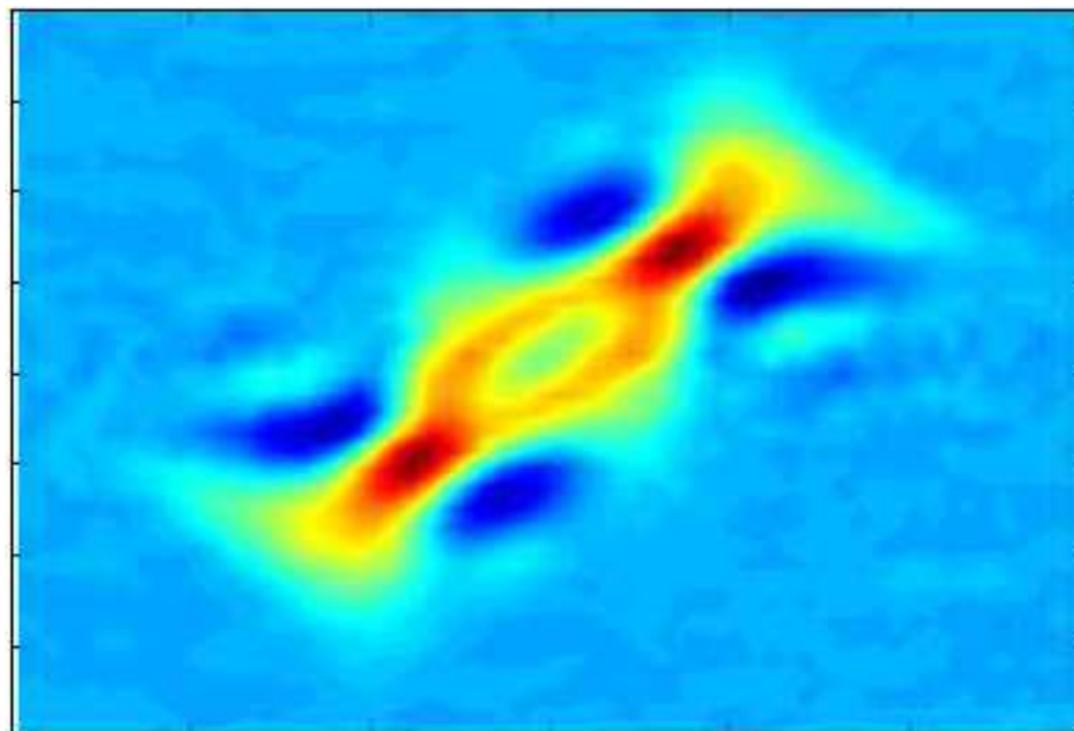


System of identical Fermions



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On the simulation of indistinguishable fermions in the many-body Wigner formalism

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The many-body Wigner equation

$$\frac{\partial f_W}{\partial t}(\mathbf{x}; \mathbf{p}; t) + \sum_{k=1}^n \frac{\mathbf{p}_k}{m_k} \cdot \nabla_{\mathbf{x}_k} f_W = \int d\mathbf{p} f_W(\mathbf{x}; \mathbf{p}; t) V_W(\mathbf{x}; \mathbf{p}; t)$$

$$V_W(\mathbf{x}; \mathbf{p}; t) = \frac{i}{\pi^{dn} \hbar^{dn+1}} \int d\mathbf{x}' e^{-(\frac{2i}{\hbar}) \sum_{k=1}^n \mathbf{x}'_k \cdot \mathbf{p}_k} \times \\ \times \left[V\left(\mathbf{x} + \frac{\mathbf{x}'}{2}; t\right) - V\left(\mathbf{x} - \frac{\mathbf{x}'}{2}; t\right) \right]$$

$$\int d\mathbf{x}' = \int d\mathbf{x}'_1 \int d\mathbf{x}'_2 \dots \int d\mathbf{x}'_n$$

$$\int d\mathbf{p} = \int d\mathbf{p}_1 \int d\mathbf{p}_2 \dots \int d\mathbf{p}_n$$

Physical interpretation of

$$\langle A \rangle_0 + \langle A \rangle_1 + \langle A \rangle_2$$

$$\begin{aligned}
\sum_{s=0}^2 \langle A \rangle_s &= \int_0^\tau dt' \int dx_i \sum_{M'=-\infty}^{+\infty} f_i(x_i, M') e^{-\int_0^{t'} \gamma(x_i(y)) dy} \times \\
&[A(x_1(t), M, t) \delta(t' - \tau) + \int_{t'}^\tau dt_1 \sum_{M=-\infty}^{+\infty} \Gamma(x_1, M_1, M') e^{-\int_{t'}^{t_1} \gamma(x_1(y)) dy} \times \\
&[A(x_2(t), M, t) \delta(t_1 - \tau) + \int_{t_1}^\tau dt_2 \sum_{M=-\infty}^{+\infty} \Gamma(x_2, M_2, M) e^{-\int_{t_1}^{t_2} \gamma(x_2(y)) dy} A(x_3(t), M_2, t) \delta(t_2 - \tau)]]
\end{aligned}$$

The iteration expansion of $\langle A \rangle$ branches.

Interpretation: creation of two new particle (+,-)

$$\frac{\Gamma(x, M, M')}{\gamma(x)} = \frac{V_W^+(x, M - M')}{\gamma(x)} - \frac{V_W^+(x, M' - M)}{\gamma(x)} + \delta_{M, M'}$$

Identical Fermions in the Wigner formalism

$$\Psi(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_i, \dots, \mathbf{x}_j, \dots, \mathbf{x}_n; t) = -\Psi(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_j, \dots, \mathbf{x}_i, \dots, \mathbf{x}_n; t)$$

$$f_W(\mathbf{x}; \mathbf{p}; t) = \frac{1}{(\hbar\pi)^{d \cdot n}} \int d\mathbf{x}' e^{-\frac{i}{\hbar} \sum_{k=1}^n \mathbf{x}'_k \cdot \mathbf{p}_k} \Psi\left(\mathbf{x} + \frac{\mathbf{x}'}{2}; t\right) \Psi^*\left(\mathbf{x} - \frac{\mathbf{x}'}{2}; t\right)$$

$$\Psi^-(\mathbf{x}_1, \dots, \mathbf{x}_n) = \frac{1}{\sqrt{n!}} \begin{vmatrix} \phi_1(\mathbf{x}_1) & \phi_2(\mathbf{x}_1) & \dots & \phi_n(\mathbf{x}_1) \\ \phi_1(\mathbf{x}_2) & \phi_2(\mathbf{x}_2) & \dots & \phi_n(\mathbf{x}_2) \\ \dots & \dots & \dots & \dots \\ \phi_1(\mathbf{x}_n) & \phi_2(\mathbf{x}_n) & \dots & \phi_n(\mathbf{x}_n) \end{vmatrix}$$

System of 2 identical Fermions

$$f_W^0(x_1, x_2; p_1, p_2) = \frac{1}{(\hbar\pi)^2} \int dx'_1 dx'_2 e^{-\frac{i}{\hbar}(x'_1 p_1 + x'_2 p_2)} \\ \Psi_0\left(x_1 + \frac{x'_1}{2}, x_2 + \frac{x'_2}{2}\right) \Psi_0^*\left(x_1 - \frac{x'_1}{2}, x_2 - \frac{x'_2}{2}\right)$$

$$\Psi_0(x_1, x_2) = \begin{vmatrix} \phi_1(x_1) & \phi_2(x_2) \\ \phi_1(x_2) & \phi_2(x_1) \end{vmatrix}$$

$$\phi_1(x) = N_1 e^{-\frac{1}{2}(\frac{x-x_1^0}{\sigma})^2} e^{ip_1^0 x},$$

$$\phi_2(x) = N_2 e^{-\frac{1}{2}(\frac{x-x_2^0}{\sigma})^2} e^{ip_2^0 x},$$

