

# **EE-612:**

## **Lecture 15:**

# **2D Electrostatics: Part 1**

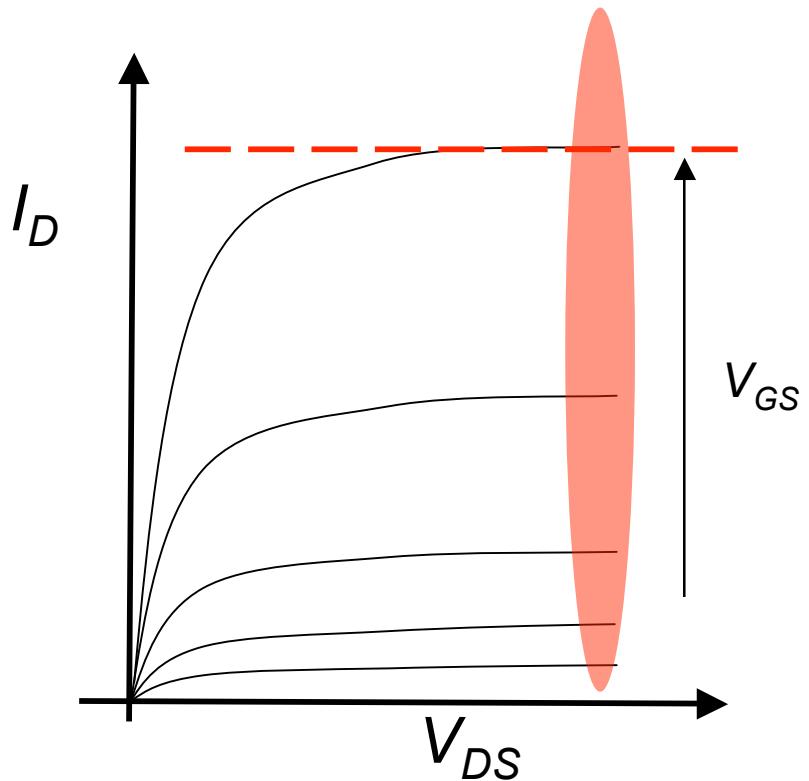
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West Lafayette, IN USA  
Fall 2006

# outline

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- 1) Consequences of 2D electrostatics
- 2) 2D Poisson equation
- 3) Barrier lowering
- 4) 2D capacitor model

# $I_D$ vs. $V_{DS}$ (long channel)



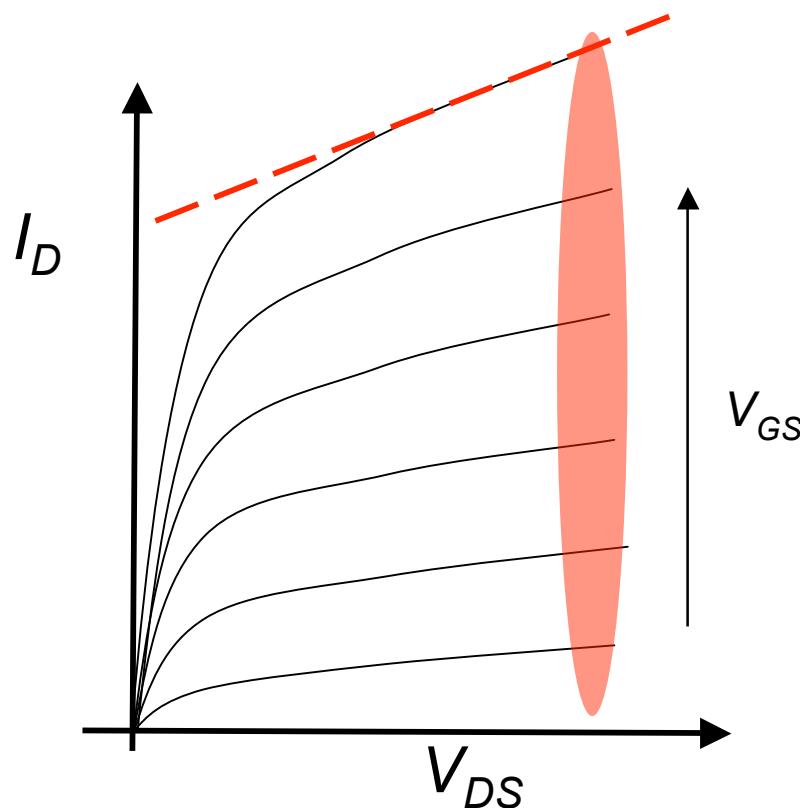
1) square law

$$I_D = \frac{W}{2L} \mu_{eff} C_{ox} \frac{(V_{GS} - V_T)^2}{m}$$

2) low output conductance

$$g_d = \frac{\partial I_D}{\partial V_{DS}}$$

# $I_D$ vs. $V_{DS}$ (short channel)



1) linear with  $V_{GS}$

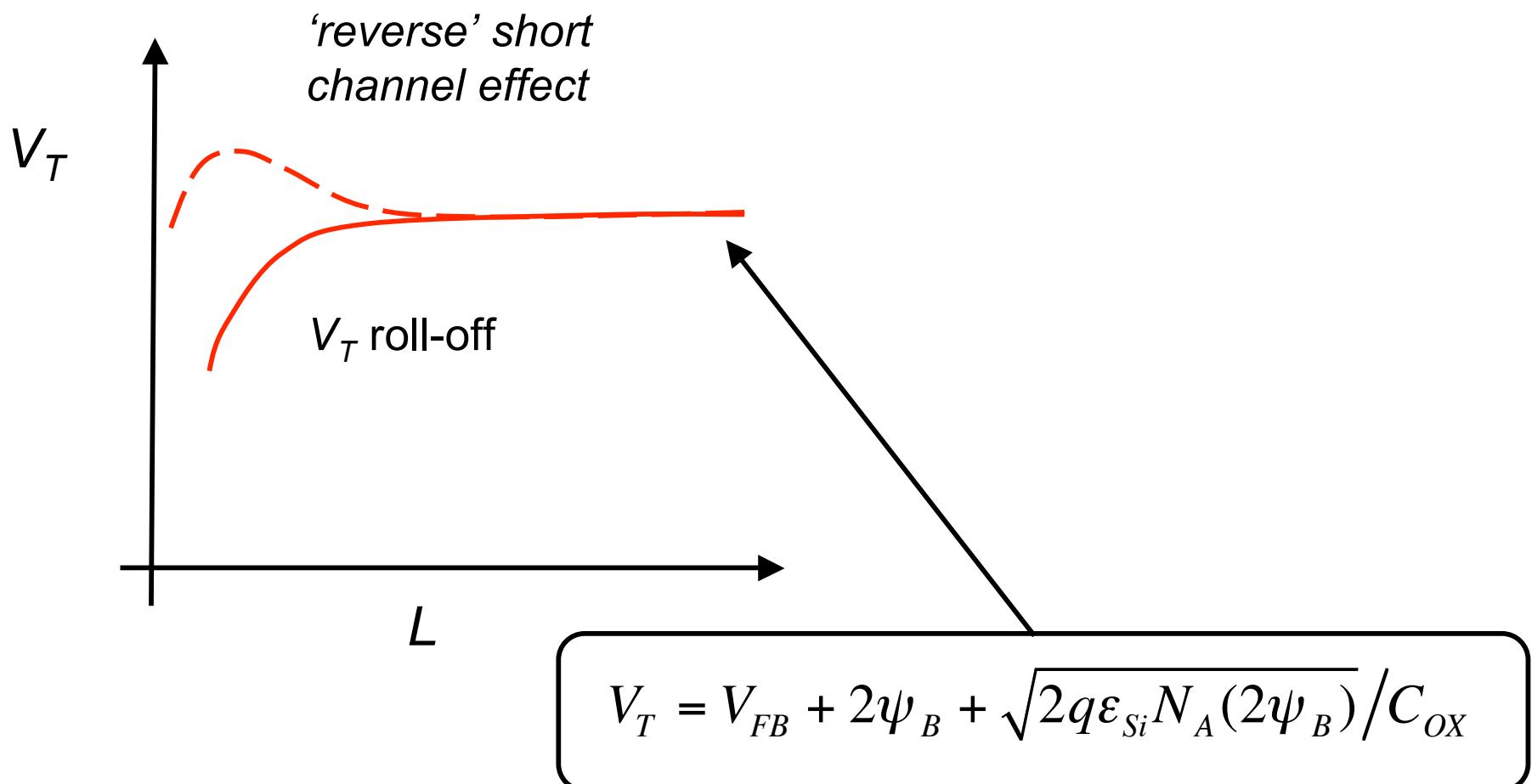
$$I_D = Wv_{sat}C_{ox}(V_{GS} - V_T)$$

2) high output conductance

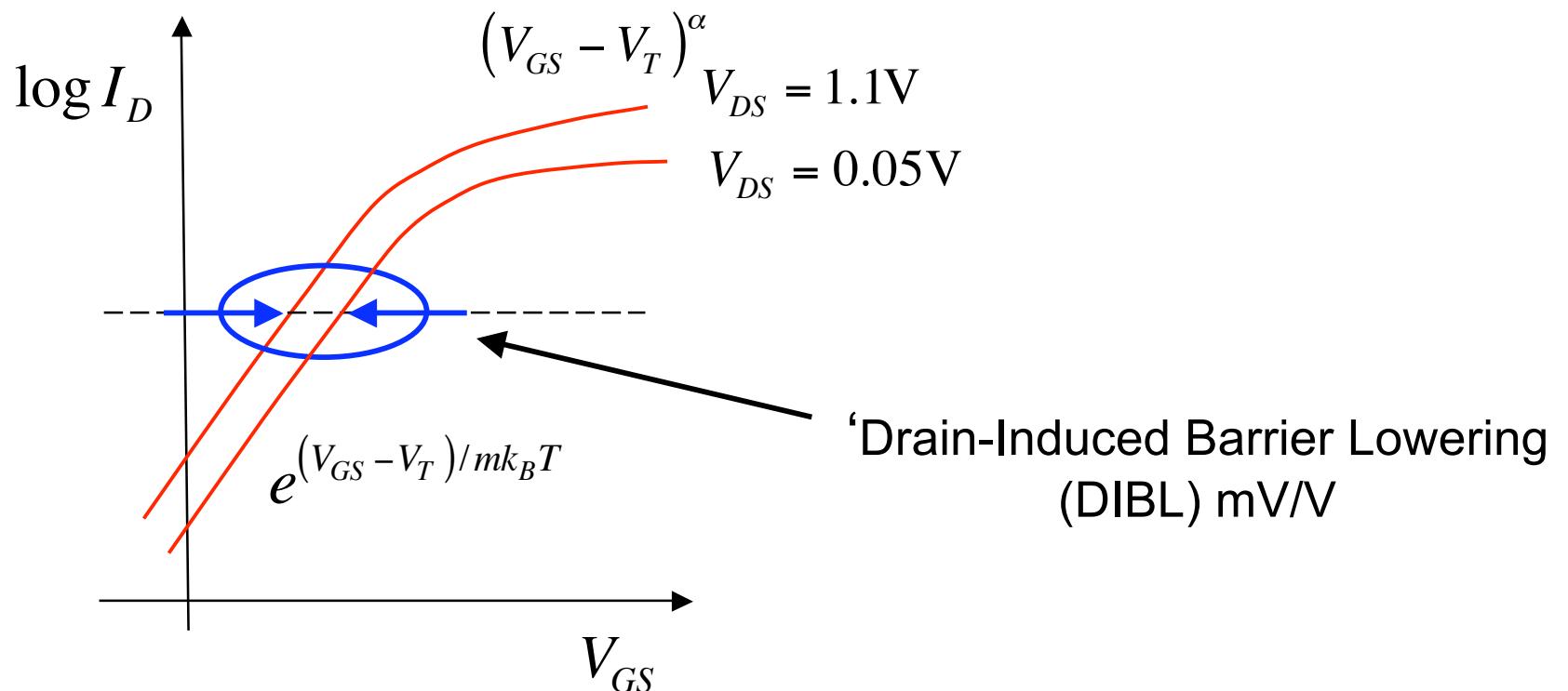
$$g_d = \frac{\partial I_D}{\partial V_{DS}}$$

see Taur and Ning, pp. 154-158

## $V_T$ roll-off

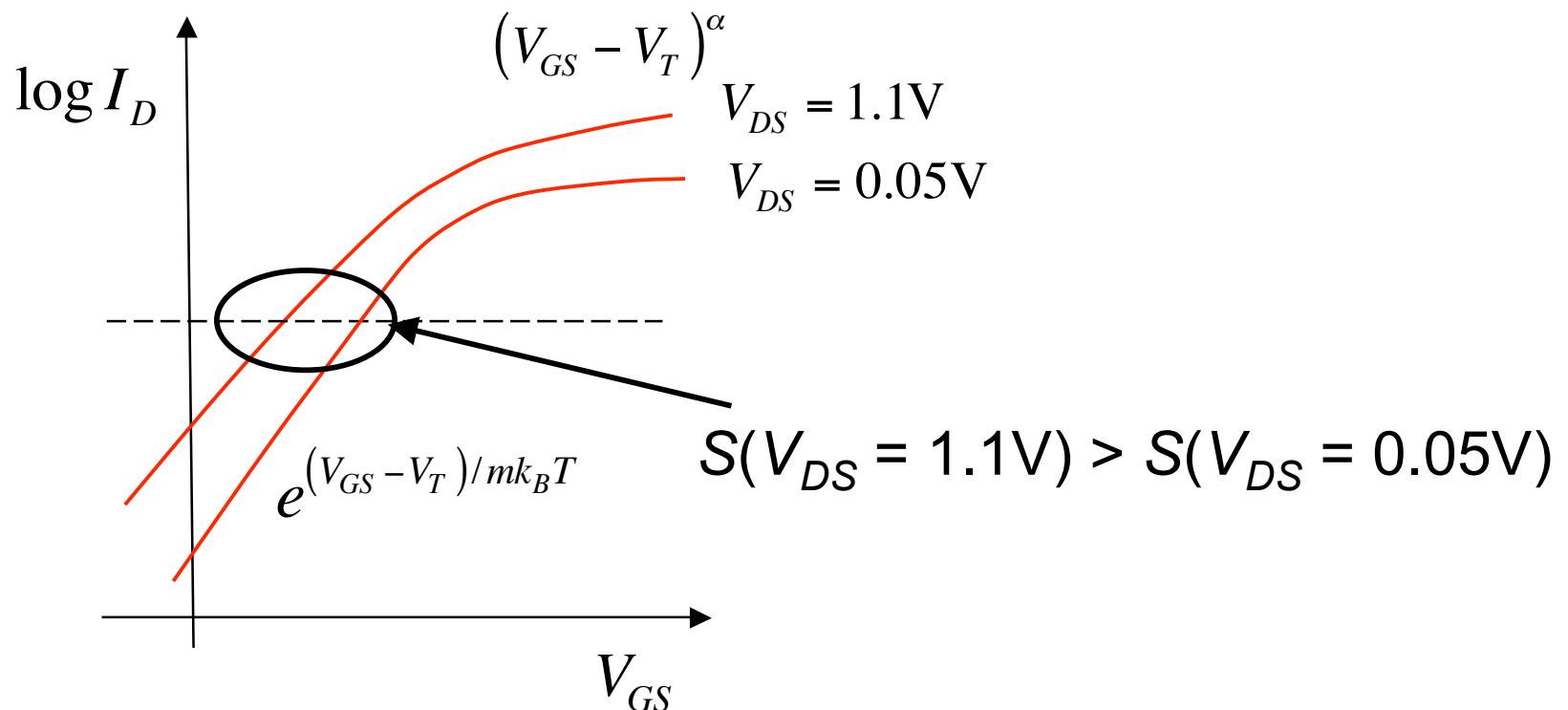


# DIBL



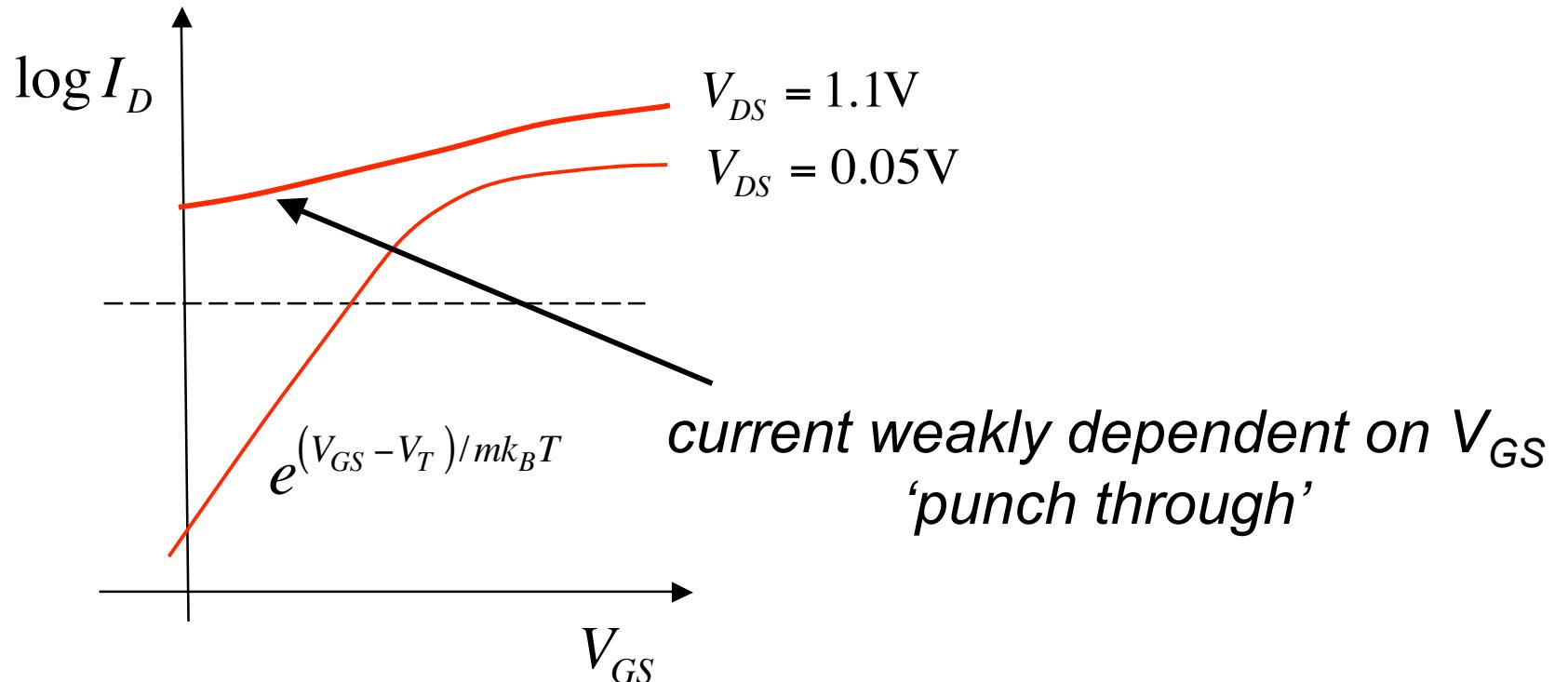
# stronger short channel effects

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# severe short channel effects

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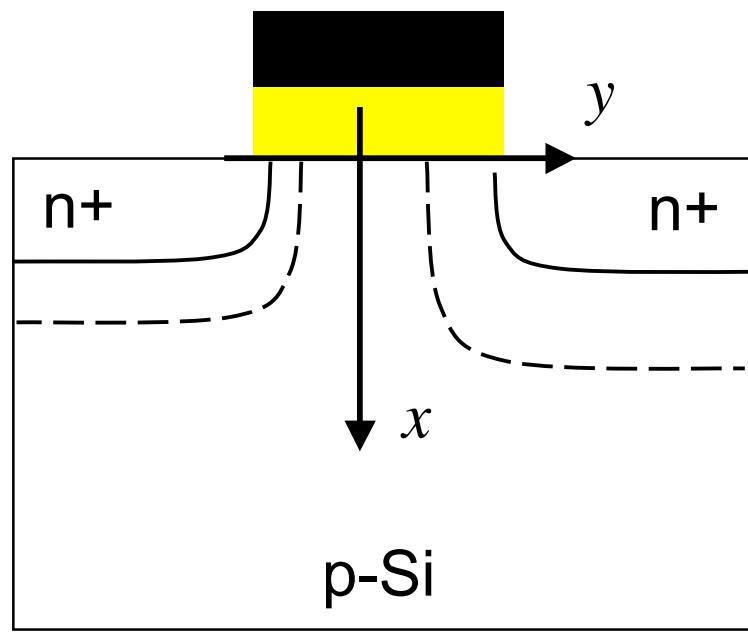
# outline

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- 1) Consequences of 2D electrostatics
- 2) 2D Poisson equation**
- 3) Charge sharing model
- 4) Barrier lowering viewpoint
- 5) 2D capacitor model

# 2D Poisson equation

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1) MOS Capacitor:

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{\rho}{\epsilon_{Si}} = \frac{qN_A}{\epsilon_{Si}} \quad (\text{below } V_T)$$

2) MOSFET:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \frac{qN_A}{\epsilon_{Si}} \quad (\text{below } V_T)$$

## 2D Poisson equation (ii)

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1) Long channel MOSFET below threshold:

$$\frac{\partial^2 \psi}{\partial x^2} \gg \frac{\partial^2 \psi}{\partial y^2}$$

gradual channel approximation:

$$Q_i(y) = -C_G [V_G - V_T - mV(y)]$$

## 2D Poisson equation (iii)

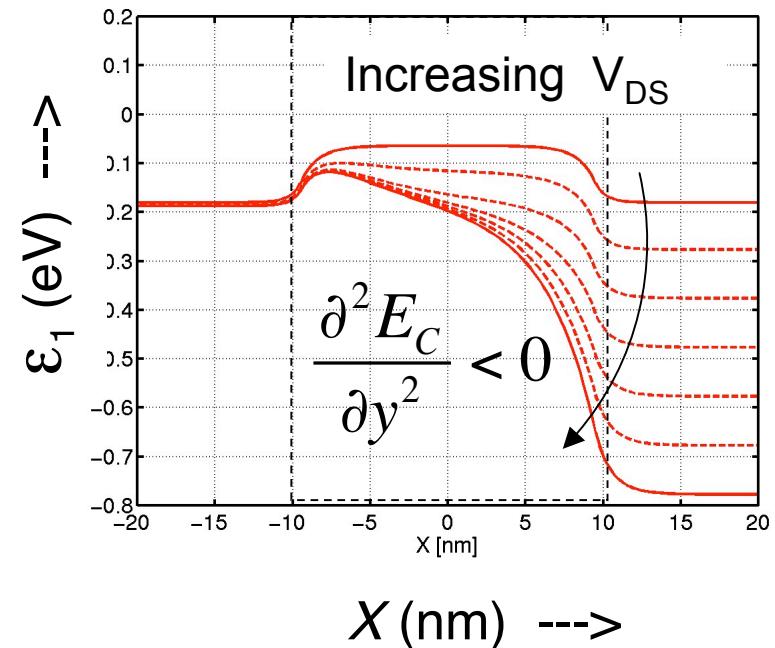
1) Short channel MOSFET below threshold:

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{qN_A}{\epsilon_{Si}} - \frac{\partial^2 \psi}{\partial y^2}$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{q N_A|_{eff}}{\epsilon_{Si}}$$

$$N_A|_{eff} < N_A$$

$V_T < V_T$  (long channel)



**explains  $V_T$  roll-off**

## 2D Poisson equation (iii)

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alternatively:

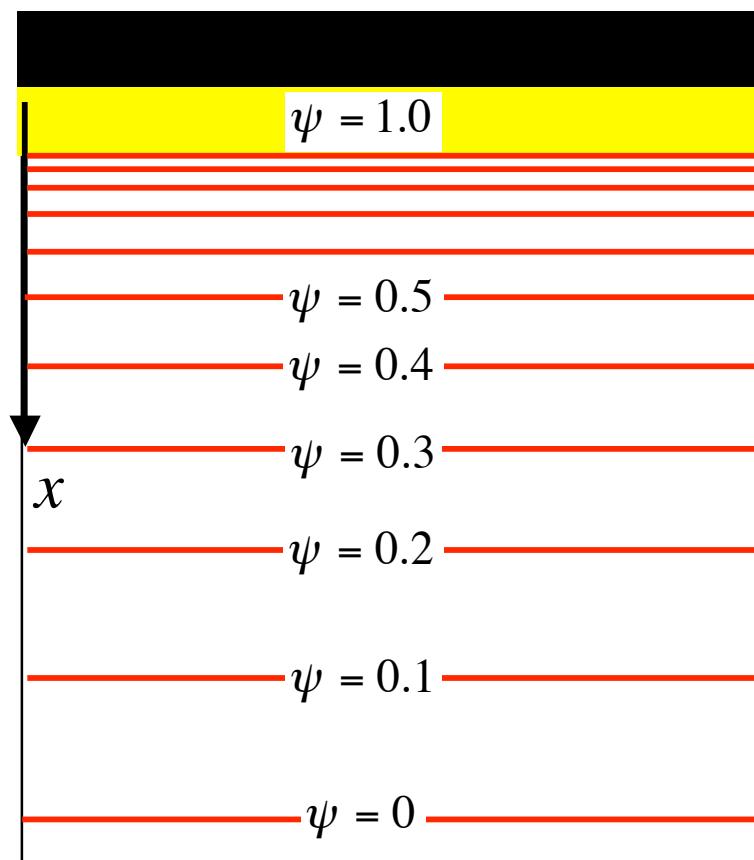
$$\frac{\partial^2 \psi}{\partial x^2} = \frac{q N_A|_{eff}}{\epsilon_{Si}} \quad N_A|_{eff} < N_A$$

$$V_T = V_{FB} + 2\psi_B + \gamma \sqrt{4q\epsilon_{Si}N_A\psi_B}/C_{ox}$$

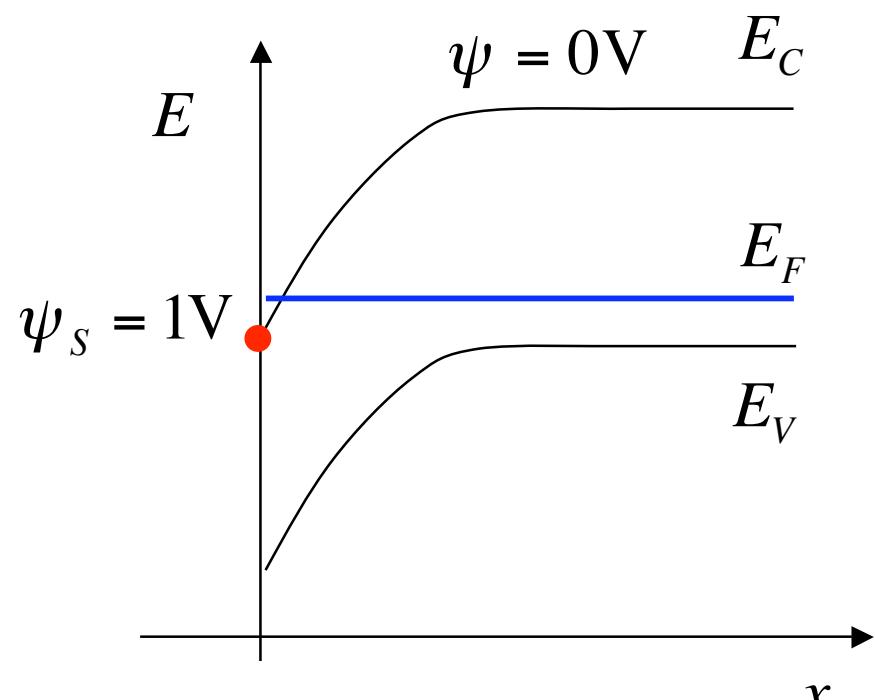
$$\gamma < 1$$

$$V_T < V_T(\text{long channel})$$

# 2D potential contours

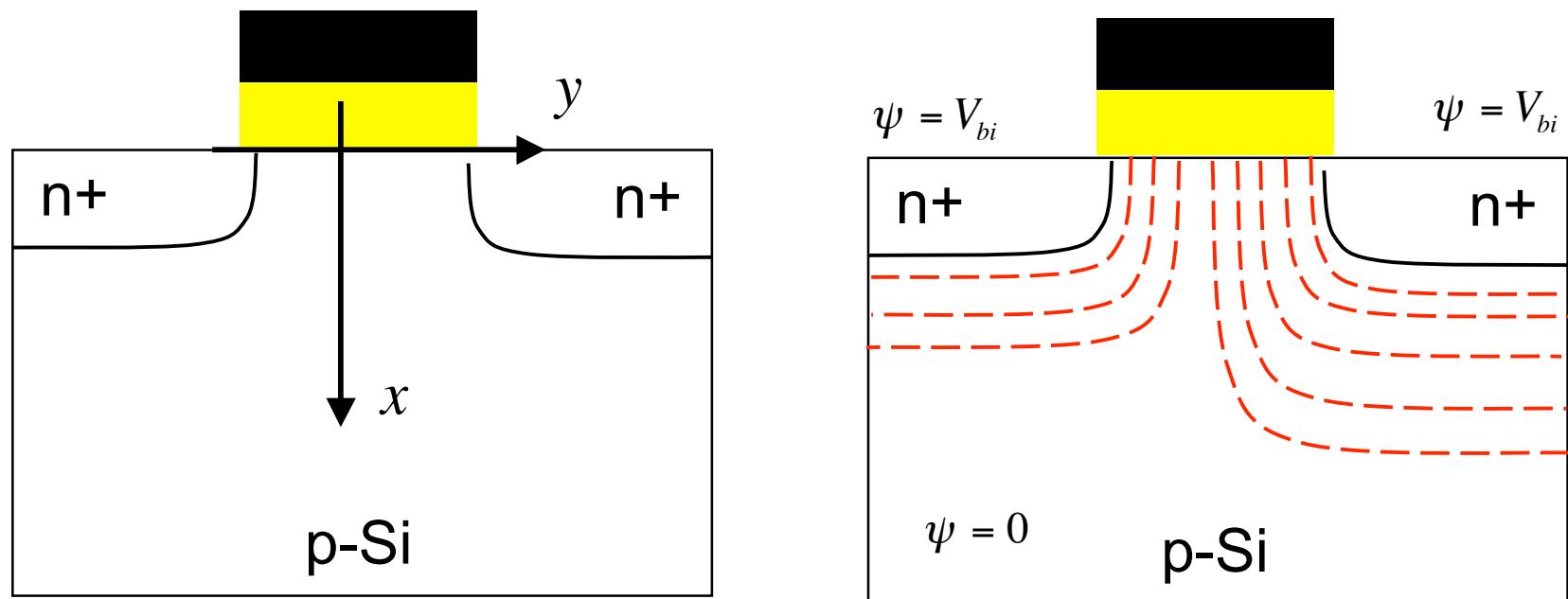


p-Si



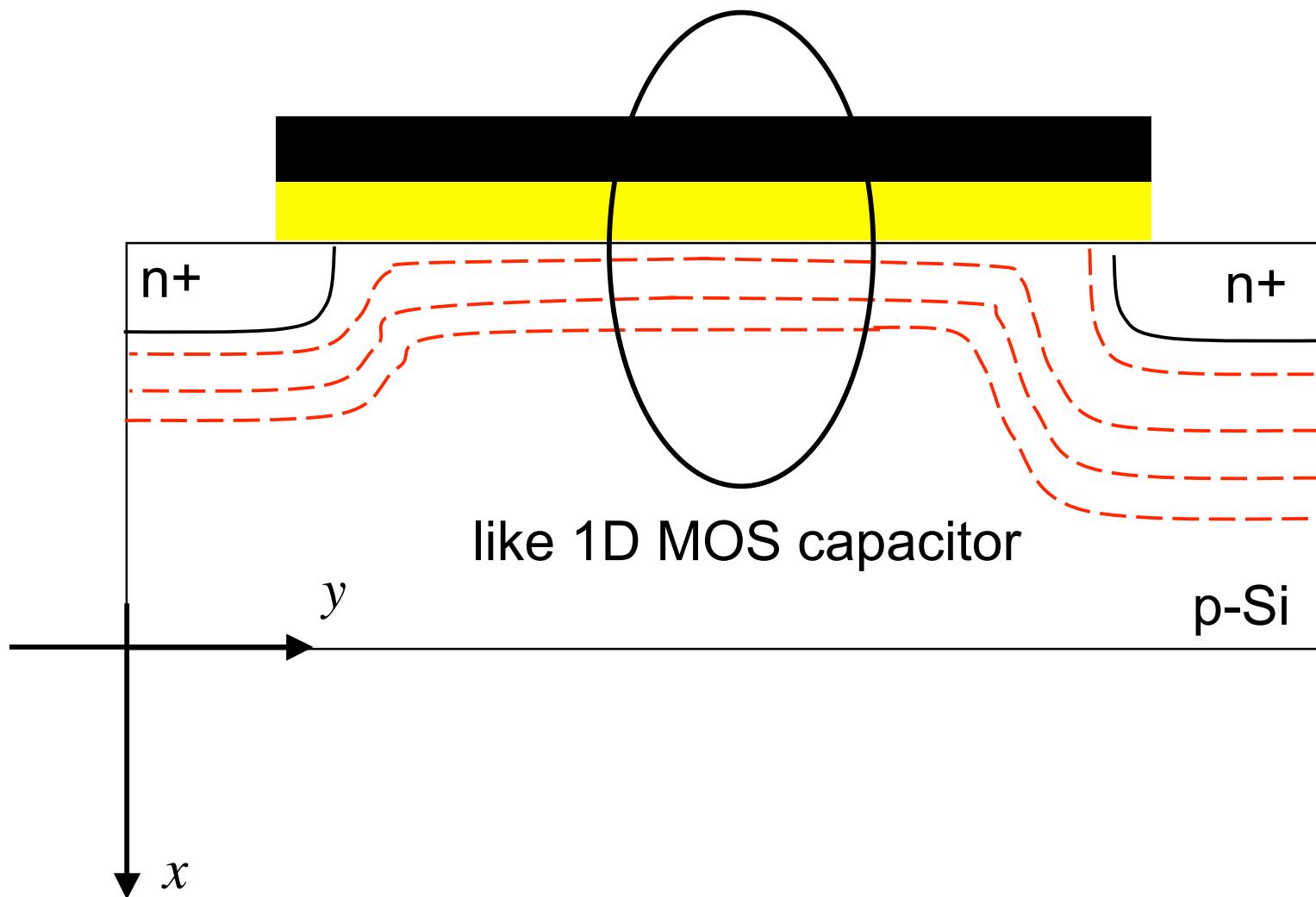
## 2D potential contours ( $V_G = V_{FB}$ )

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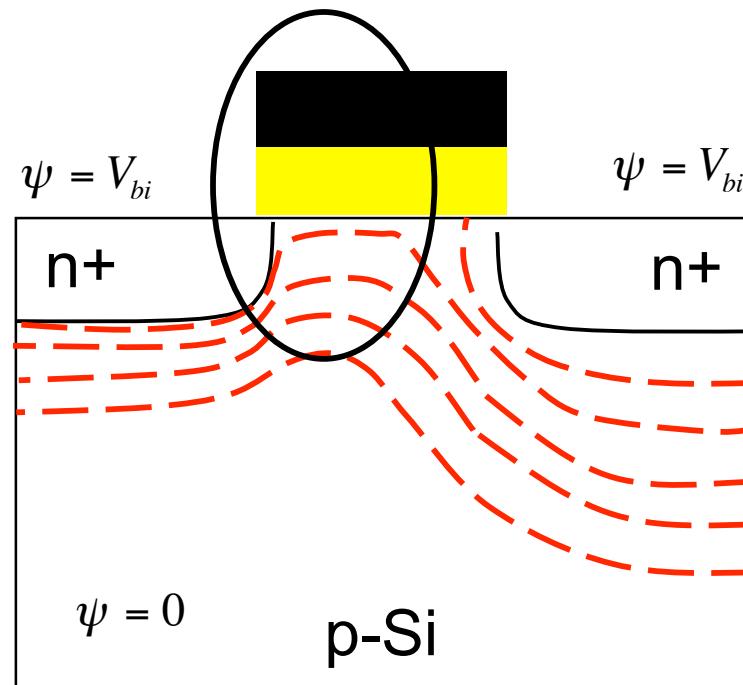
## 2D potential contours (long channel)

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## 2D potential contours (short channel)

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(See Fig. 3.18 of Taur and Ning)

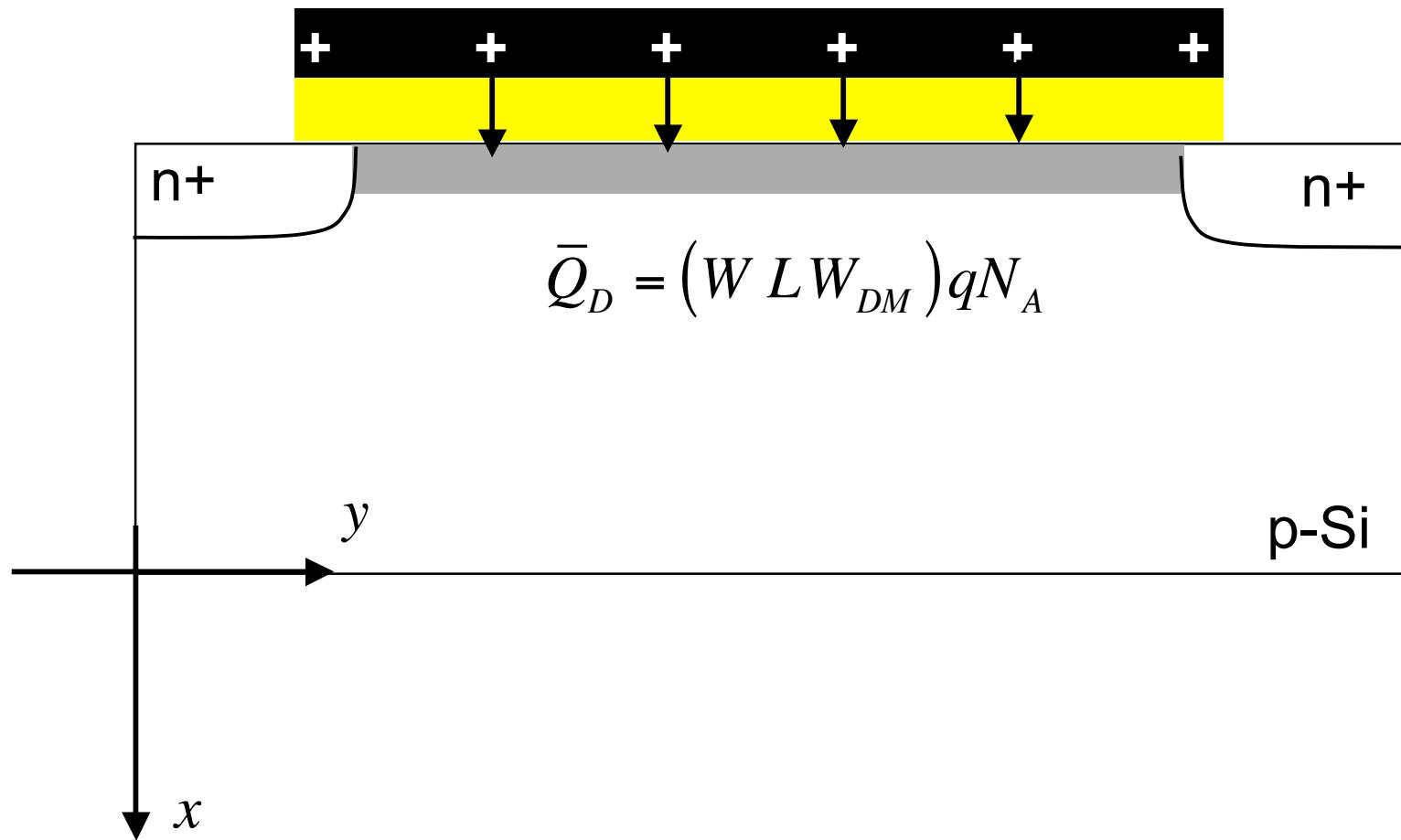
# outline

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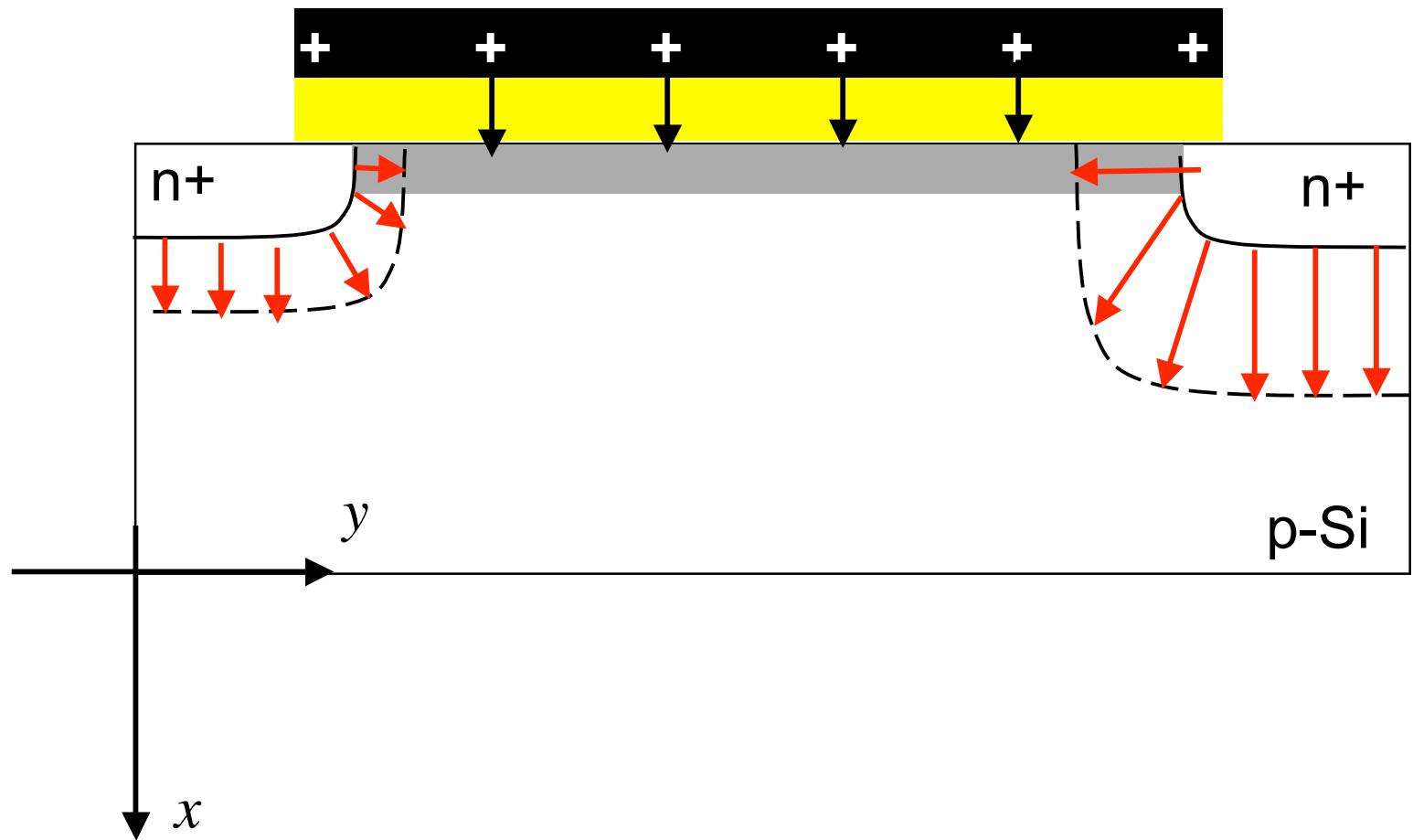
- 1) Consequences of 2D electrostatics
- 2) 2D Poisson equation
- 3) Charge sharing model**
- 4) Barrier lowering viewpoint
- 5) 2D capacitor model
- 6) Geometric scale length
- 7) Discussion

# charge sharing model

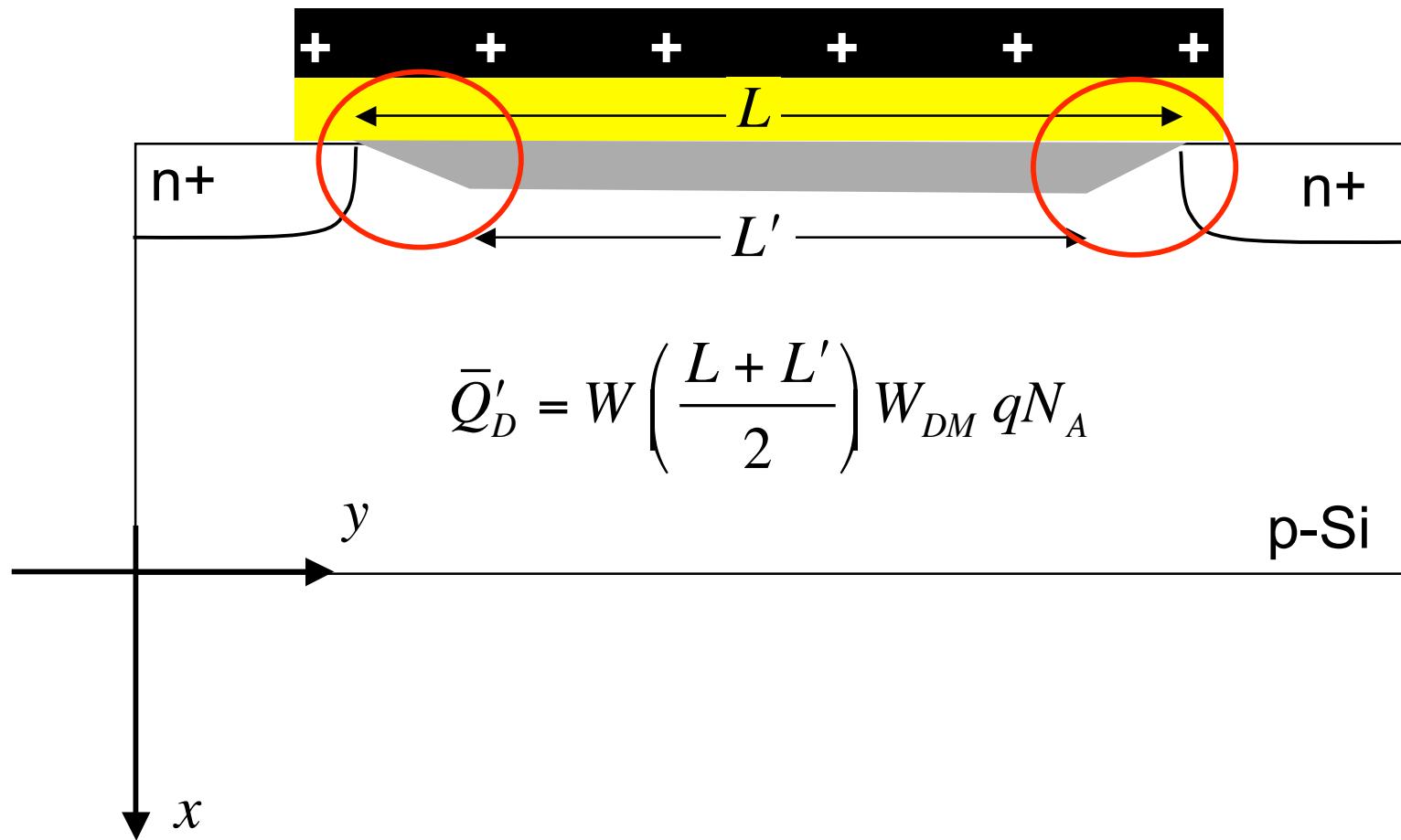
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## charge sharing model (ii)



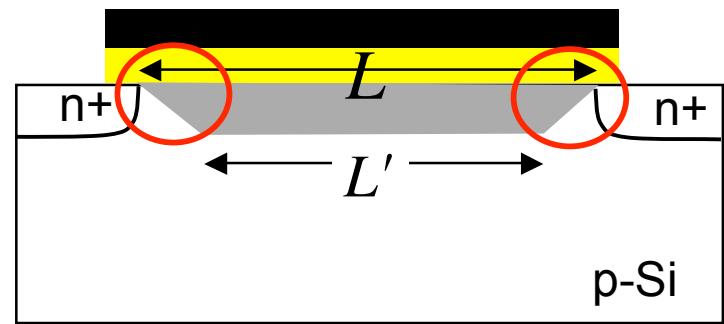
## charge sharing model (ii)



## charge sharing model (iii)

$$\bar{Q}_D = (W L W_{DM}) q N_A$$

$$\bar{Q}'_D = W \left( \frac{L + L'}{2} \right) W_{DM} q N_A$$



$$\frac{\bar{Q}'_D}{\bar{Q}_D} = \gamma = \frac{1}{2} \left( 1 + \frac{L}{L'} \right) = \frac{L + L'}{2L'} < 1$$

$$V_T = V_{FB} + 2\psi_B - \gamma \frac{Q_D}{C_{OX}} < V_T (\text{long channel})$$

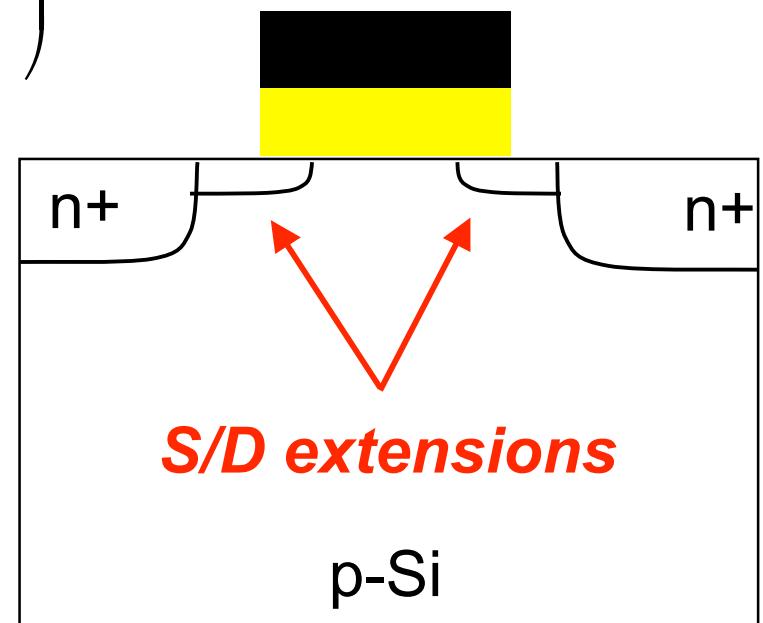
## charge sharing model (iii)

$$V_T = V_{FB} + 2\psi_B - \gamma \frac{Q_D}{C_{OX}} < V_T \text{ (long channel)}$$

$$\gamma = \frac{L + L'}{2L} = 1 - \frac{x_j}{L} \left( \sqrt{1 + \frac{2W_{DM}}{x_j}} - 1 \right)$$

(prob. 3.6, Taur and Ning)

for  $\gamma \sim 1$ , need:  $\begin{cases} x_j \ll L \\ W_{DM} \ll x_j \end{cases}$

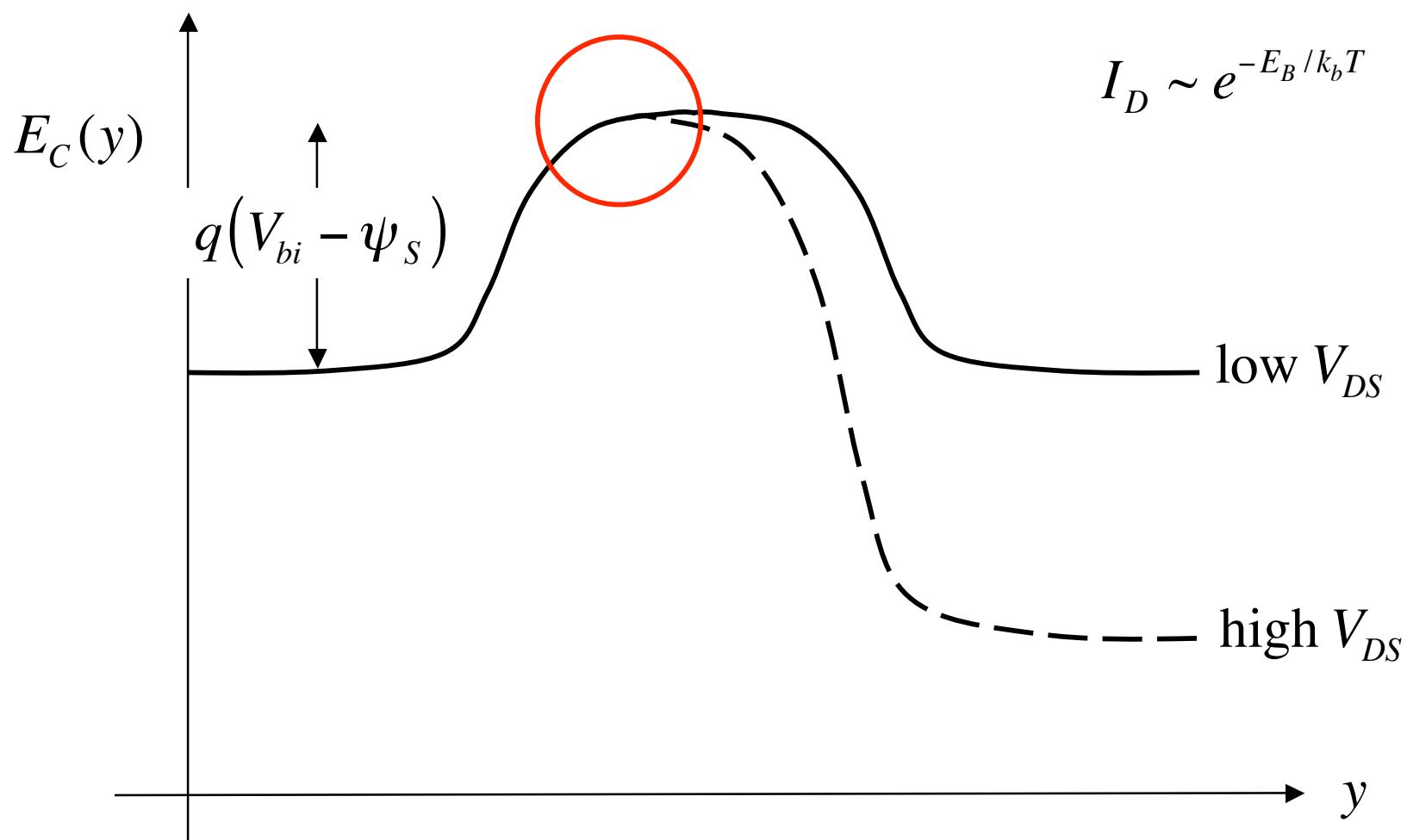


# outline

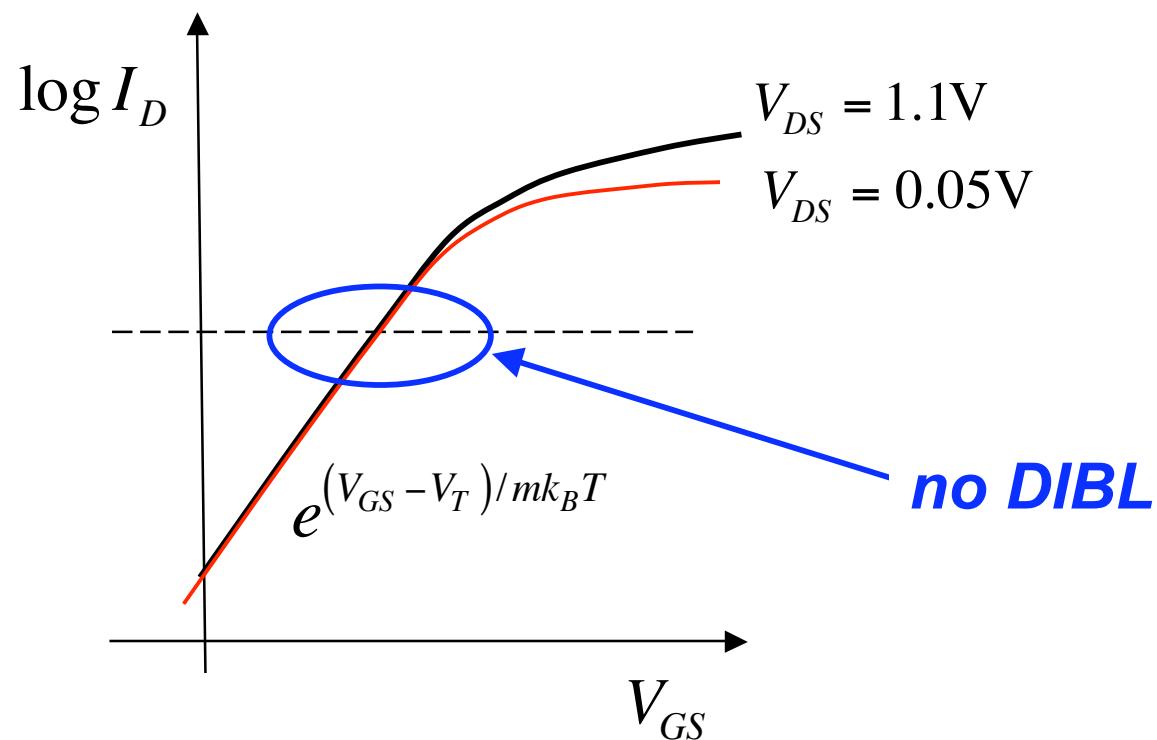
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- 4) 2D capacitor model

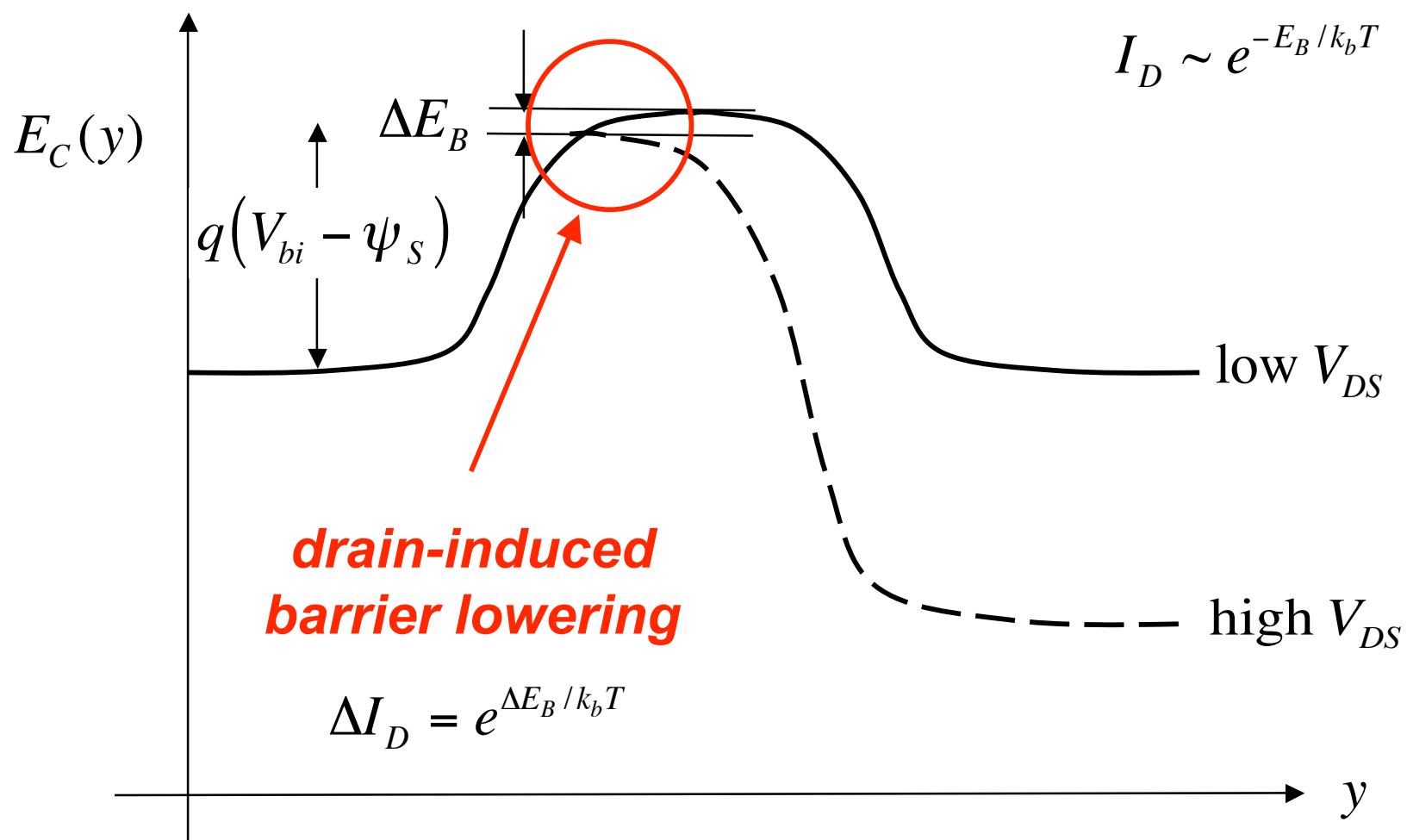
# barrier lowering



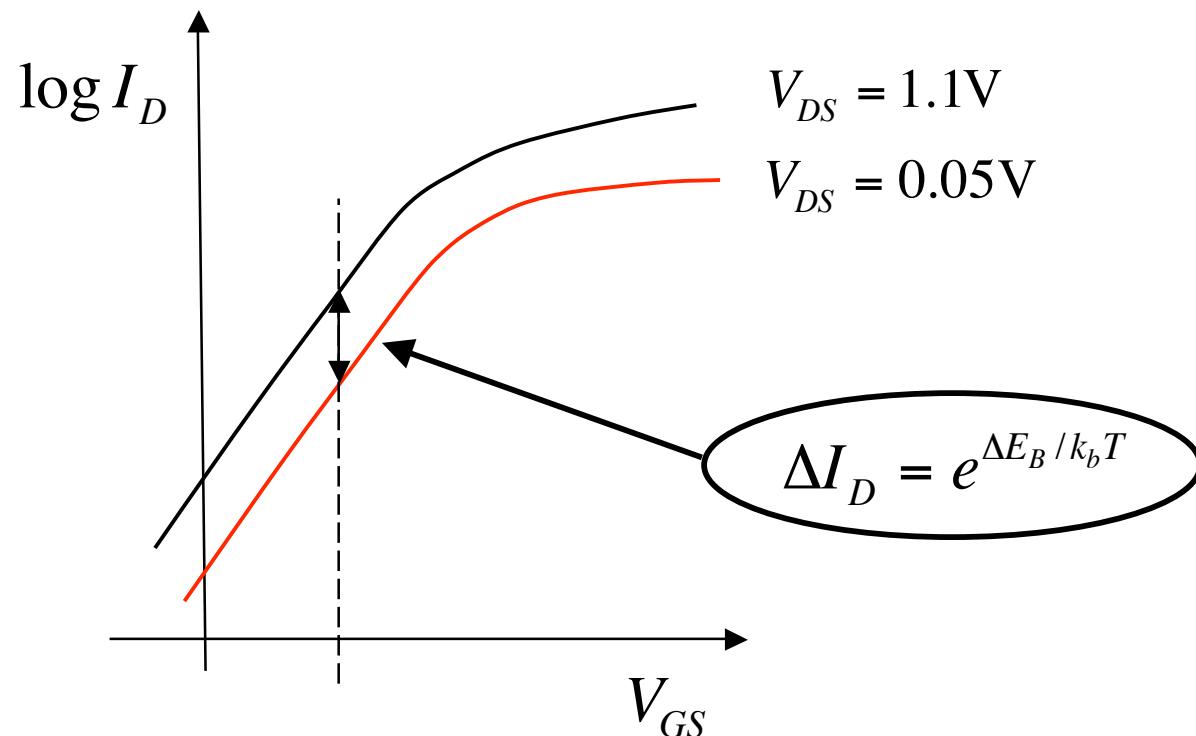
## barrier lowering (ii)



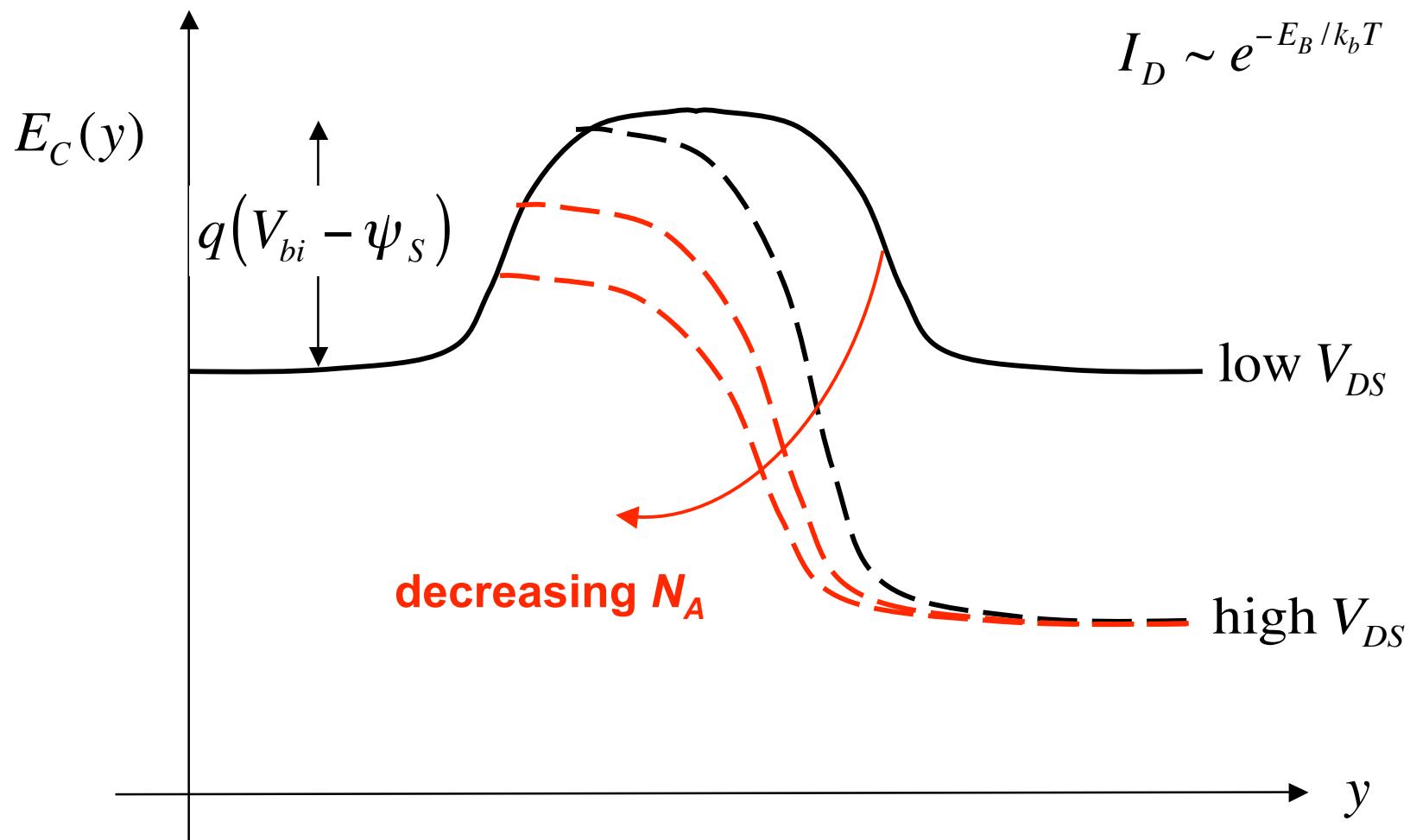
## barrier lowering (iii)



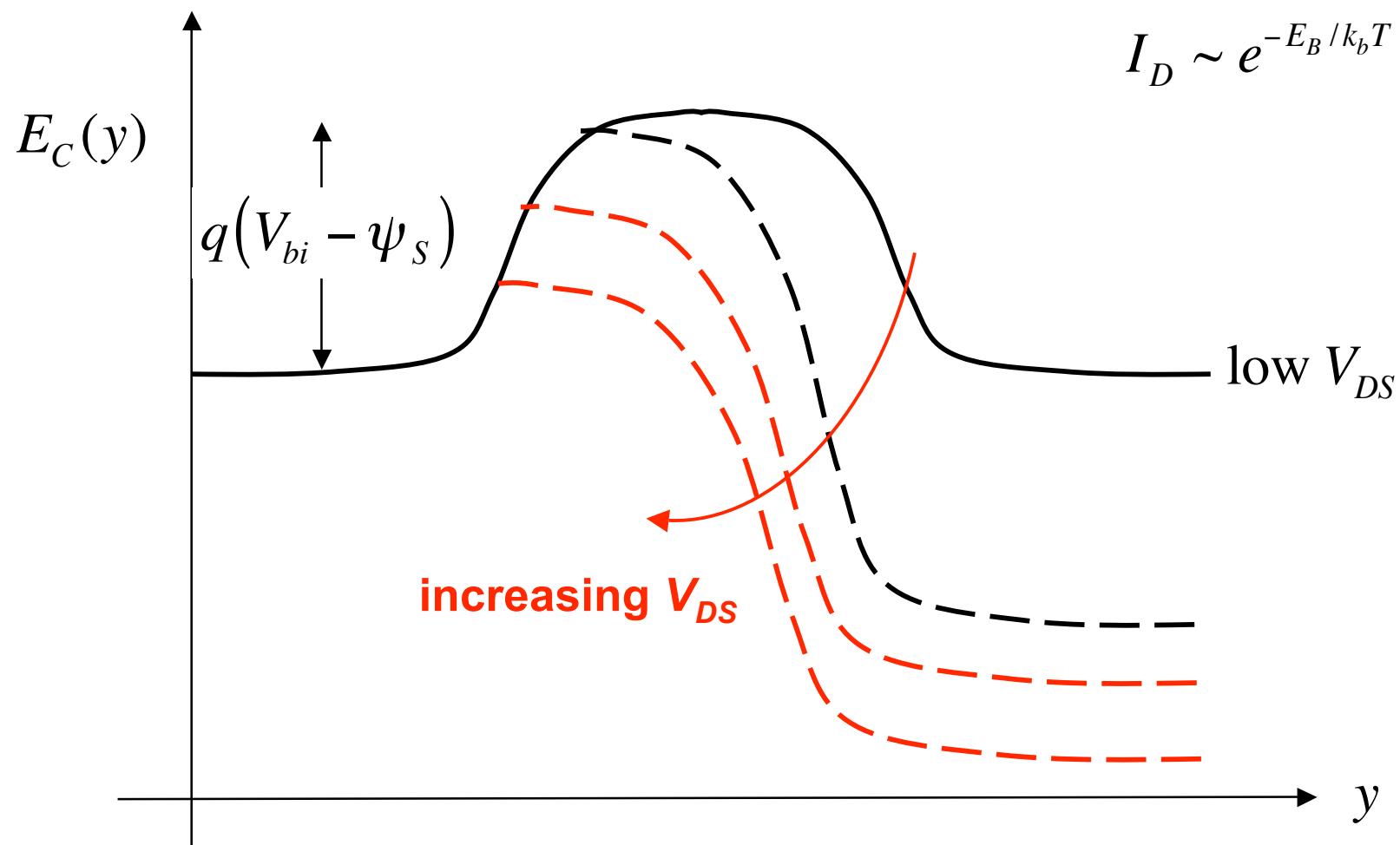
## barrier lowering (iv)



# punchthrough



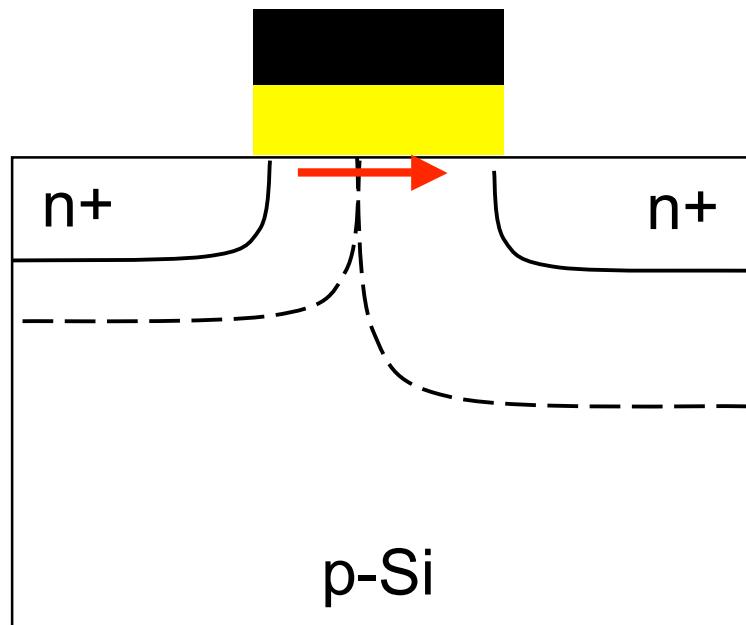
## punchthrough (ii)



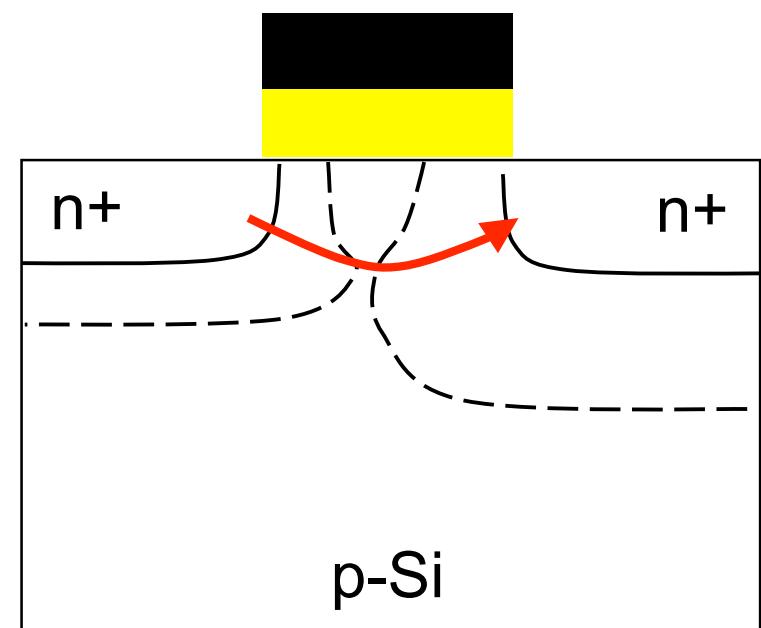
# punchthrough (iii)

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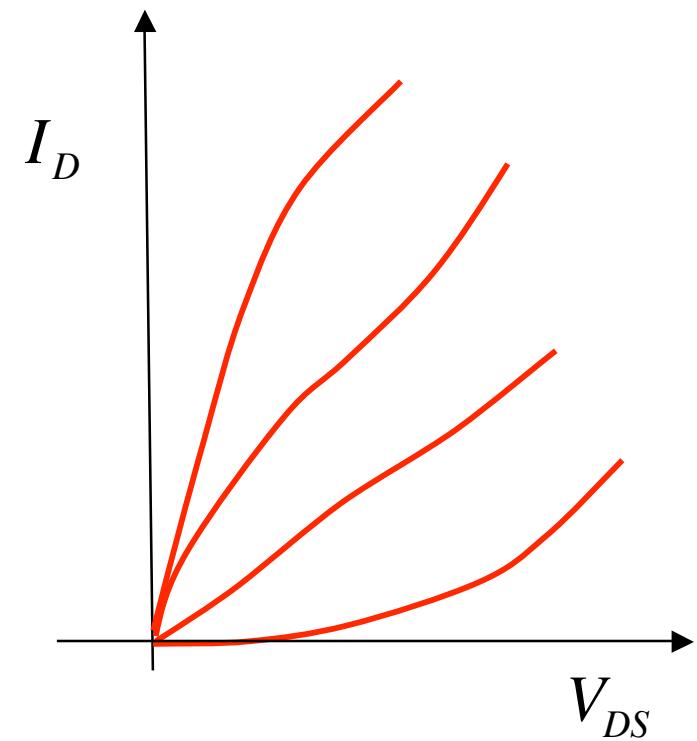
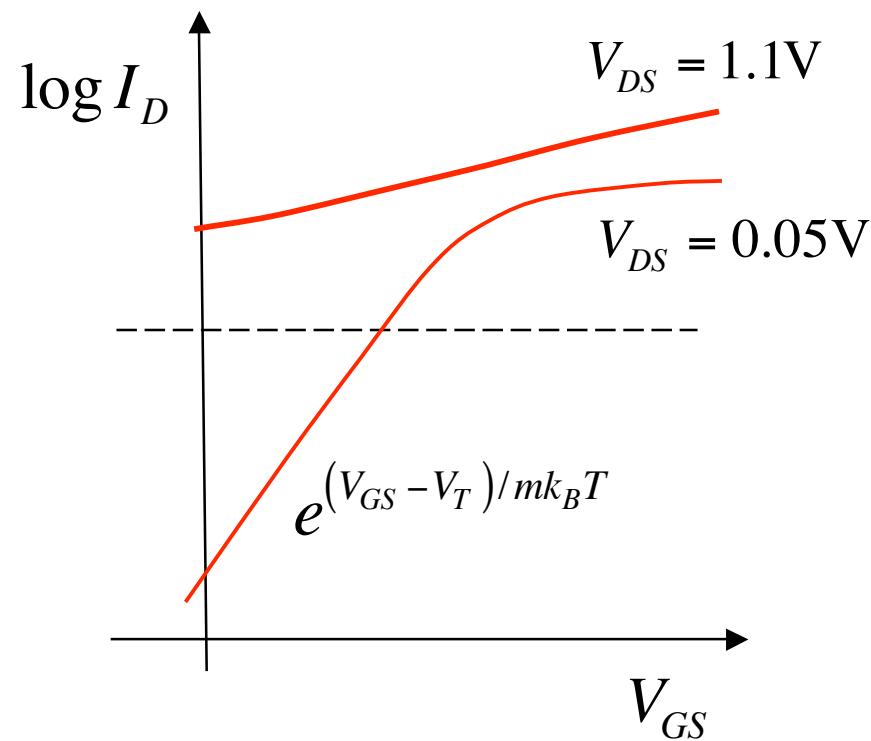
surface punchthrough



bulk punchthrough



# punchthrough (iv)



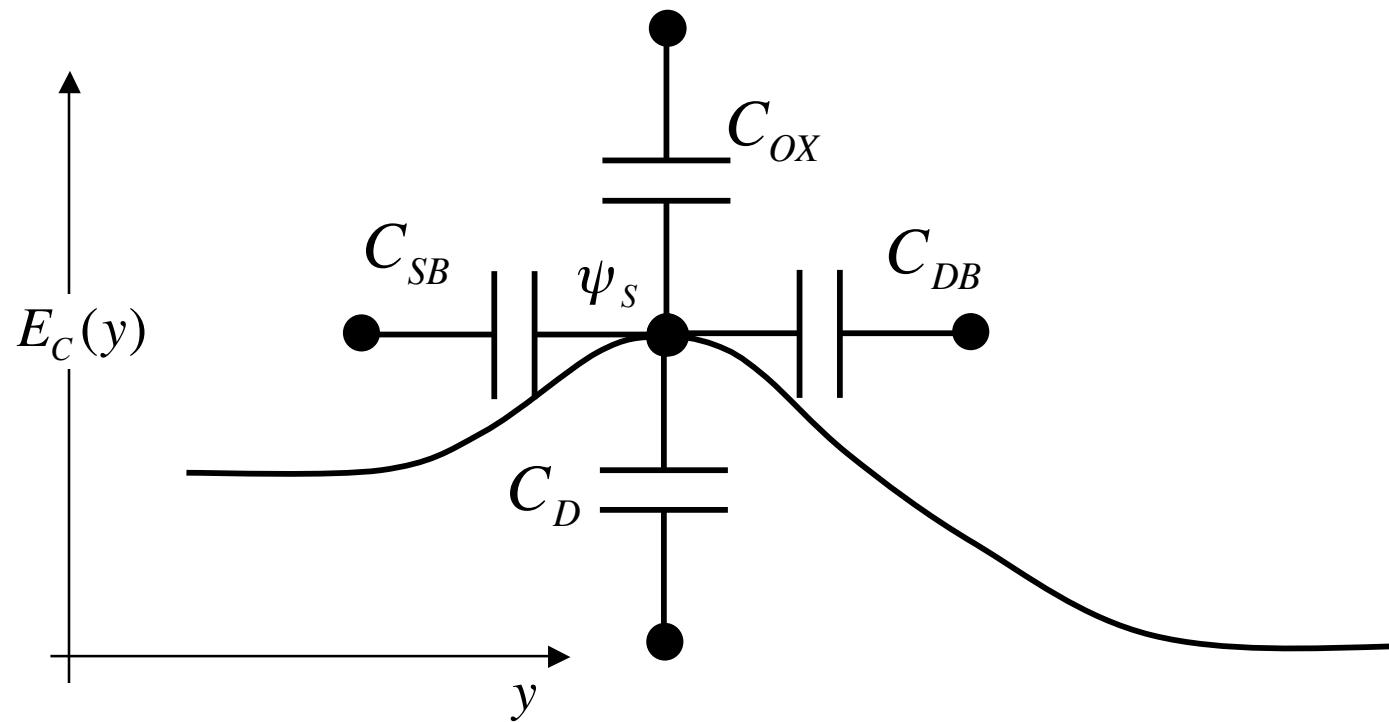
# outline

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- 1) Consequences of 2D electrostatics
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- 4) 2D capacitor model**

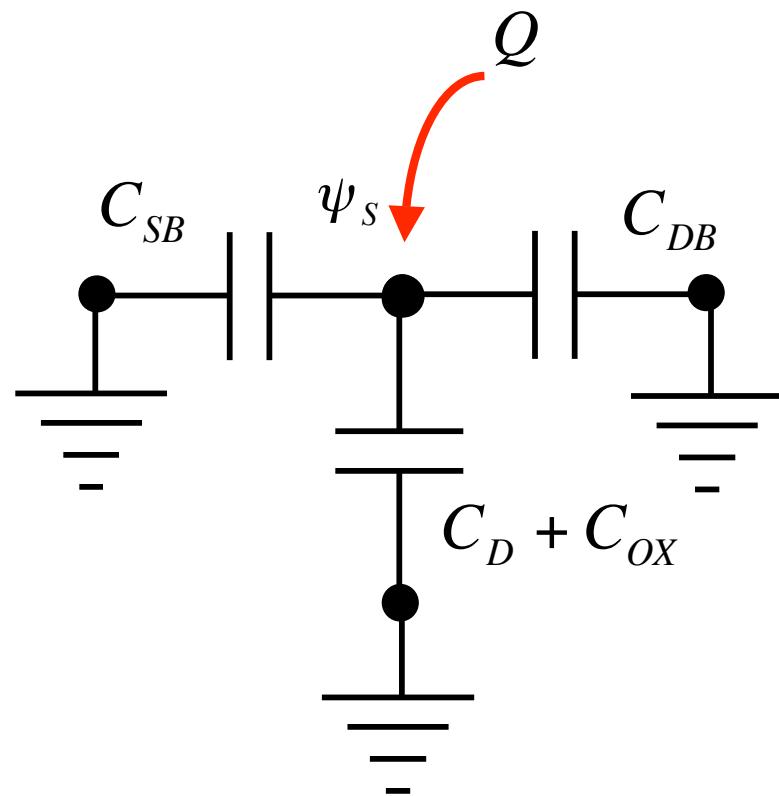
# 2D capacitor model

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## 2D capacitor model ( $V = 0$ )

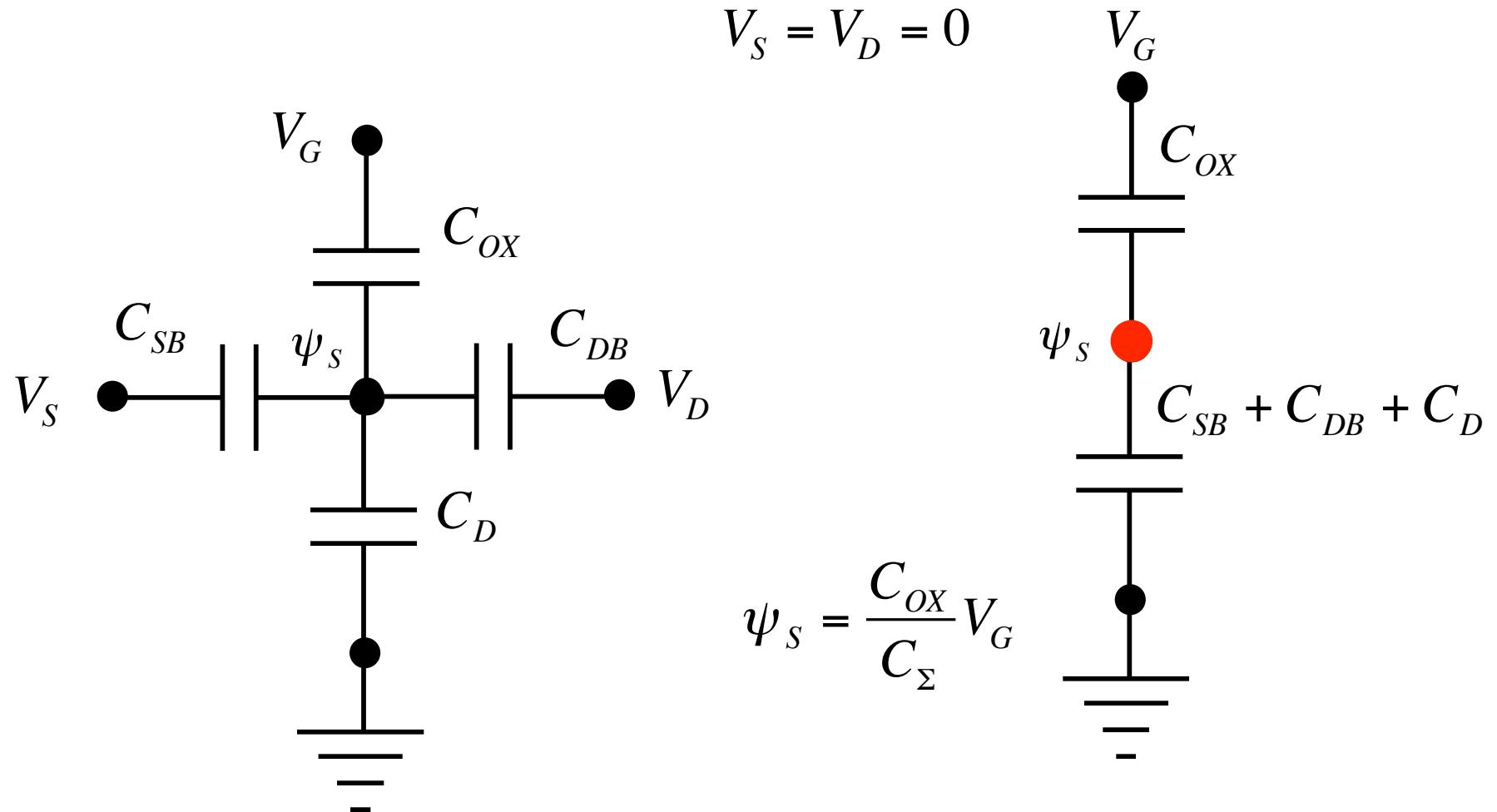
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$$\psi_s = \frac{Q}{C_\Sigma}$$

$$C_\Sigma = C_G + C_{SB} + C_{DB} + C_D$$

## 2D capacitor model ( $Q = 0$ )



## 2D capacitor model (general solution)

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$$\psi_s = \frac{C_{ox}}{C_\Sigma} V_G + \frac{C_{sb}}{C_\Sigma} V_s + \frac{C_{db}}{C_\Sigma} V_d + \frac{Q}{C_\Sigma}$$

*recall:*

$$V_G = \psi_s - \frac{Q}{C_{ox}}$$

## 2D capacitor model (general solution)

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$$\psi_s = \frac{C_{ox}}{C_\Sigma} V_G + \frac{C_{sb}}{C_\Sigma} V_s + \frac{C_{db}}{C_\Sigma} V_d + \frac{Q}{C_\Sigma}$$

$$\frac{\partial \psi_s}{\partial V_G} = \frac{C_{ox}}{C_\Sigma} \quad \frac{\partial \psi_s}{\partial V_d} = \frac{C_{db}}{C_\Sigma}$$

$$\frac{\partial \psi_s}{\partial V_G} \gg \frac{\partial \psi_s}{\partial V_d} \Rightarrow C_{ox} \gg C_{db}$$

need  $t_{ox} \ll L$

# 2D capacitor model and S

$$I_D \propto e^{q\psi_S/k_B T}$$

$$\frac{\partial I_D}{\partial V_G} \propto \frac{e^{q\psi_S/k_B T}}{k_B T / q} \frac{\partial \psi_S}{\partial V_G} = \frac{I_D}{k_B T / q} \left( \frac{C_{OX}}{C_\Sigma} \right)$$

$$I_D \propto e^{qV_G/mk_B T}$$

$$\frac{\partial I_D}{\partial V_G} \propto e^{qV_G/mk_B T} \frac{q}{mk_B T} = \frac{I_D}{k_B T / q} \left( \frac{1}{m} \right)$$

$$m = C_\Sigma / C_{OX}$$

$$\text{if } C_\Sigma = C_{OX} + C_D$$

$$\begin{aligned} m &= (C_{OX} + C_D) / C_{OX} \\ &= (1 + C_D / C_{OX}) \end{aligned}$$

**2D electrostatics  
( $C_{DB}$  not negligible)  
increases S.**

## 2D capacitor model and S

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$$I_D \propto e^{q\psi_S/k_B T}$$

$$\frac{\partial \log_{10} I_D}{\partial V_D} \propto \frac{1}{2.3k_B T / q} \frac{\partial \psi_S}{\partial V_D} = \frac{1}{2.3k_B T / q} \left( \frac{C_D}{C_\Sigma} \right)$$

$$\Delta \log_{10} I_D = \frac{\Delta V_D}{2.3k_B T / q} \left( \frac{C_D}{C_\Sigma} \right)$$
$$\Delta \log_{10} I_D = \frac{\Delta V_G}{S}$$

$$DIBL = C_{DB}/C_{OX}$$

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