 \]

- Both in the metal and dielectric:
  \[ k_{sp} = k_x = \sqrt{\varepsilon_i \left( \frac{\omega}{c} \right)^2 - k_{zi}^2} \]

\[ \frac{k_{zm}}{\varepsilon_m} = \frac{k_{zd}}{\varepsilon_d} \]

Dispersion relation

\[ k_x = \frac{\omega}{c} \left( \frac{\varepsilon_m \varepsilon_d}{\varepsilon_m + \varepsilon_d} \right)^{1/2} \]

homework 😊
Dispersion Relation Surface-Plasmon Polaritons

Plot of the dispersion relation

• Last page: \[ k_x = \frac{\omega}{c} \left( \frac{\varepsilon_m \varepsilon_d}{\varepsilon_m + \varepsilon_d} \right)^{1/2} \]

• Plot dielectric constants

• Low \( \omega \): \[ k_x = \frac{\omega}{c} \lim_{\varepsilon_m \to -\infty} \left( \frac{\varepsilon_m \varepsilon_d}{\varepsilon_m + \varepsilon_d} \right)^{1/2} \approx \frac{\omega}{c} \sqrt{\varepsilon_d} \]

• At \( \omega = \omega_{sp} \) (when \( \varepsilon_m = -\varepsilon_d \)): \( k_x \to \infty \)

• Note: Solution lies below the light line
Dispersion relation plasma modes and SPP

- Note: Higher index medium on metal results in lower $\omega_{sp}$

$$\omega = \omega_{sp} \text{ when: } \epsilon_m = 1 - \frac{\omega_p^2}{\omega^2} = -\epsilon_d \quad \Rightarrow \quad \omega^2 - \omega_p^2 = -\epsilon_d \omega^2 \quad \Rightarrow \quad \omega^2 = \frac{\omega_p^2}{1 + \epsilon_d} \quad \Rightarrow \quad \omega = \frac{\omega_p}{\sqrt{1 + \epsilon_d}}$$
Excitation Surface-Plasmon Polaritons with Electrons

Excitation with electrons

• First experiments with high energy electrons
• Measurement: Energy loss

  Direction of e’s:

• Whole dispersion relation can be investigated

• Low k’s are hard!

Example: 50 keV has a $\lambda = 0.005$ nm $\ll \lambda_{\text{light}}$

$\Rightarrow k_{\text{electron}} \gg k_{\text{light}}$

$\Rightarrow$ Stringent requirement on divergence e-beam
The operating speed of data transporting and processing systems

- The ever-increasing need for faster information processing and transport is undeniable

- Electronic components are running out of steam due to issues with RC-delay times

Nanophotonics with Plasmonics: A logical next step?
As data rates AND component packing densities INCREASE, electrical interconnects become progressively limited by \( RC \)-delay:

\[
R \propto \frac{L}{A} \quad \text{and} \quad C \propto L \quad \Rightarrow \quad B_{\text{max}} \propto \frac{1}{RC} \propto \frac{A}{L^2}
\]

\[
\Rightarrow B_{\text{max}} \leq 10^{15} \times \frac{A}{L^2} \quad \text{(bit/s)} (A \ll L^2!)
\]

Electronics is aspect-ratio limited in speed!
The bit rate in optical communications is fundamentally limited only by the carrier frequency: $B_{\text{max}} < f \sim 100$ Tbit/s (!), but light propagation is subjected to diffraction:

\[ n_{\text{core}} = n_{\text{clad}} + \delta n = n + \delta n \Rightarrow V = \frac{2\pi}{\lambda} a \sqrt{n_{\text{core}}^2 - n_{\text{clad}}^2} \approx \frac{2\vartheta}{\varepsilon} a \sqrt{2n \delta n} \]

well–guided mode: $V \propto \pi \Rightarrow a \approx \lambda / 2\sqrt{2n \delta n}$ — mode size: $\delta n \ll 1$ (!)

Photonics is diffraction- limited in size!
Why Plasmonics?

Dispersion Relation for SPPs:

$$k_{sp} = \frac{\omega}{c} \sqrt{\frac{\epsilon_m \epsilon_d}{\epsilon_m + \epsilon_d}}$$

SP wavelengths can reach nanoscale at optical frequencies! SPPs are “x-ray waves” with optical frequencies!
Why nanophotonics needs plasmons?

- Graph of the operating regimes of different technologies

- Plasmonics will enable an improved synergy between electronic and photonic devices
  - Plasmonics naturally interfaces with similar size electronic components
  - Plasmonics naturally interfaces with similar operating speed photonic networks

Courtesy of M. Brongersma