INFO AND EQUATION SHEETS

Sheets with relevant equations and a listing of Si material parameters, physical constants, and key conversion factors were routinely supplied at the end of closed-book test booklets. Rather than provide the info/equation sheets with each test, we have collected on the following pages† a set of sheets that can be used with any of the closed-book Module-C test booklets.

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† Pages C0-2 through C0-4 and the equations on pages C0-5 through C0-9 were reproduced from *PIERRET, ROBERT F., SEMICONDUCTOR DEVICE FUNDAMENTALS, 1st edition, © 1996. Reprinted by permission of Pearson Education, Inc., Upper Saddle River, NJ.*

SILICON MATERIAL PARAMETERS

$E_G = 1.12$ eV $(T = 300\text{K})$

$n_i = 10^{10}/\text{cm}^3$ $(T = 300\text{K})$

$K_S = 11.8$

$K_O = 3.9$

PHYSICAL CONSTANTS

Symbol	Name	Value
q	Electronic charge (magnitude)	1.60×10^{-19} coul
ε_0	Permittivity of free space	8.85×10^{-14} farad/cm
k	Boltzmann constant	8.617×10^{-5} eV/K
h	Planck constant	6.63×10^{-34} joule-sec
m_0	Electron rest mass	9.11×10^{-31} kg
kT	Thermal energy	0.0259 eV $(T = 300\text{ K})$
kT/q	Thermal voltage	0.0259 V $(T = 300\text{ K})$

CONVERSION FACTORS

1Å $= 10^{-8}$ cm $= 10^{-10}$ m

1 μm $= 10^{-4}$ cm $= 10^{-6}$ m

1 eV $= 1.60 \times 10^{-19}$ joules

Table 2.4 Carrier Modeling Equation Summary.

Density of States and Fermi Function

$$g_c(E) = \frac{m_n^*\sqrt{2m_n^*(E - E_c)}}{\pi^2\hbar^3}, \quad E \geq E_c$$

$$g_v(E) = \frac{m_p^*\sqrt{2m_p^*(E_v - E)}}{\pi^2\hbar^3}, \quad E \leq E_v$$

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}}$$

Carrier Concentration Relationships

$$n = N_C \frac{2}{\sqrt{\pi}} F_{1/2}(\eta_c) \qquad N_C = 2\left[\frac{m_n^* kT}{2\pi\hbar^2}\right]^{3/2}$$

$$p = N_V \frac{2}{\sqrt{\pi}} F_{1/2}(\eta_v) \qquad N_V = 2\left[\frac{m_p^* kT}{2\pi\hbar^2}\right]^{3/2}$$

$$n = N_C e^{(E_F-E_c)/kT}$$

$$p = N_V e^{(E_v-E_F)/kT}$$

$$n = n_i e^{(E_F-E_i)/kT}$$

$$p = n_i e^{(E_i-E_F)/kT}$$

n_i, np-Product, and Charge Neutrality

$$n_i = \sqrt{N_C N_V}\, e^{-E_G/2kT} \qquad np = n_i^2 \qquad p - n + N_D - N_A = 0$$

n, p, and Fermi Level Computational Relationships

$$n = \frac{N_D - N_A}{2} + \left[\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2\right]^{1/2}$$

$$E_i = \frac{E_c + E_v}{2} + \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right)$$

$$n \simeq N_D, \quad p \simeq n_i^2/N_D \qquad N_D \gg N_A,\ N_D \gg n_i$$

$$E_F - E_i = kT\ln(n/n_i) = -kT\ln(p/n_i)$$

$$p \simeq N_A, \quad n \simeq n_i^2/N_A \qquad N_A \gg N_D,\ N_A \gg n_i$$

$$E_F - E_i = kT\ln(N_D/n_i) \quad N_D \gg N_A,\ N_D \gg n_i$$

$$E_i - E_F = kT\ln(N_A/n_i) \quad N_A \gg N_D,\ N_A \gg n_i$$

Table 3.3 Carrier Action Equation Summary.

Equations of State
$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \cdot \mathbf{J}_N + \left.\frac{\partial n}{\partial t}\right\|_{\substack{\text{thermal}\\ \text{R–G}}} + \left.\frac{\partial n}{\partial t}\right\|_{\substack{\text{other}\\ \text{processes}}} \qquad \frac{\partial \Delta n_p}{\partial t} = D_N\frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G_L$$
$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \cdot \mathbf{J}_P + \left.\frac{\partial p}{\partial t}\right\|_{\substack{\text{thermal}\\ \text{R–G}}} + \left.\frac{\partial p}{\partial t}\right\|_{\substack{\text{other}\\ \text{processes}}} \qquad \frac{\partial \Delta p_n}{\partial t} = D_P\frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G_L$$
Current and R–G Relationships
$$\mathbf{J}_N = \mathbf{J}_{N\|\text{drift}} + \mathbf{J}_{N\|\text{diff}} = q\mu_n n\mathscr{E} + qD_N\nabla n \qquad \left.\frac{\partial n}{\partial t}\right\|_{\substack{\text{i-thermal}\\ \text{R–G}}} = -\frac{\Delta n}{\tau_n}$$
⇕ drift ⇕ diffusion
$$\mathbf{J}_P = \mathbf{J}_{P\|\text{drift}} + \mathbf{J}_{P\|\text{diff}} = q\mu_p p\mathscr{E} - qD_P\nabla p \qquad \left.\frac{\partial p}{\partial t}\right\|_{\substack{\text{i-thermal}\\ \text{R–G}}} = -\frac{\Delta p}{\tau_p}$$
$$\mathbf{J} = \mathbf{J}_N + \mathbf{J}_P$$
Key Parametric Relationships
$$L_N \equiv \sqrt{D_N\tau_n} \qquad \frac{D_N}{\mu_n} = \frac{kT}{q} \qquad \tau_n = \frac{1}{c_n N_T}$$
$$L_P \equiv \sqrt{D_P\tau_p} \qquad \frac{D_P}{\mu_p} = \frac{kT}{q} \qquad \tau_p = \frac{1}{c_p N_T}$$
Resistivity and Electrostatic Relationships
$$\rho = \frac{1}{q(\mu_n n + \mu_p p)}$$
$$\rho = \frac{1}{q\mu_n N_D}$$ . . . *n*-type semiconductor
$$\rho = \frac{1}{q\mu_p N_A}$$ . . . *p*-type semiconductor
$$\mathscr{E} = \frac{1}{q}\frac{dE_c}{dx} = \frac{1}{q}\frac{dE_v}{dx} = \frac{1}{q}\frac{dE_i}{dx} \qquad V = -\frac{1}{q}(E_c - E_{ref})$$
Quasi-Fermi Level Relationships
$$F_N \equiv E_i + kT\ln\left(\frac{n}{n_i}\right) \qquad \mathbf{J}_N = \mu_n n\nabla F_N$$
$$F_P \equiv E_i - kT\ln\left(\frac{p}{n_i}\right) \qquad \mathbf{J}_P = \mu_p p\nabla F_P$$

DIODES

BUILT-IN VOLTAGE

$$J_N = q\mu_n n\mathcal{E} + qD_N \frac{dn}{dx} = 0$$

$$\mathcal{E} = -\frac{D_N}{\mu_n}\frac{dn/dx}{n} = -\frac{kT}{q}\frac{dn/dx}{n}$$

$$V_{bi} = -\int_{-x_p}^{x_n} \mathcal{E}\,dx = \frac{kT}{q}\int_{n(-x_p)}^{n(x_n)} \frac{dn}{n} = \frac{kT}{q}\ln\left[\frac{n(x_n)}{n(-x_p)}\right]$$

$$n(x_n) = N_D$$

$$n(-x_p) = n_i^2/N_A$$

$$\boxed{V_{bi} = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)}$$

STEP JUNCTION

For $-x_p \le x \le 0 \ldots$

$$\mathscr{E}(x) = -\frac{qN_A}{K_S\varepsilon_0}(x_p + x)$$

$$V(x) = \frac{qN_A}{2K_S\varepsilon_0}(x_p + x)^2$$

$$x_p = \left[\frac{2K_S\varepsilon_0}{q}\frac{N_D}{N_A(N_A + N_D)}(V_{bi} - V_A)\right]^{1/2}$$

For $0 \le x \le x_n \ldots$

$$\mathscr{E}(x) = -\frac{qN_D}{K_S\varepsilon_0}(x_n - x)$$

$$V(x) = V_{bi} - V_A - \frac{qN_D}{2K_S\varepsilon_0}(x_n - x)^2$$

$$x_n = \left[\frac{2K_S\varepsilon_0}{q}\frac{N_A}{N_D(N_A + N_D)}(V_{bi} - V_A)\right]^{1/2}$$

and

$$\boxed{W = \left[\frac{2K_S\varepsilon_0}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)(V_{bi} - V_A)\right]^{1/2}}$$

LINEARLY GRADED JUNCTION

$$\rho(x) = \begin{cases} qax & \ldots -W/2 \le x \le W/2 \\ 0 & \ldots x \le -W/2 \text{ and } x \ge W/2 \end{cases}$$

$$x_p = x_n = \frac{W}{2}$$

$$\mathcal{E}(x) = \frac{qa}{2K_S\varepsilon_0}\left[x^2 - \left(\frac{W}{2}\right)^2\right] \qquad \ldots -\frac{W}{2} \le x \le \frac{W}{2}$$

$$V(x) = \frac{qa}{6K_S\varepsilon_0}\left[2\left(\frac{W}{2}\right)^3 + 3\left(\frac{W}{2}\right)^2 x - x^3\right] \qquad \ldots -\frac{W}{2} \le x \le \frac{W}{2}$$

$$\boxed{W = \left[\frac{12K_S\varepsilon_0}{qa}(V_{bi} - V_A)\right]^{1/3}}$$

Ideal Diode Equation – 1

$$n(x_n)p(x_n) = p(x_n)N_D = n_i^2\, e^{qV_A/kT}$$

$$p(x_n) = \frac{n_i^2}{N_D} e^{qV_A/kT}$$

$$\boxed{\Delta p_n(x_n) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)}$$

$$0 = D_P \frac{d^2 \Delta p_n}{dx'^2} - \frac{\Delta p_n}{\tau_p} \qquad \ldots x' \geq 0$$

$$\Delta p_n(x' \to \infty) = 0$$

$$\Delta p_n(x'=0) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)$$

Ideal Diode Equation – 2

$$\Delta p_n(x') = A_1 e^{-x'/L_P} + A_2 e^{x'/L_P} \qquad \ldots x' \geq 0$$

$$L_P = \sqrt{D_P \tau_p}$$

$$\boxed{\Delta p_n(x') = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right) e^{-x'/L_P}} \qquad \ldots x' \geq 0$$

$$\boxed{J_P(x') = -qD_P \frac{d\Delta p_n}{dx'} = q\frac{D_P}{L_P}\frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right) e^{-x'/L_P}} \qquad \ldots x' \geq 0$$

$$\boxed{\Delta n_p(x'') = \frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right) e^{-x''/L_N}} \qquad \ldots x'' \geq 0$$

Ideal Diode Eq. – 3

$$\boxed{J_N(x'') = -qD_N \frac{d\Delta n_p}{dx''} = q\frac{D_N}{L_N}\frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right) e^{-x''/L_N}} \qquad \ldots x'' \geq 0$$

$$J_N(x=-x_p) = J_N(x''=0) = q\frac{D_N}{L_N}\frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right)$$

$$J_P(x=x_n) = J_P(x'=0) = q\frac{D_P}{L_P}\frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)$$

$$I = AJ = qA\left(\frac{D_N}{L_N}\frac{n_i^2}{N_A} + \frac{D_P}{L_P}\frac{n_i^2}{N_D}\right)\left(e^{qV_A/kT} - 1\right)$$

$$\boxed{\begin{aligned} I &= I_0\left(e^{qV_A/kT} - 1\right) \\ I_0 &\equiv qA\left(\frac{D_N}{L_N}\frac{n_i^2}{N_A} + \frac{D_P}{L_P}\frac{n_i^2}{N_D}\right) \end{aligned}}$$

Junction Capacitance

$$Y = G + j\omega C$$

$$C_J = \frac{K_S \varepsilon_0 A}{W}$$

$$W = \left[\frac{2K_S\varepsilon_0}{qN_B}(V_{bi} - V_A)\right]^{1/2}$$ ... asymmetrical step junction

$$W = \left[\frac{12K_S\varepsilon_0}{qa}(V_{bi} - V_A)\right]^{1/3}$$ ... linearly-graded junction

$$N_B(x) = bx^m$$ ... $x > 0$

Schottky I–V

$$I_{M \bullet \to S}(V_A) = I_{M \bullet \to S}(V_A = 0)$$

$$I_{M \bullet \to S}(V_A = 0) = -I_{S \bullet \to M}(V_A = 0) = -A\mathcal{a}^* T^2 e^{-\Phi_B/kT}$$

$$I = I_{S \bullet \to M} + I_{M \bullet \to S} = I_{S \bullet \to M} + I_{M \bullet \to S}(V_A = 0)$$

$$I = I_s (e^{qV_A/kT} - 1)$$

$$I_s \equiv A\mathcal{a}^* T^2 e^{-\Phi_B/kT}$$

MOS

$$\phi_S = \frac{1}{q}\left[E_i(\text{bulk}) - E_i(\text{surface})\right]$$

$$\phi_F = \frac{1}{q}\left[E_i(\text{bulk}) - E_F\right]$$

$$\phi_F = \begin{cases} \frac{kT}{q}\ln(N_A/n_i) & \ldots\ p\text{-type semiconductor} \\ -\frac{kT}{q}\ln(N_D/n_i) & \ldots\ n\text{-type semiconductor} \end{cases}$$

$\phi_S = 2\phi_F$ at the depletion–inversion transition point

$$C = \begin{cases} C_O & \text{acc} \\ \dfrac{C_O}{1 + \dfrac{K_O W}{K_S x_o}} & \text{depl} \\ C_O & \text{inv } (\omega \to 0) \\ \dfrac{C_O}{1 + \dfrac{K_O W_T}{K_S x_o}} & \text{inv } (\omega \to \infty) \end{cases}$$

$$W_T = \left[\frac{2K_S\varepsilon_0}{qN_A}(2\phi_F)\right]^{1/2}$$

$$V_G = \phi_S + \frac{K_S}{K_O}x_o\mathscr{E}_S$$

$$C = \frac{C_O}{\sqrt{1 + \dfrac{V_G}{V_\delta}}} \quad \text{(depletion biases)}$$

$$V_\delta \equiv \frac{q}{2}\frac{K_S x_o^2}{K_O^2\varepsilon_0}N_A$$

$$V_T = 2\phi_F - \frac{K_S x_o}{K_O}\sqrt{\frac{4qN_D}{K_S\varepsilon_0}(-\phi_F)} \quad \ldots \text{ ideal } p\text{-channel } (n\text{-bulk}) \text{ devices}$$

$$V_T = 2\phi_F + \frac{K_S x_o}{K_O}\sqrt{\frac{4qN_A}{K_S\varepsilon_0}\phi_F} \quad \ldots \text{ ideal } n\text{-channel } (p\text{-bulk}) \text{ devices}$$

$$I_D = -\frac{Z\bar{\mu}_n}{L}\int_0^{V_D} Q_N\,d\phi$$

$$I_D = \frac{Z\bar{\mu}_n C_o}{L}\left[(V_G - V_T)V_D - \frac{V_D^2}{2}\right] \quad \begin{pmatrix} 0 \le V_D \le V_{Dsat} \\ V_G \ge V_T \end{pmatrix}$$

$$C_o \equiv \frac{C_O}{A_G} = \frac{K_O\varepsilon_0}{x_o}$$

$$I_{Dsat} = \frac{Z\bar{\mu}_n C_o}{2L}(V_G - V_T)^2$$

$$V_{Dsat} = V_G - V_T$$

$$\Delta V_G = (V_G - V_G')\big|_{\text{same }\phi_S \text{ (or same } C)} = \phi_{MS}$$

$$\phi_{MS} \equiv \frac{1}{q}(\Phi_M - \Phi_S) = \frac{1}{q}\left[\Phi_M' - \chi' - (E_c - E_F)_{FB}\right]$$

$$\Delta V_G\begin{pmatrix}\text{mobile}\\ \text{ions}\end{pmatrix} = -\frac{1}{K_O\varepsilon_0}\int_0^{x_o} x\rho_{ion}(x)\,dx$$

$$\Delta V_G\begin{pmatrix}\text{fixed}\\ \text{charge}\end{pmatrix} = -\frac{Q_F}{C_o}$$

$$\Delta V_G = (V_G - V_G')\big|_{\text{same }\phi_S} = \phi_{MS} - \frac{Q_F}{C_o} - \frac{Q_M\gamma_M}{C_o} - \frac{Q_{IT}(\phi_S)}{C_o}$$

$$\Delta V_G\begin{pmatrix}\text{interfacial}\\ \text{traps}\end{pmatrix} = -\frac{Q_{IT}(\phi_S)}{C_o}$$

$$\gamma_M \equiv \frac{\int_0^{x_o} x\rho_{ion}(x)\,dx}{x_o\int_0^{x_o}\rho_{ion}(x)\,dx}$$

BJT

$$I_E = I_B + I_C$$

$$V_{EB} + V_{BC} + V_{CE} = 0 \qquad (V_{CE} = -V_{BC})$$

$$\gamma = \frac{I_{Ep}}{I_E} = \frac{I_{Ep}}{I_{Ep} + I_{En}} \quad \ldots \text{ pnp BJT}$$

$$\alpha_T = \frac{I_{Cp}}{I_{Ep}} \quad \ldots \text{ pnp BJT}$$

$$\alpha_{dc} = \gamma\alpha_T$$

$$I_{En} = -qAD_E \left.\frac{d\Delta n_E}{dx''}\right|_{x''=0} \qquad I_{Cp} = -qAD_B \left.\frac{d\Delta p_B}{dx}\right|_{x=W}$$

$$I_{Ep} = -qAD_B \left.\frac{d\Delta p_B}{dx}\right|_{x=0} \qquad I_{Cn} = qAD_C \left.\frac{d\Delta n_C}{dx'}\right|_{x'=0}$$

$$\beta_{dc} = \frac{I_C}{I_B} \qquad \beta_{dc} = \frac{\alpha_{dc}}{1 - \alpha_{dc}}$$

$$\Delta p_B(x) = \Delta p_B(0)\frac{\sinh[(W-x)/L_B]}{\sinh(W/L_B)} + \Delta p_B(W)\frac{\sinh(x/L_B)}{\sinh(W/L_B)}$$

$$\Delta p_B(x) = \Delta p_B(0) + [\Delta p_B(W) - \Delta p_B(0)]\frac{x}{W}$$

$$I_{En} = qA\frac{D_E}{L_E} n_{E0}(e^{qV_{EB}/kT} - 1)$$

$$I_{Ep} = qA\frac{D_B}{L_B} p_{B0}\left[\frac{\cosh(W/L_B)}{\sinh(W/L_B)}(e^{qV_{EB}/kT} - 1) - \frac{1}{\sinh(W/L_B)}(e^{qV_{CB}/kT} - 1)\right]$$

$$I_{Cn} = -qA\frac{D_C}{L_C} n_{C0}(e^{qV_{CB}/kT} - 1)$$

$$I_{Cp} = qA\frac{D_B}{L_B} p_{B0}\left[\frac{1}{\sinh(W/L_B)}(e^{qV_{EB}/kT} - 1) - \frac{\cosh(W/L_B)}{\sinh(W/L_B)}(e^{qV_{CB}/kT} - 1)\right]$$

$$\gamma = \frac{1}{1 + \frac{D_E}{D_B}\frac{N_B}{N_E}\frac{W}{L_E}}$$

$$\alpha_T = \frac{1}{1 + \frac{1}{2}\left(\frac{W}{L_B}\right)^2}$$

$$\alpha_{dc} = \frac{1}{1 + \frac{D_E}{D_B}\frac{N_B}{N_E}\frac{W}{L_E} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2}$$

$$\beta_{dc} = \frac{1}{\frac{D_E}{D_B}\frac{N_B}{N_E}\frac{W}{L_E} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2}$$

$$I_E = I_{F0}(e^{qV_{EB}/kT} - 1) - \alpha_R I_{R0}(e^{qV_{CB}/kT} - 1)$$

$$I_C = \alpha_F I_{F0}(e^{qV_{EB}/kT} - 1) - I_{R0}(e^{qV_{CB}/kT} - 1)$$

$$V_{CE0} = V_{CB0}(1 - \alpha_{dc})^{1/m} = \frac{V_{CB0}}{(\beta_{dc} + 1)^{1/m}}$$