

# **EE-612:**

# **Lecture 14:**

# **Effective Mobility**

**Mark Lundstrom**  
Electrical and Computer Engineering  
Purdue University  
West Lafayette, IN USA  
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# outline

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- 1) Review of mobility
- 2) Effective mobility
- 3) Physics of the effective mobility
- 4) Measuring effective mobility
- 5) Discussion

# mobility

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$$\mu = \frac{q\tau}{m^*}$$

$1/\tau$  = probability per second of scattering ('scattering rate')

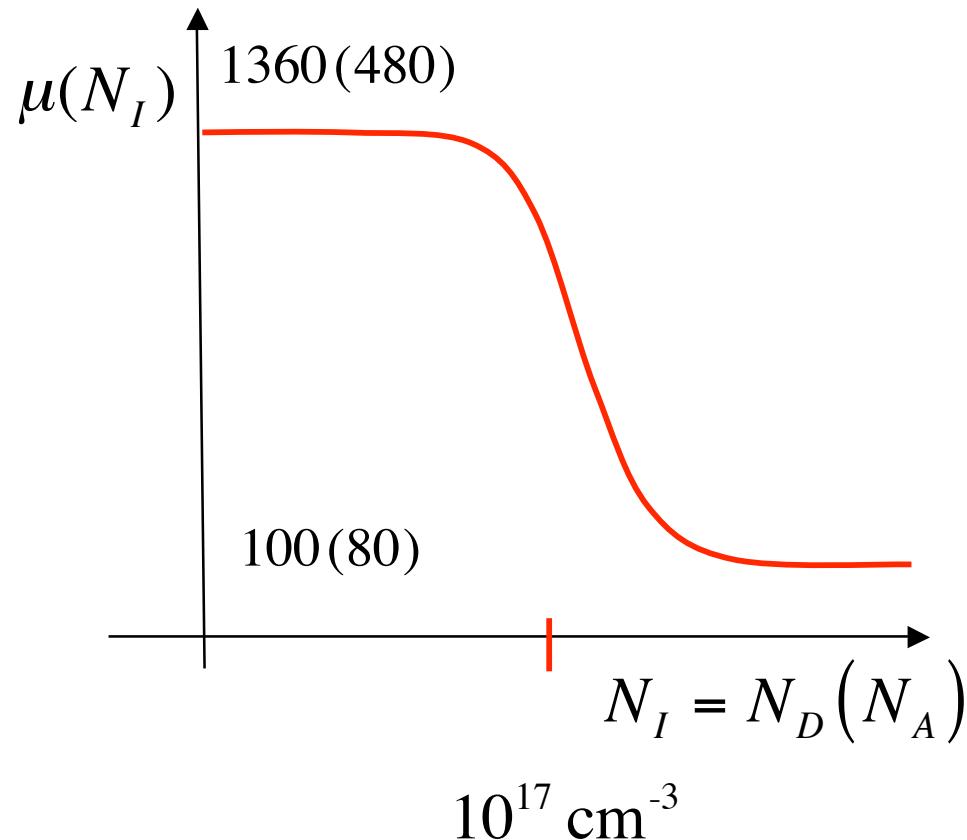
$$\mu_1 = \frac{q\tau_1}{m^*} \quad \mu_2 = \frac{q\tau_2}{m^*} \quad \mu_{12} = ?$$

$$\mu_{12} \neq \mu_1 + \mu_2$$

$$\mu_{12} = \left( \frac{1}{\mu_1} + \frac{1}{\mu_2} \right)^{-1}$$

# ionized impurity scattering (silicon)

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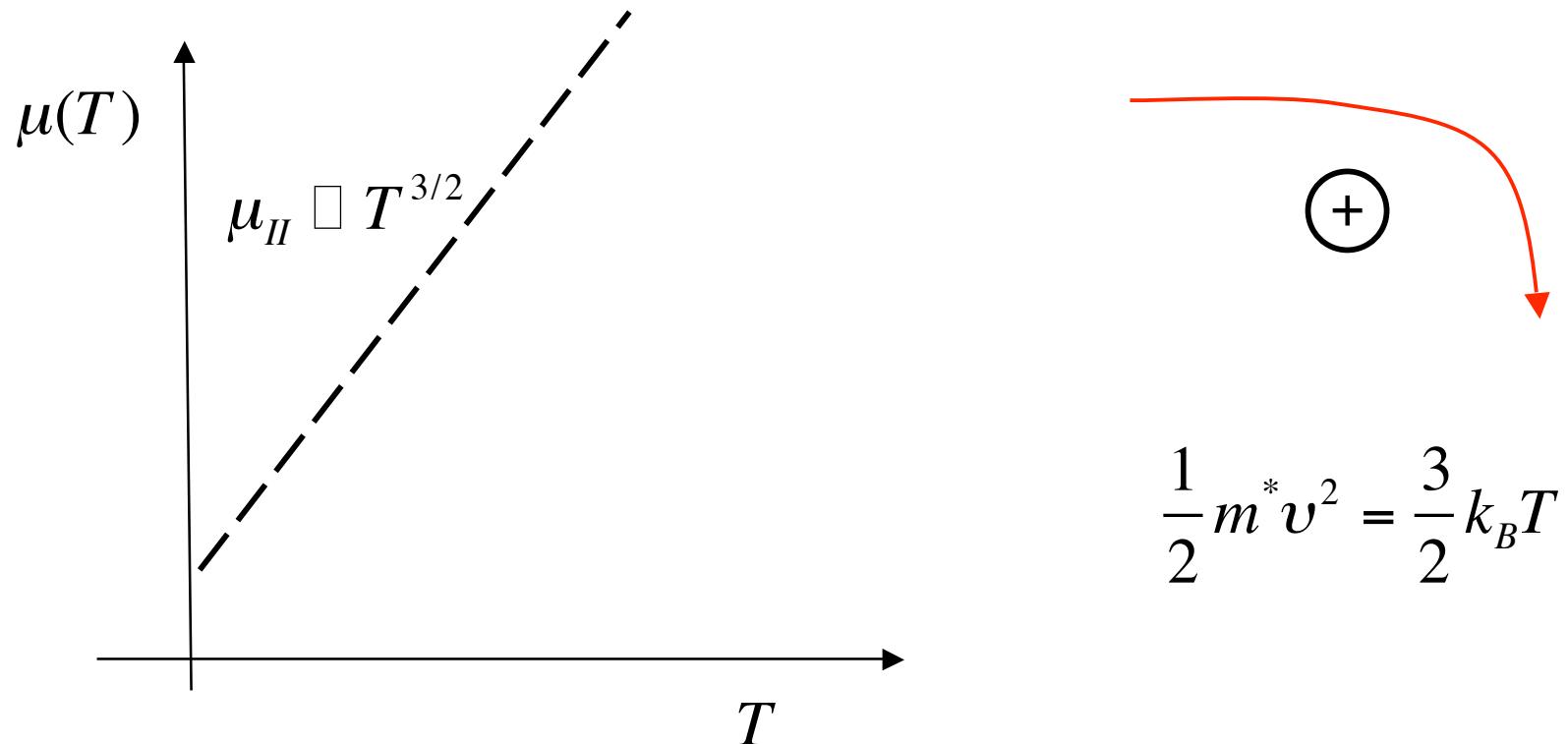


$$\mu_{II} \square 1/N_I$$

$$\mu = \left( \frac{1}{\mu_L} + \frac{1}{\mu_{II}} \right)^{-1}$$

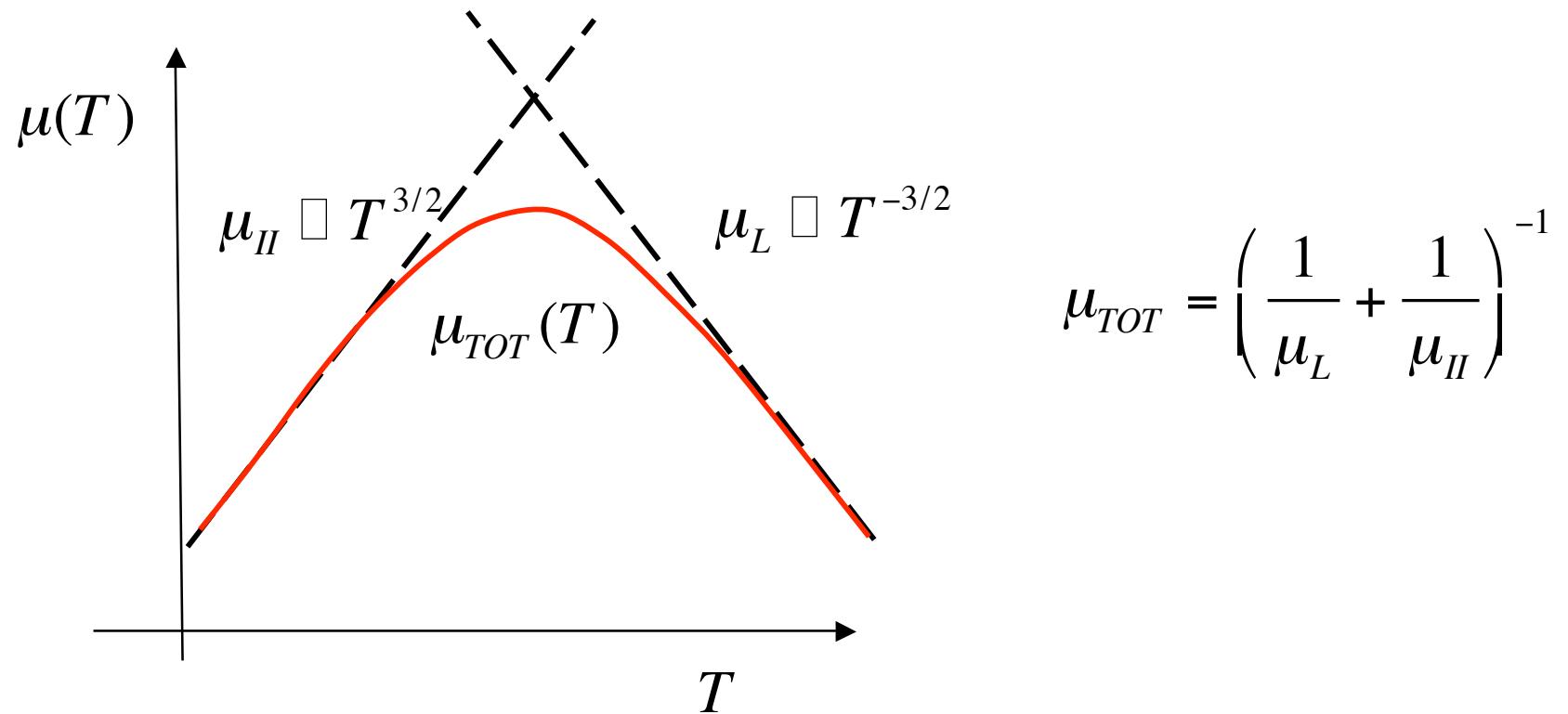
$$\mu_I (N_I \ll N_{CR}) \approx \mu_L$$

# $\mu_{II}$ temperature dependence (silicon)



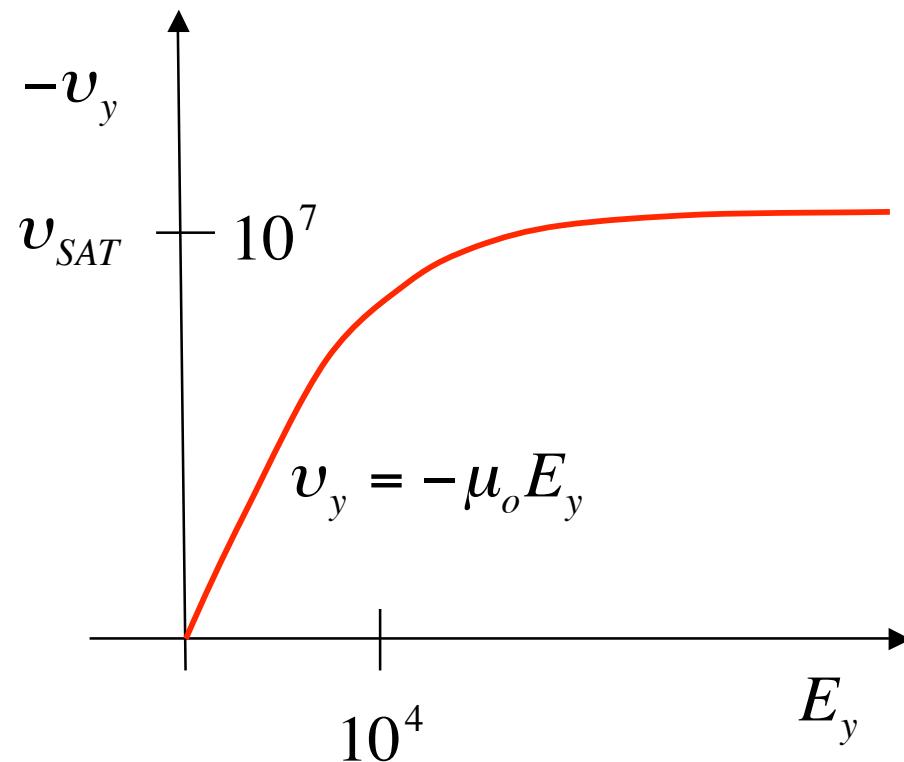
# lattice scattering (silicon)

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# field-dependent mobility (silicon)

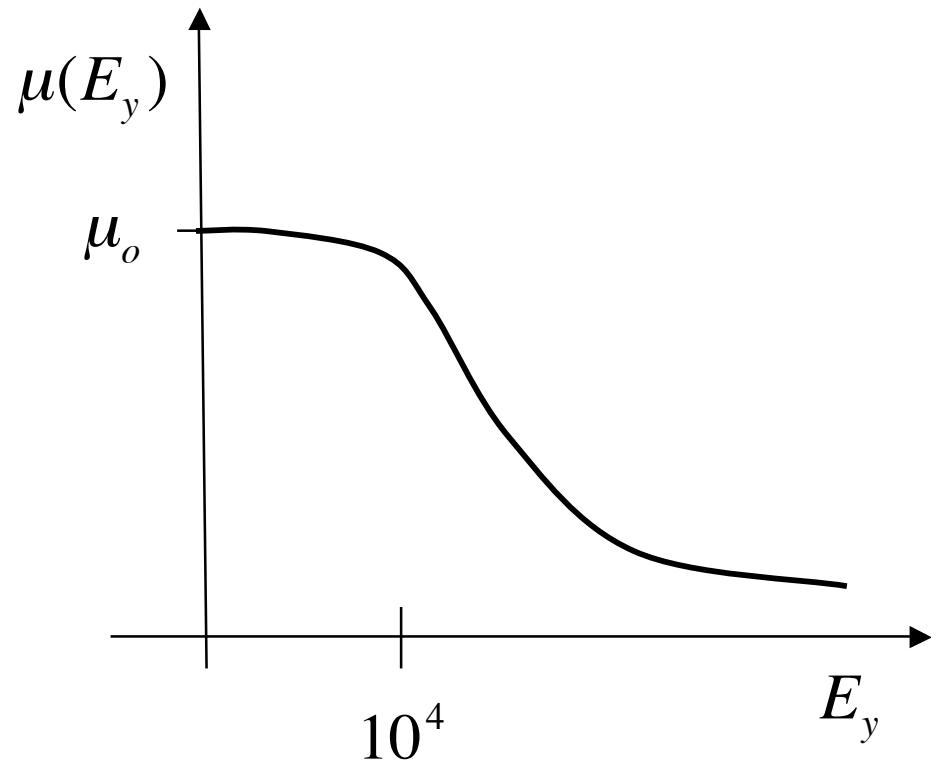
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$$v_y = \frac{-\mu_o E_y}{\sqrt{1 + (E_y/E_{cr})^2}}$$

## field-dependent mobility (ii)

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$$\begin{aligned}\mu(E_y) &\equiv \frac{-v_y}{E_y} \\ &= \frac{\mu_o}{\sqrt{1 + (E_y/E_{cr})^2}}\end{aligned}$$

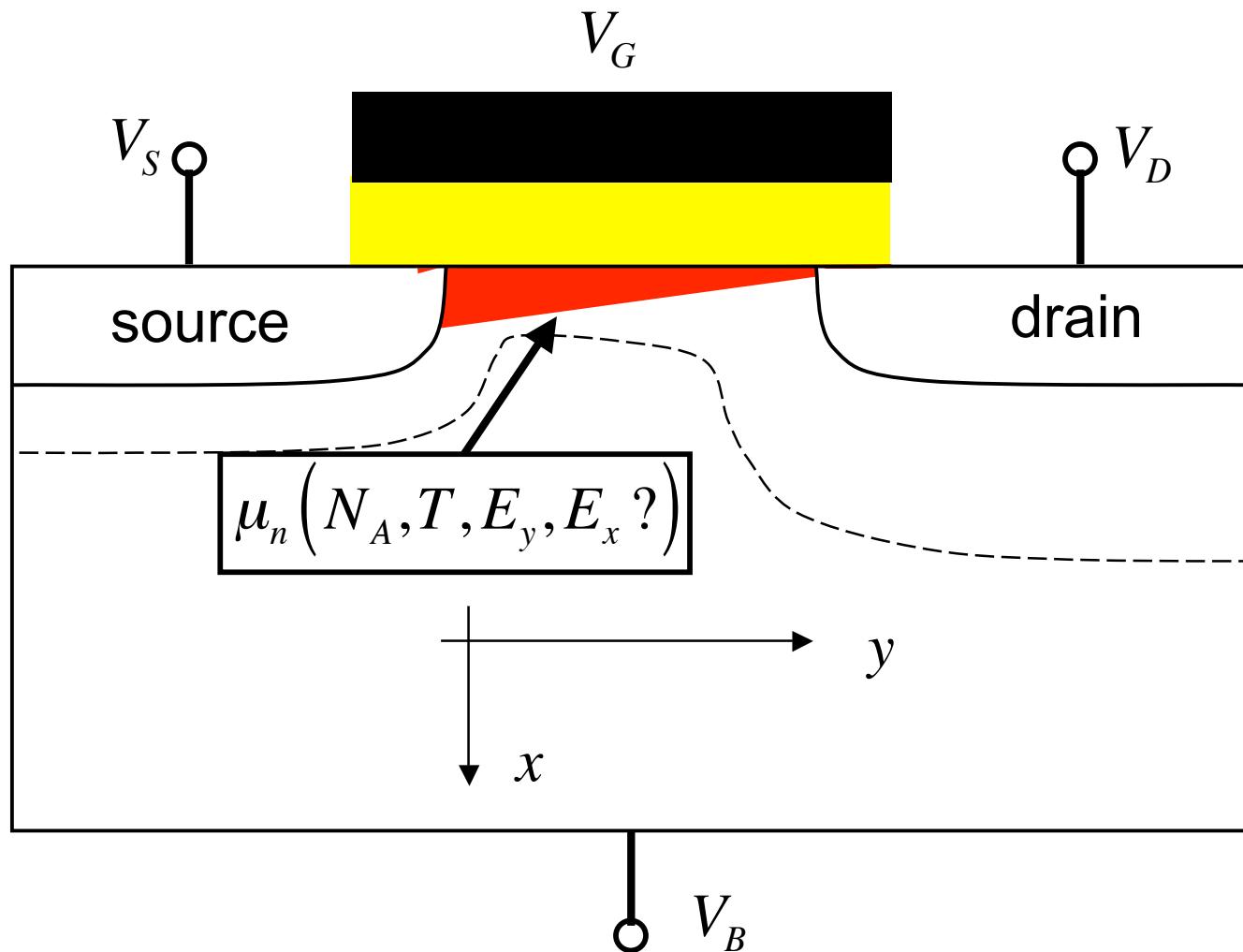
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# mobility in an inversion layer

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# effective mobility

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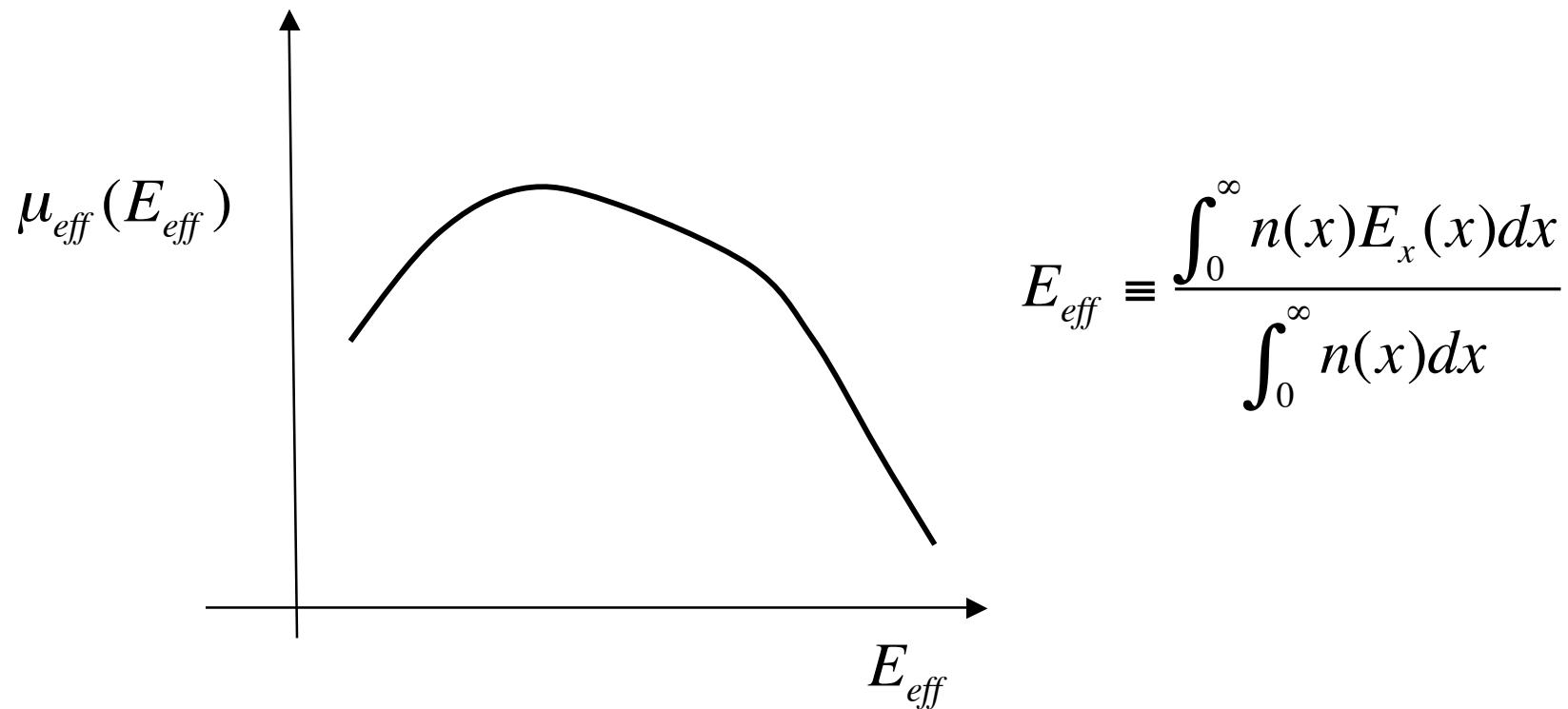
$$I_D = -WQ_i(y)v_y = WC_G[V_G - V_T - mV(y)]\mu_n(x,y)\frac{dV}{dy}$$

$$I_D \int_0^L dy = WC_G \mu_{eff} \int_0^{V_{DS}} [V_G - V_T - mV(y)] dV$$

$$\mu_{eff} = \frac{\int_0^\infty n(x)\mu_n(x)dx}{\int_0^\infty n(x)dx}$$

## effective normal field

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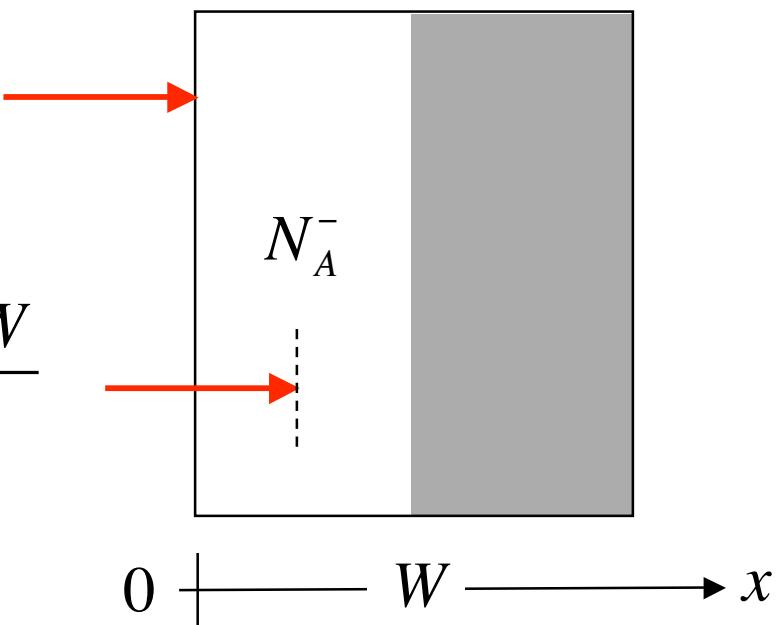


# effective normal field

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$$E_S = \frac{qN_A W}{\epsilon_{Si}}$$

$$E(W/2) = \frac{qN_A W}{2\epsilon_{Si}}$$



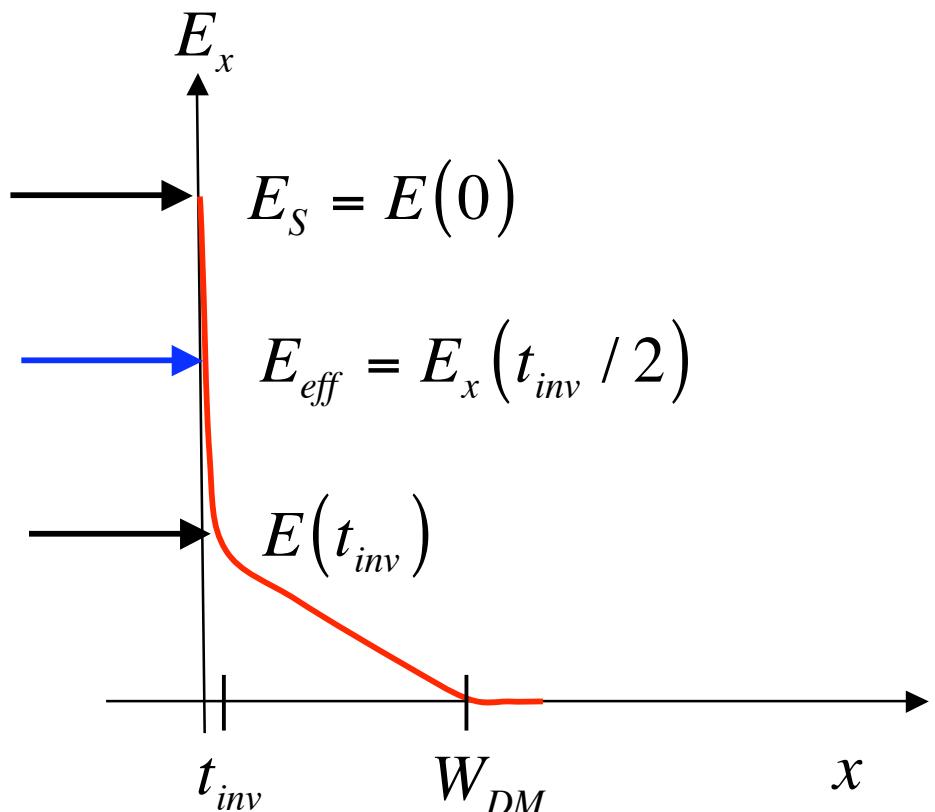
## effective normal field (ii)

$$E_S = \frac{|Q_S|}{\epsilon_{Si}}$$

$$E_S = \frac{1}{\epsilon_{Si}} (|Q_{DM}| + |Q_i|)$$

$$E(t_{inv}) = \frac{|Q_{DM}|}{\epsilon_{Si}}$$

$$E_{eff} = \frac{1}{\epsilon_{Si}} \left( |Q_{DM}| + \frac{|Q_i|}{2} \right)$$



## effective normal field (ii)

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$$E_{eff} = \frac{1}{\epsilon_{Si}} \left( |Q_{DM}| + \frac{|Q_i|}{2} \right) \quad (1)$$

$$V_T = V_{FB} + 2\psi_B + \frac{|Q_{DM}|}{C_{OX}}$$

$$|Q_i| = C_G (V_{GS} - V_T)$$

$$|Q_{DM}| = C_{OX} (V_T - V_{FB} - 2\psi_B)$$

$$|Q_i| \approx C_{OX} (V_{GS} - V_T)$$

$$E_{eff} = \frac{C_{OX}}{\epsilon_{Si}} \left( V_T - V_{FB} - 2\psi_B + \frac{V_G}{2} - \frac{V_T}{2} \right)$$

## effective normal field (ii)

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$$E_{eff} = \frac{C_{OX}}{\varepsilon_{Si}} \left( V_T - V_{FB} - 2\psi_B + \frac{V_G}{2} - \frac{V_T}{2} \right)$$

$$V_{FB} = -\frac{E_G}{2q} - \psi_B$$

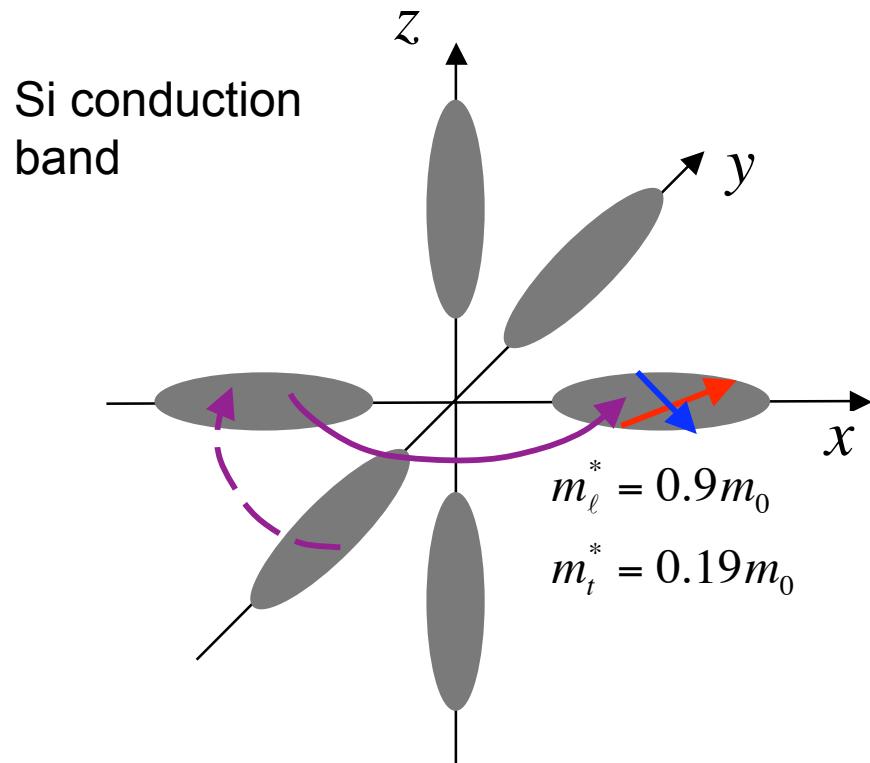
$$E_{eff} = \frac{\varepsilon_{OX}}{\varepsilon_{Si} t_{OX}} \left[ \left( \frac{V_G + V_T}{2} \right) + \frac{E_G}{2q} - \psi_B \right]$$

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# transport in bulk Si



***under low (and modest) fields:***

- 6 equivalent ellipsoids
- $n/6$  electrons in each one

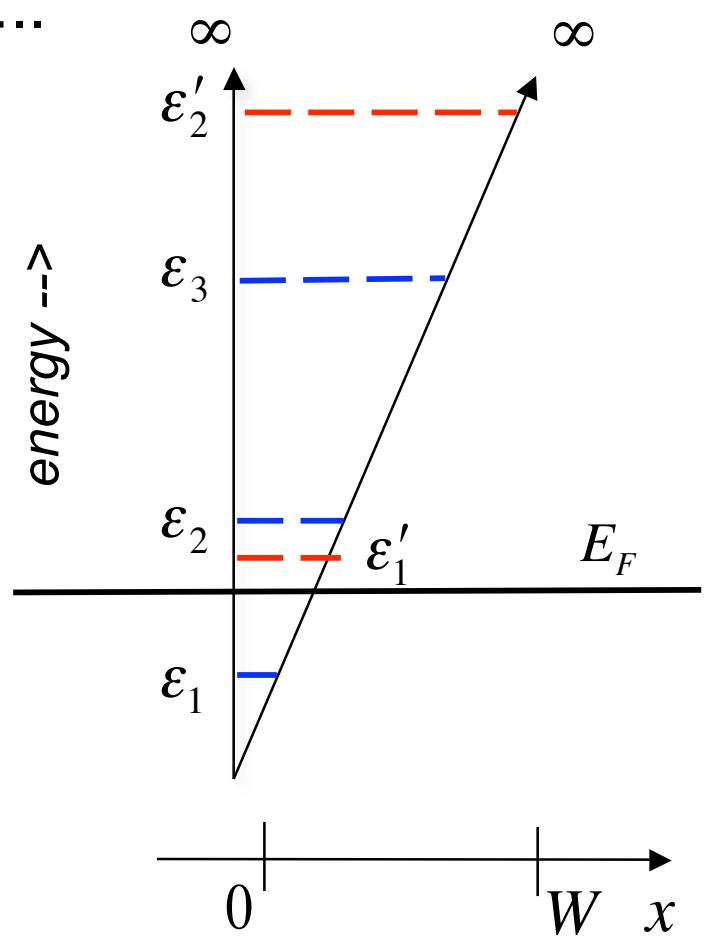
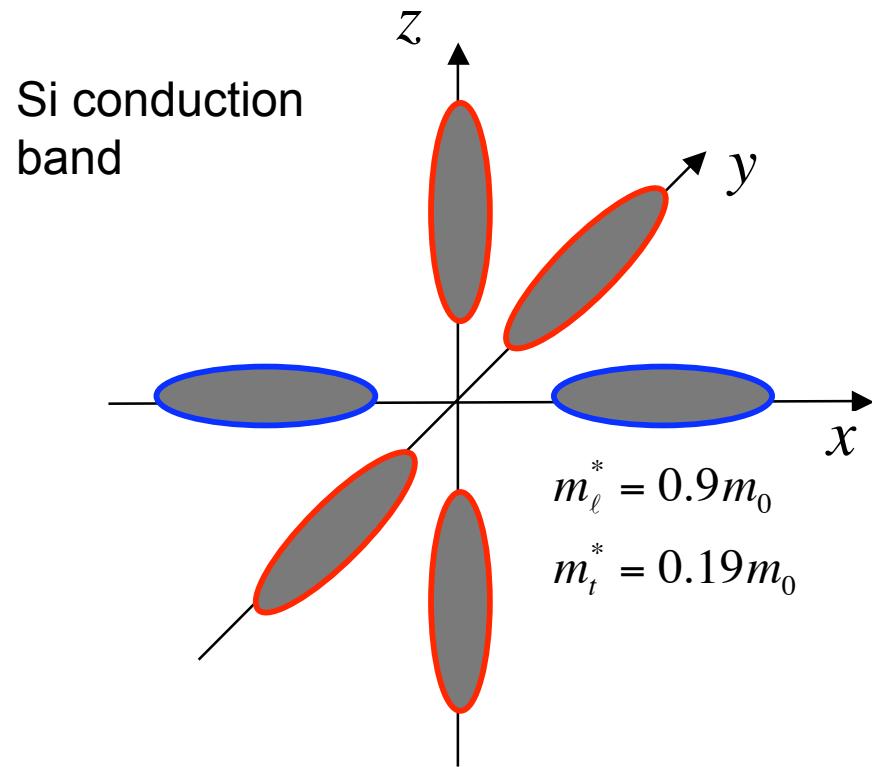
$$m_C^* = \left( \frac{2}{6m_\ell^*} + \frac{4}{6m_t^*} \right)^{-1} = 0.26m_0$$

**dominant scattering processes:**  
(low-field, room temperature)

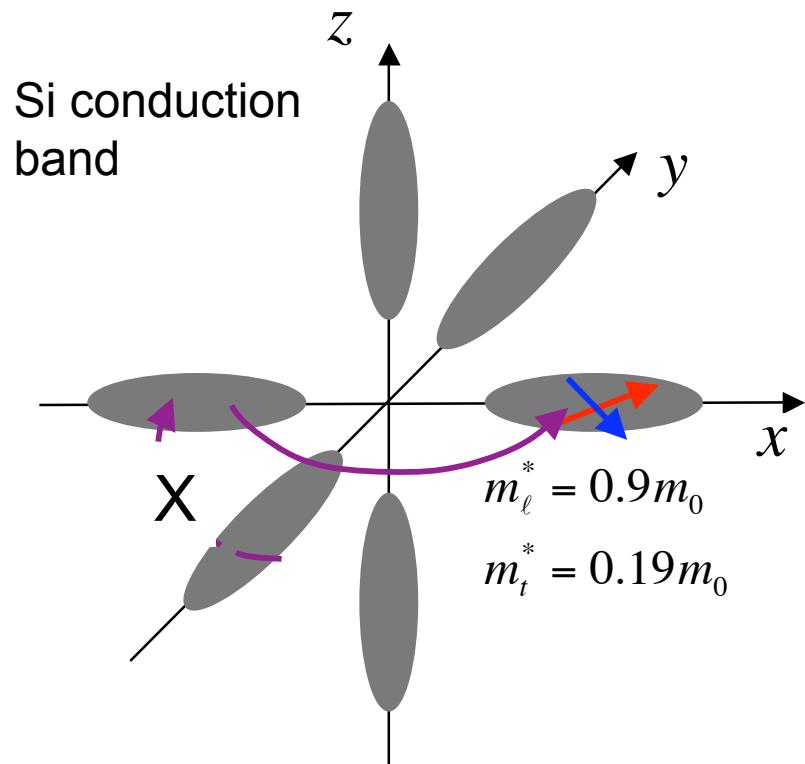
- acoustic phonons (ADP)
- ionized impurities (II)
- intervalley phonons (IV)

# transport in Si inversion layers

is different from transport in bulk Si.....



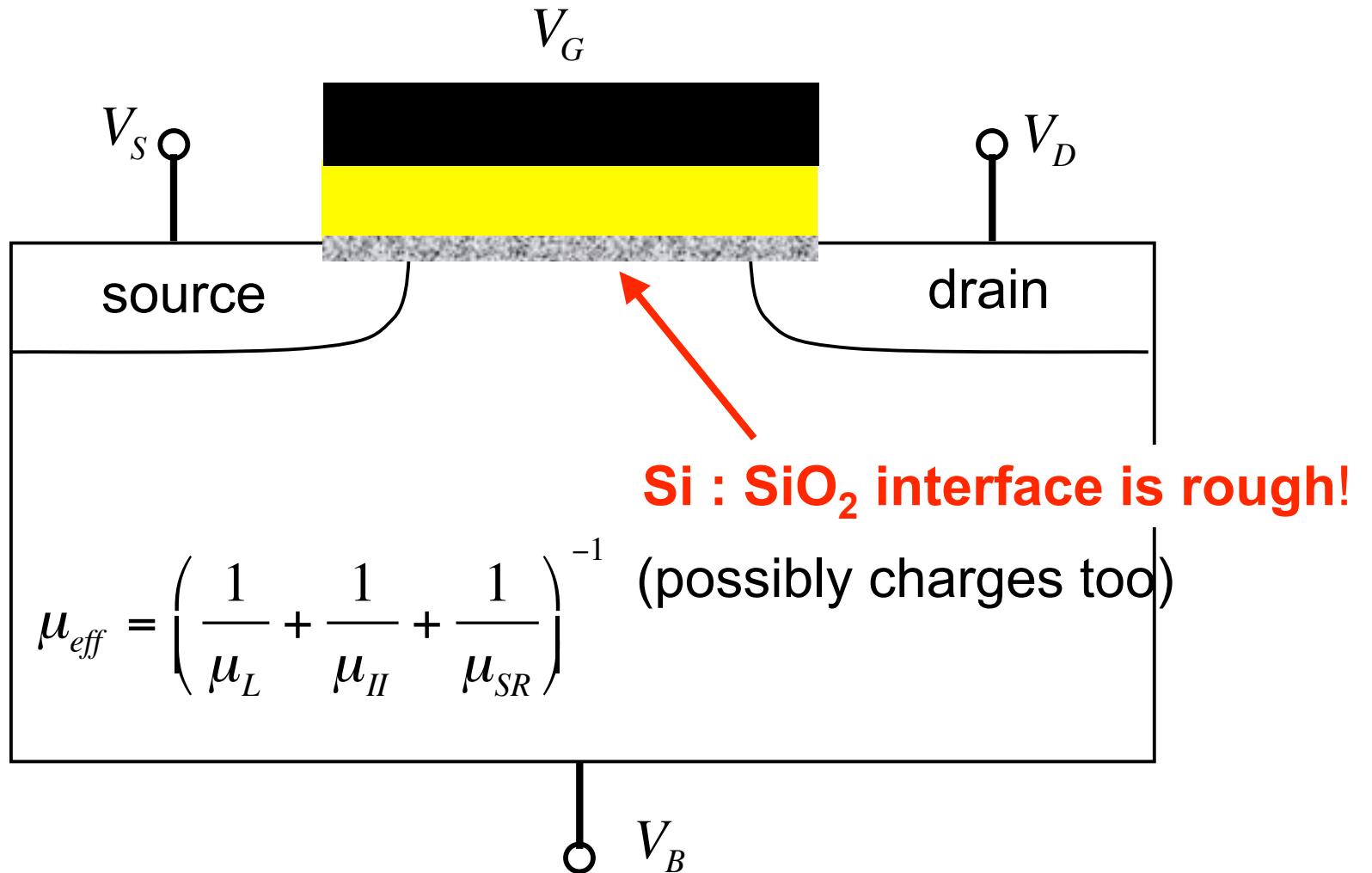
# transport in Si inversion layers



## ***expectations:***

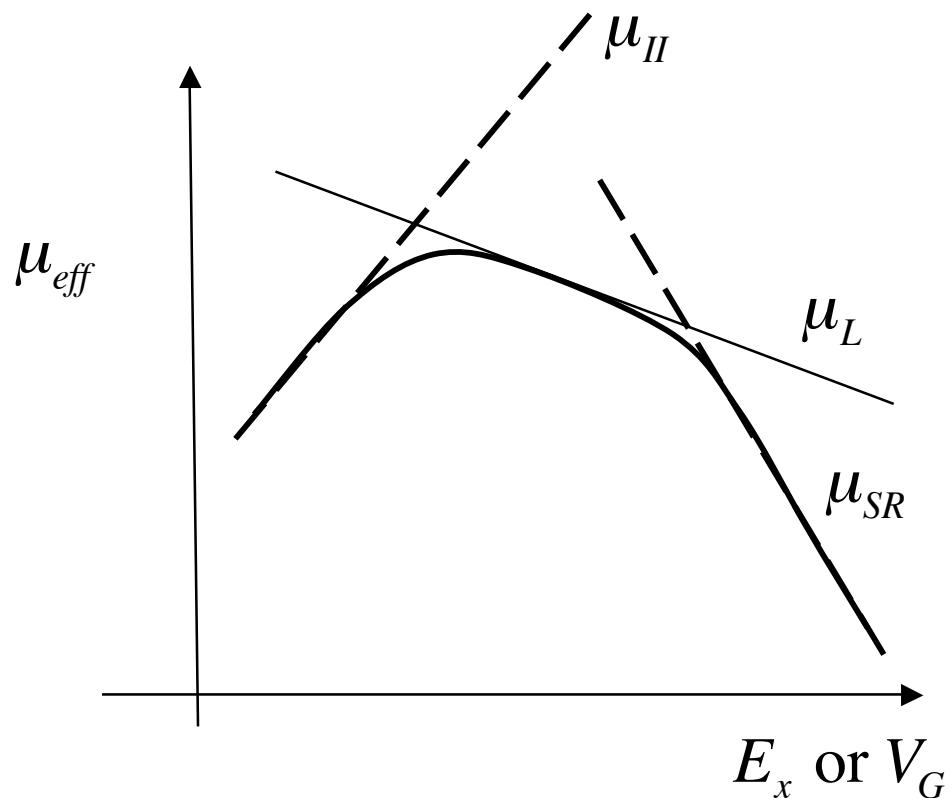
- >  $n/3$  carriers in unprimed subbands
- lighter conductivity  $m^*$   
 $m_c^* \approx 0.19m_0$
- suppressed intersubband scattering
- enhanced intra subband phonon scattering

# surface mobility



# effective mobility vs. field

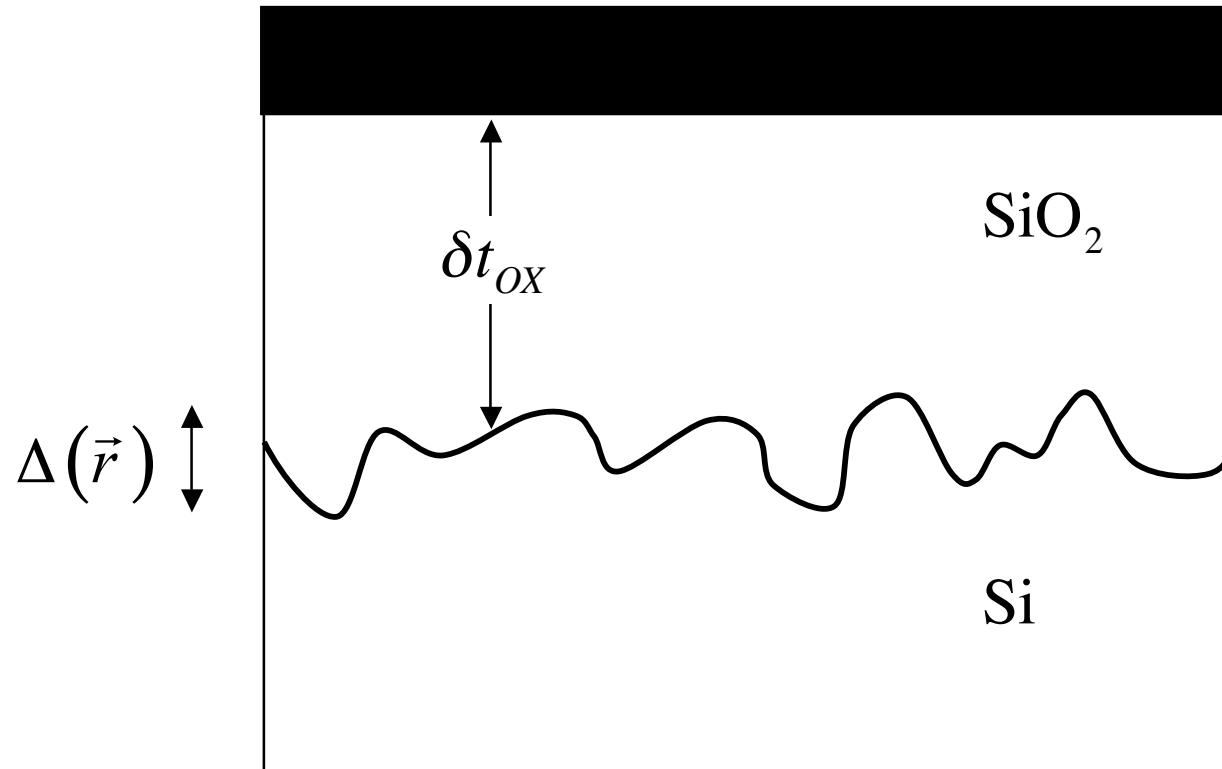
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- 1) screening
- 2) electron confinement
- 3) proximity to the surface

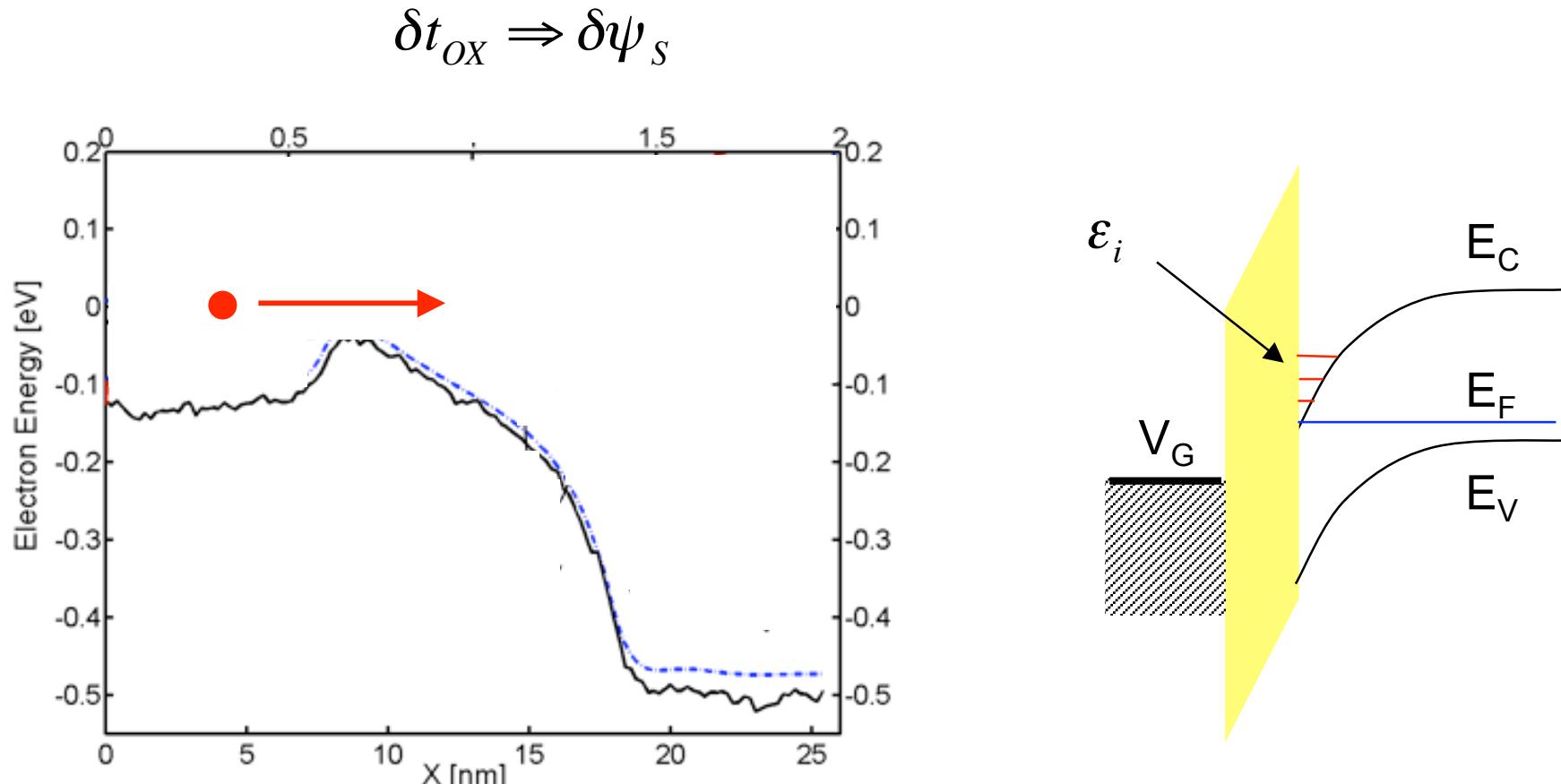
# surface roughness scattering

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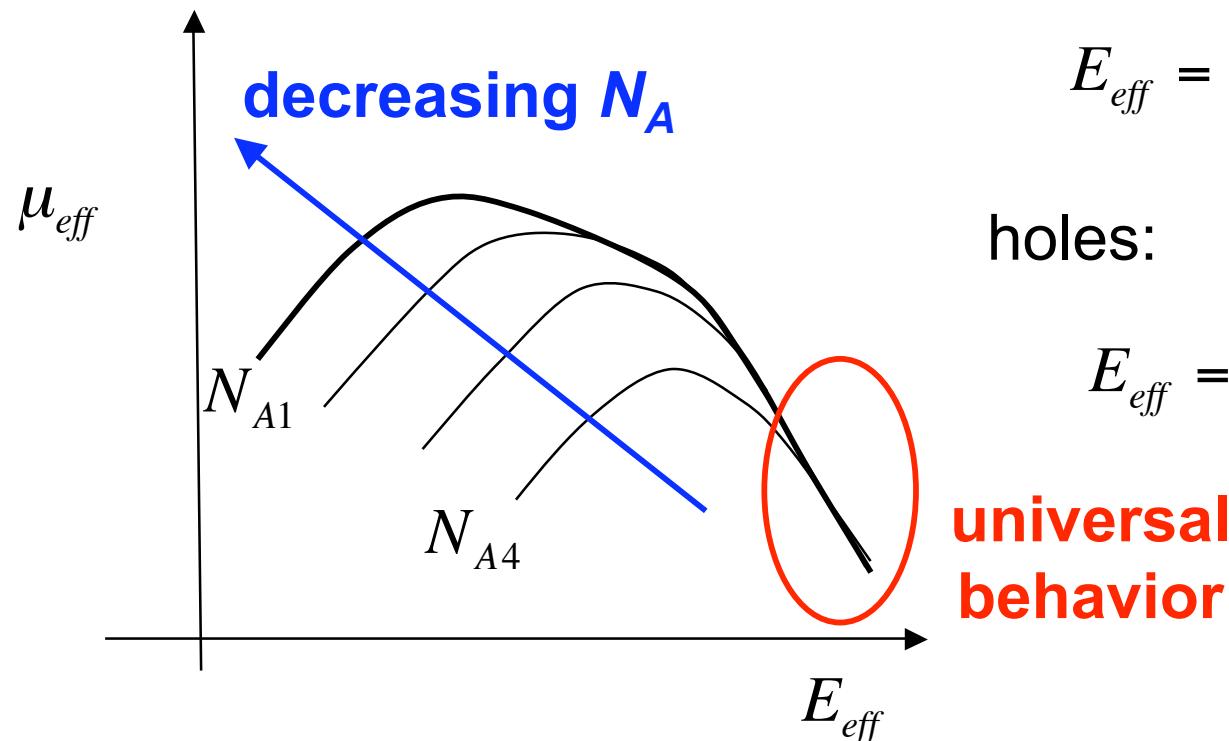
$$V_G = V_{FB} + \psi_s - Q_s/C_{OX} \quad \delta t_{OX} \Rightarrow \delta\psi_s$$

## surface roughness scattering (ii)



from Jing Wang, et al., *Appl. Phys. Lett.*, Aug. 2005,

# ‘universal’ mobility



electrons:

$$E_{eff} = \frac{1}{\epsilon_{Si}} \left( |Q_{DM}| + \frac{|Q_i|}{2} \right)$$

holes:

$$E_{eff} = \frac{1}{\epsilon_{Si}} \left( |Q_{DM}| + \frac{|Q_i|}{3} \right)$$

**universal  
behavior**

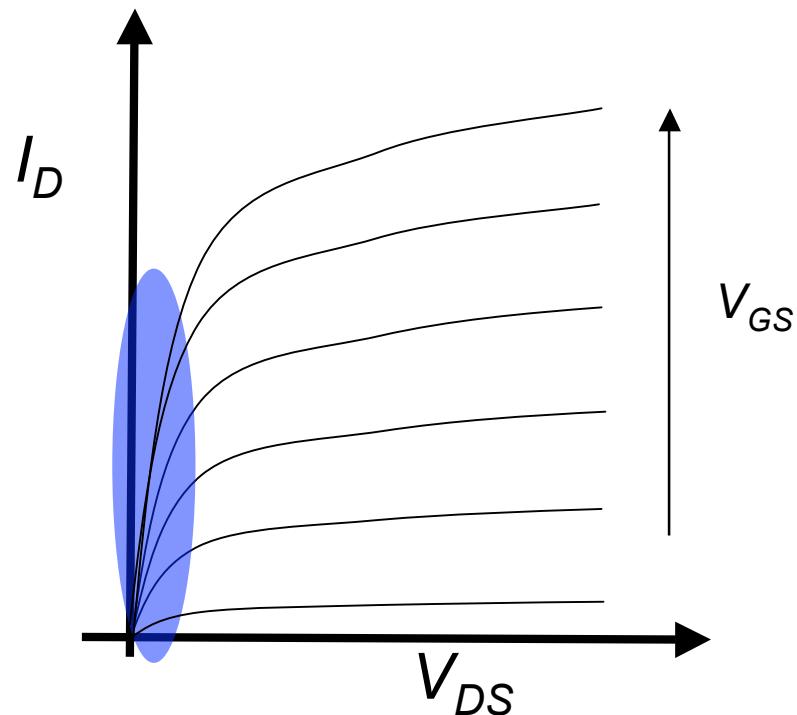
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# measuring $\mu_{eff}$

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$$I_D = \frac{W}{L} \mu_{eff} Q_i V_{DS}$$

$$R_{CH} = \frac{V_{DS}}{I_D} = \frac{L}{W \mu_{eff} Q_i}$$

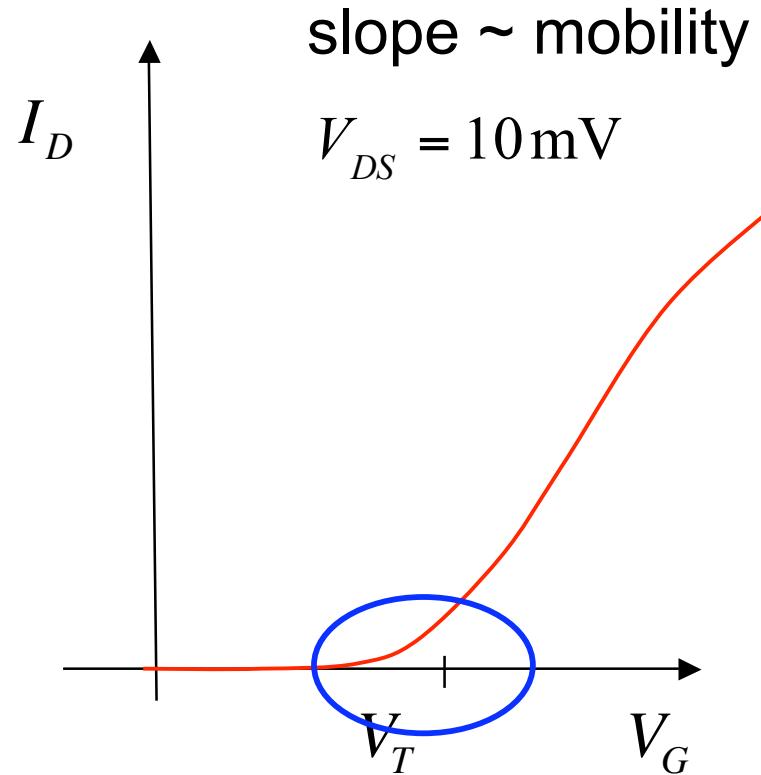
$$\mu_{eff}(V_G) = \frac{L}{W R_{CH}(V_G) Q_i(V_G)}$$

$$Q_i(V_G) \approx C_{ox} (V_{GS} - V_T)$$

$$R_{CH} \gg R_{SD}$$

## measuring $\mu_{eff}$ (ii)

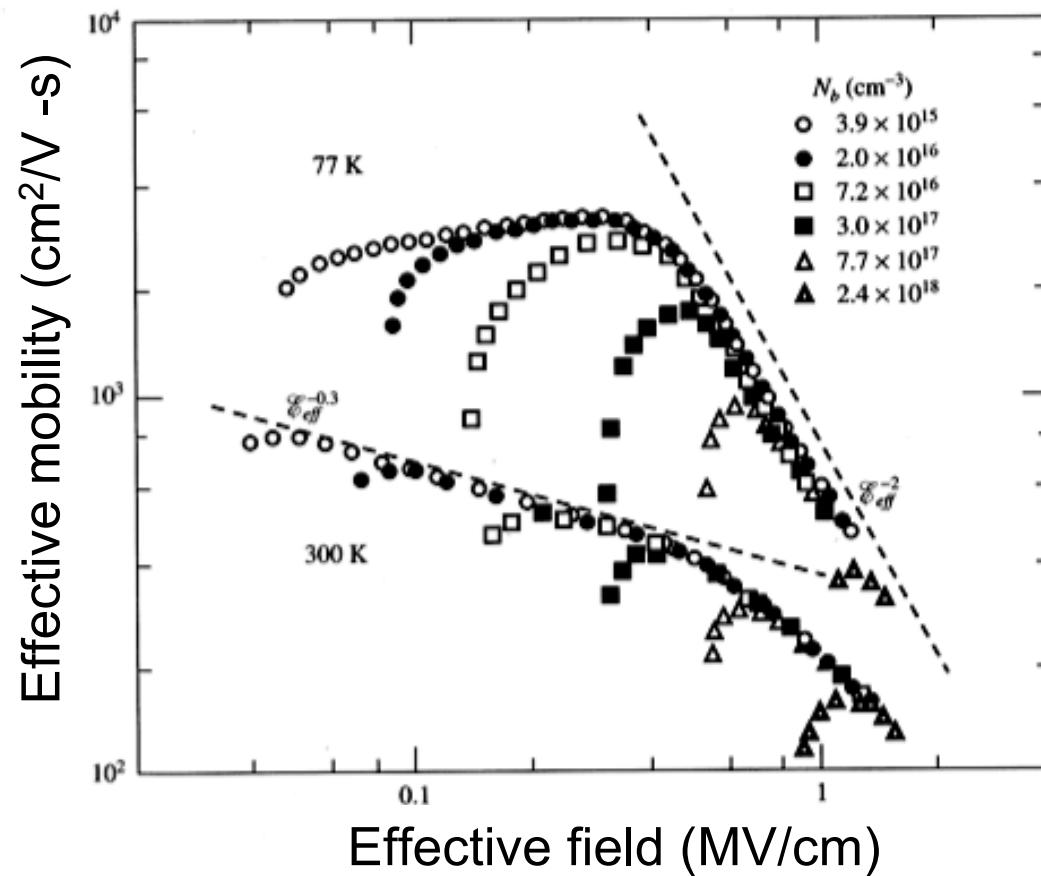
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$$Q_i(V_G) \approx C_{ox} (V_{GS} - V_T)$$

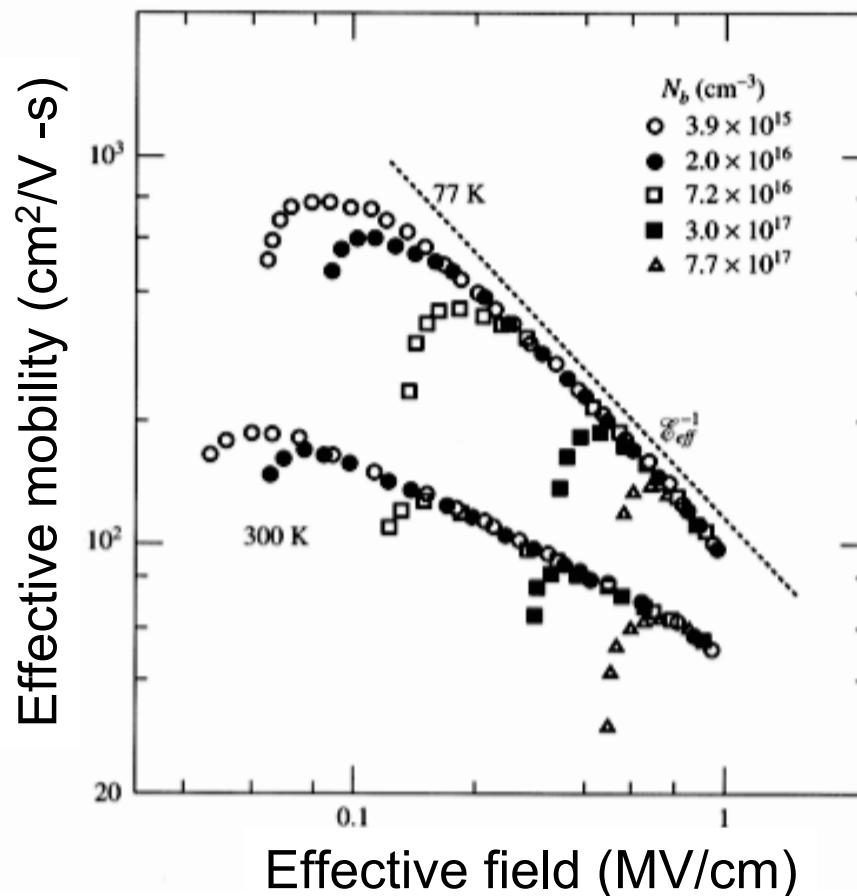
$$Q_i(V_G) = \int_{-\infty}^{V_G} C_{gs} (V_G) dV_G$$

# universal mobility for electrons



S. Takagi, A. Toriumi, M. Iwase, and H. Tango, *IEEE Trans. Electron Dev.*, **41**, pp. 2357-2362, 1994

# universal mobility for holes



S. Takagi, A. Toriumi, M. Iwase, and H. Tango, *IEEE Trans. Electron Dev.*, **41**, pp. 2357-2362, 1994

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# the tyranny of the effective mobility\*

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$$E_{eff} = \frac{\epsilon_{ox}}{\epsilon_{Si} t_{ox}} \left[ (V_G + V_T)/2 + E_G/2q - \psi_B \right]$$

each technology generation, device scaling increases  $N_A$ ,  
decreases  $t_{ox}$  -->  $E_{eff}$  increases and mobility decreases

mobility decreases each technology generation, unless we  
can find ‘new’ materials with higher mobilities (e.g. strained  
silicon).

(\* Dimitri Antoniadis at MIT describes the issue this way.)

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