A typical medium approach to Anderson localization in correlated systems.

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Outline

- Models for strongly correlated electron systems
- Review of DMFT and cluster approaches
- Anderson localization
- Extending DMFT: Typical medium theory
- Disordered Hubbard model – Do interactions and disorder co-operate or compete?
- Extending DCA: Typical medium DCA
- Benchmarks
- Metal-Insulator-Transition in a Weakly interacting Disordered Electron System

Minimal Models

Hubbard model

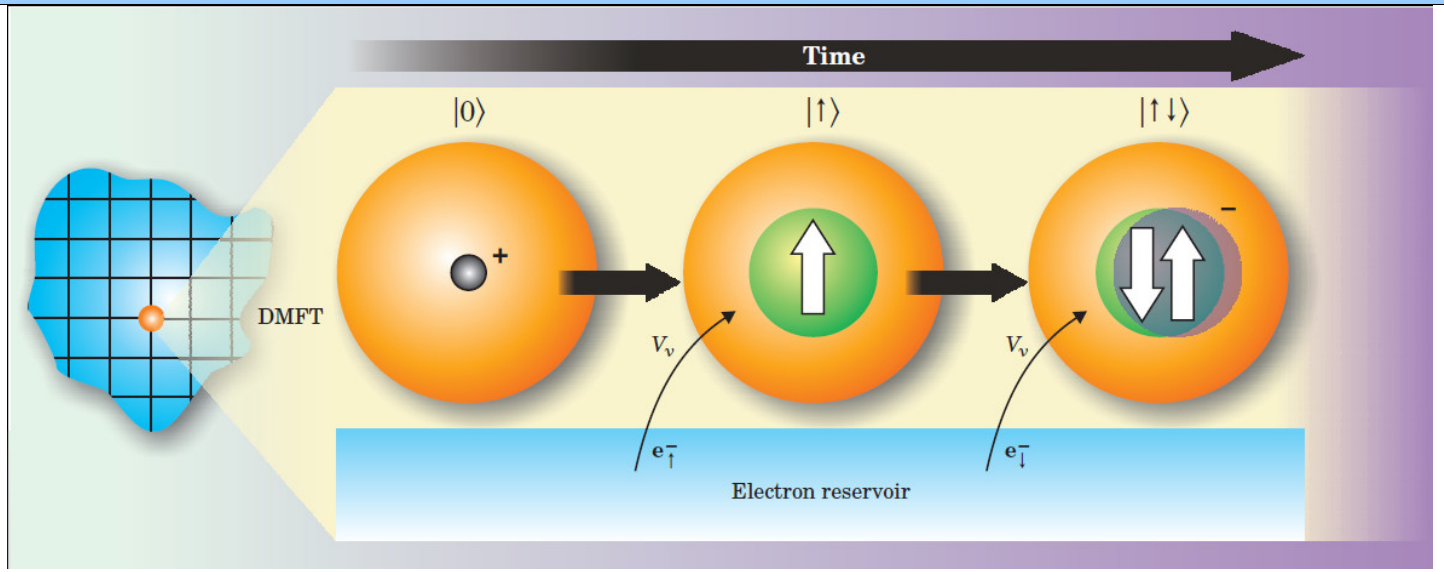
$$\hat{H} = -\sum_{<ij>,\sigma} t_{ij}(c^{\dagger}_{i\sigma}c_{j\sigma} + \text{h.}c.) + \epsilon_d \sum_{i\sigma} c^{\dagger}_{i\sigma}c_{i\sigma} + U\sum_{i} n_{i\uparrow}n_{i\downarrow}$$

Periodic Anderson Model

$$\hat{H} = \epsilon_c \sum_{i,\sigma} c^{\dagger}_{i\sigma}c_{i\sigma} - t\sum_{(i,j),\sigma} c^{\dagger}_{i\sigma}c_{j\sigma} + V\sum_{i,\sigma}(f^{\dagger}_{i\sigma}c_{i\sigma} + \text{H.c.}) + \sum_{i,\sigma}\left(\epsilon_f + \frac{U}{2}f^{\dagger}_{i\,-\sigma}f_{i\,-\sigma}\right) f^{\dagger}_{i\sigma}f_{i\sigma}.$$

Dynamical mean field theory

Mean field theory for quantum many body systems on a lattice.Maps lattice models to self-consistent impurity modelsSelf energy and Vertex function become purely local and momentum independent.Ignores spatial fluctuations but accounts for quantum local temporal fluctuations.Exact in the limit of infinite dimensions.

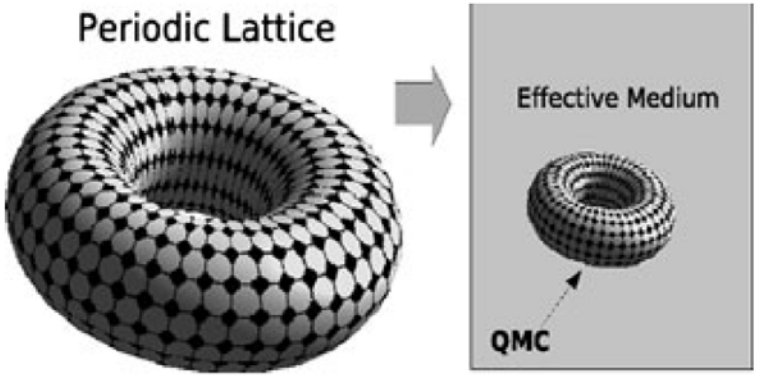


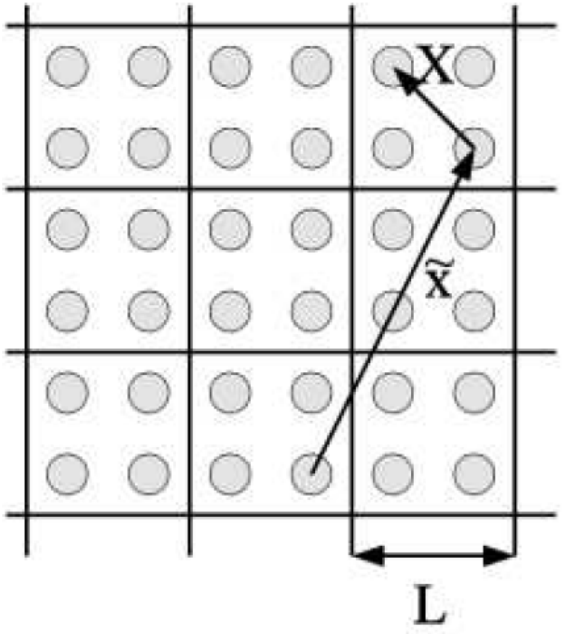
Kotliar and Vollhardt, Physics Today (March

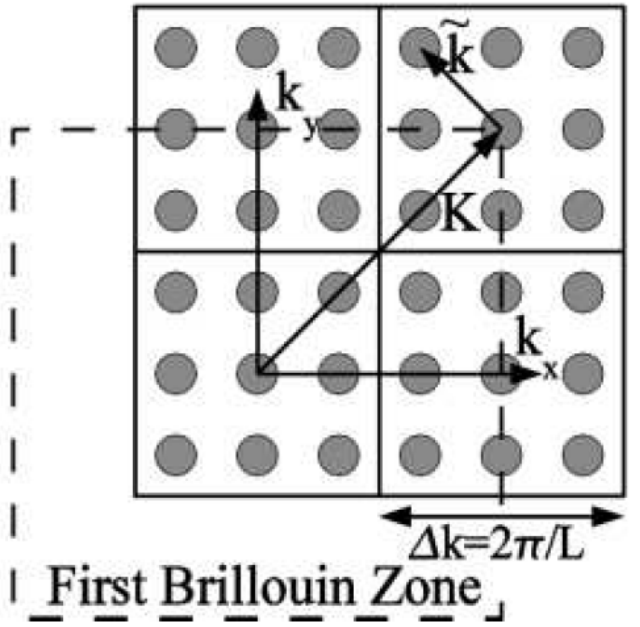
Beyond DMFT: Cluster approaches

Cluster-DMFT

Dynamical Cluster Approximation







Nc=4 Real space

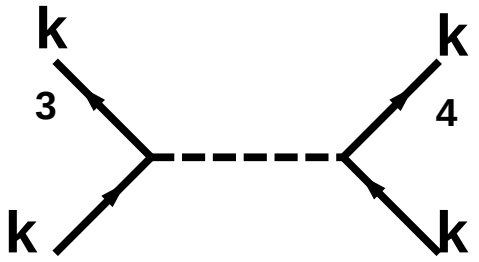
Nc=4 Momentum space

H.Fotso et al, Chapter in *Strongly Correlated Systems,* Springer (2012).
T. Maier et al, Rev. Mod. Phys **77** 1027 (2005).

Momentum space clusters
- Dynamical Cluster approximation

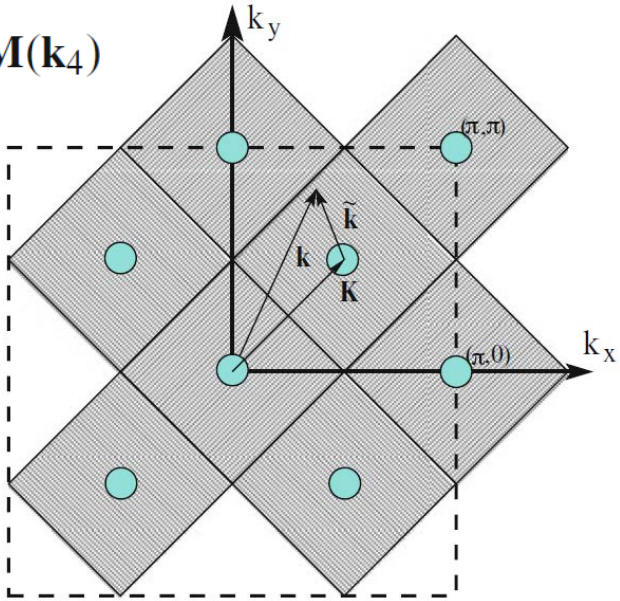
- In an exact theory, the Laue function expressing momentum conservation would be $\Delta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) = \sum_{\mathbf{r}} \exp\left[i\mathbf{r} \cdot (\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3 - \mathbf{k}_4)\right]$

$$= N\delta_{\mathbf{k}_1+\mathbf{k}_2,\mathbf{k}_3+\mathbf{k}_4},$$



- In the limit of infinite dimensions, $\Delta_{D\to\infty}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) = 1 + \mathcal{O}(1/D)$. hence momentum conservation can be ignored.
- In the dynamical cluster approximation, the Laue function may be expressed as $\Delta_{\mathrm{DCA}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) = N_c\delta_{\mathbf{M}(\mathbf{k}_1)+\mathbf{M}(\mathbf{k}_2),\mathbf{M}(\mathbf{k}_3)+\mathbf{M}(\mathbf{k}_4)}$
- **M(k):** mapping of **k** to the cluster momentum **K**
- The Green's function is given by

$$G(\mathbf{k}, z) = \frac{1}{z - (\epsilon_{\mathbf{k}} - \mu) - \bar{\Sigma}(\mathbf{M}(\mathbf{k}), z)}$$



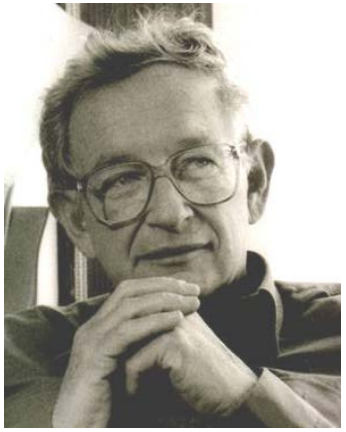
T. Maier et al, Rev. Mod. Phys **77** 1027 (2005).

Comparison of DMFT and Cluster approaches

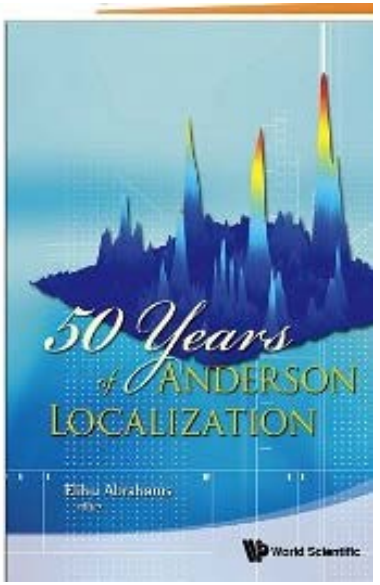
Do we incorporate?	DMFT	DCA
Strong interaction physics	✓	✓
Multiple orbitals effects	✓	✓
Real material aspects	✓	✓
Non-local dynamical fluctuations	✗	✓
Low dimensional physics	✗	✓
Symmetry broken phases with complex order parameters	✗	✓
True phase transitions (i.e avoid spurious ones)	✗	✓
Anderson Localization	✗	✗

What is Anderson localization (AL) and how do we investigate AL theoretically?

Anderson Localization



1958 – Absence of diffusion in certain random lattices – Physical Review **109** 1492-1505.



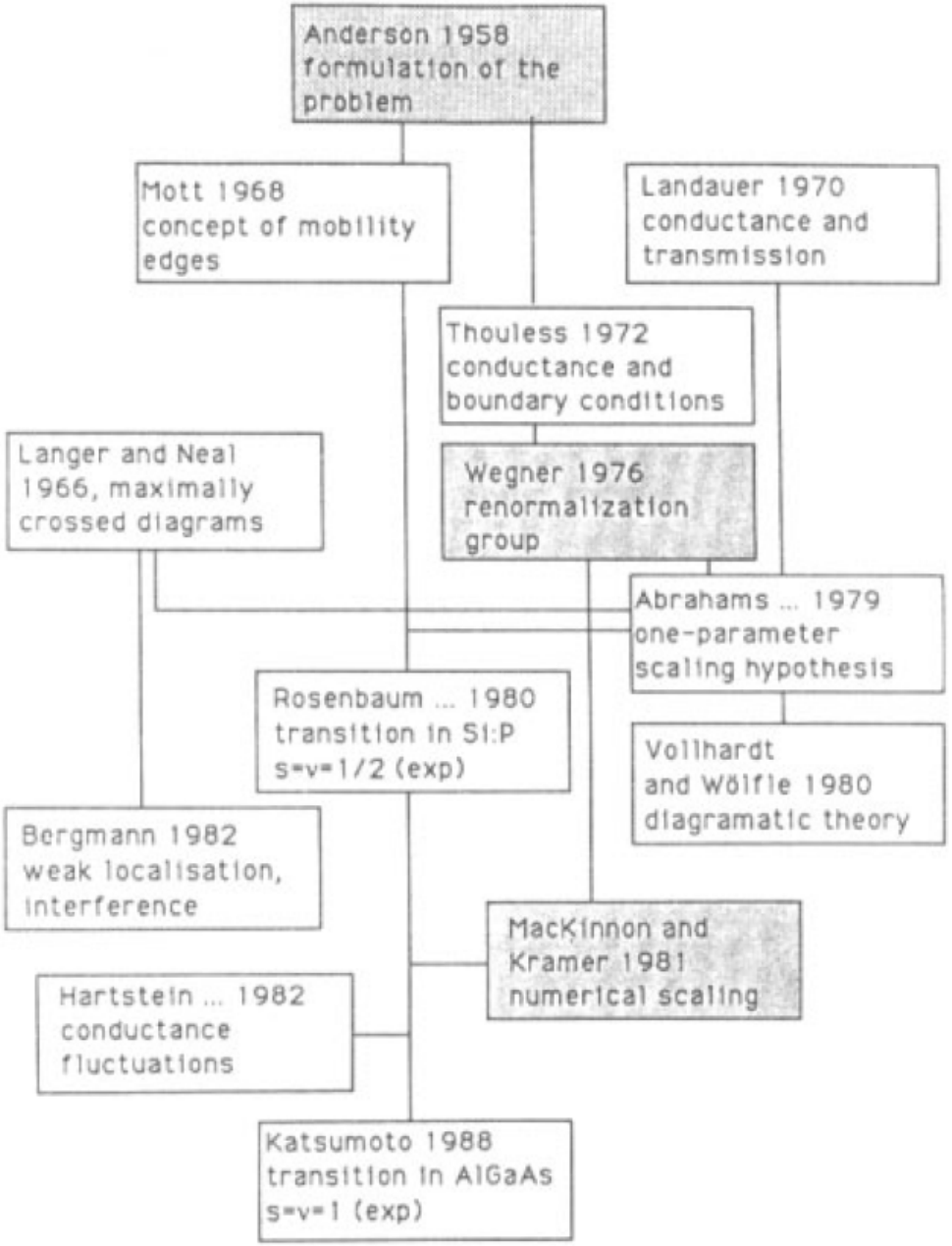
Ed. E. Abrahams, (2010).

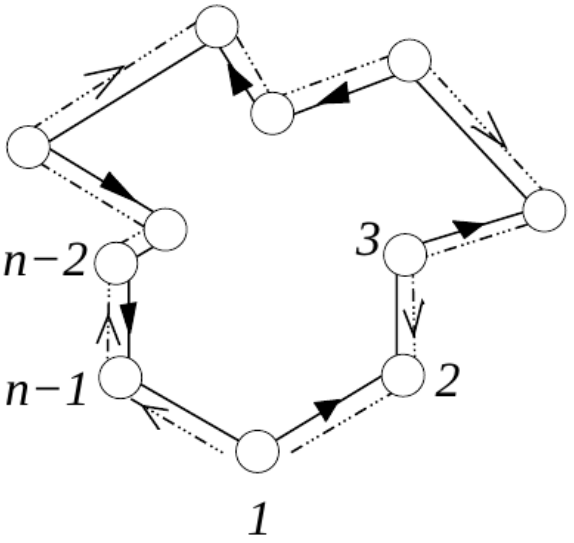
Hamiltonian

$$\hat{H}_A \equiv \hat{H}_{dis} + \hat{H}_{kin} = \sum_{i\sigma} \varepsilon_i c^{\dagger}_{i\sigma} c_{i\sigma} - t \sum_{<i,j>,\sigma} c^{\dagger}_{i\sigma} c_{j\sigma}.$$

Conclusion: Disorder induced metal-insulator transition

Anderson Localization



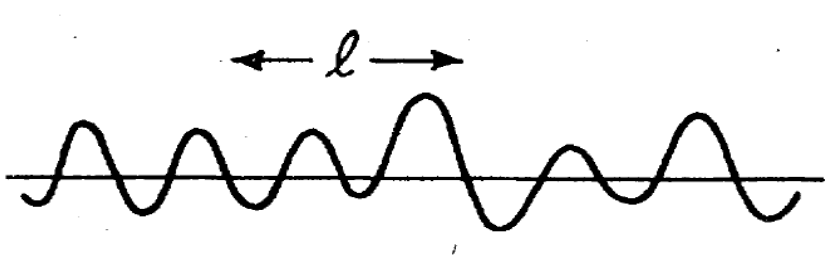


Coherent backscattering: Time reversed paths interfere constructively leading to finite return probability => Localization.

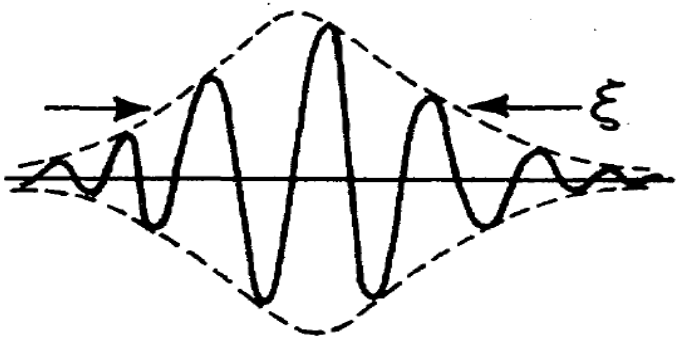
Kramer, B. and MacKinnon, A. Localization: theory and experiment. Rep. Prog. Phys. 56, 1469–1564 (1993)

Anderson Localization

- Gapless spectrum with localized states at the chemical potential
- Anderson insulator is distinct from band or Mott insulators which are either due to band-fillingor interaction-induced and have a gapped spectrum.
- Systems – Heavily doped semiconductors Si:P, a-Si, quantum Hall systems



Extended state with mean free path ℓ



Localized state with localization length ξ

Lee and Ramakrishnan Rev. Mod. Phys. **57** (1985)

Anderson Localization

Localized state wavefunctions have a complex spatial structure and exhibit multifractility

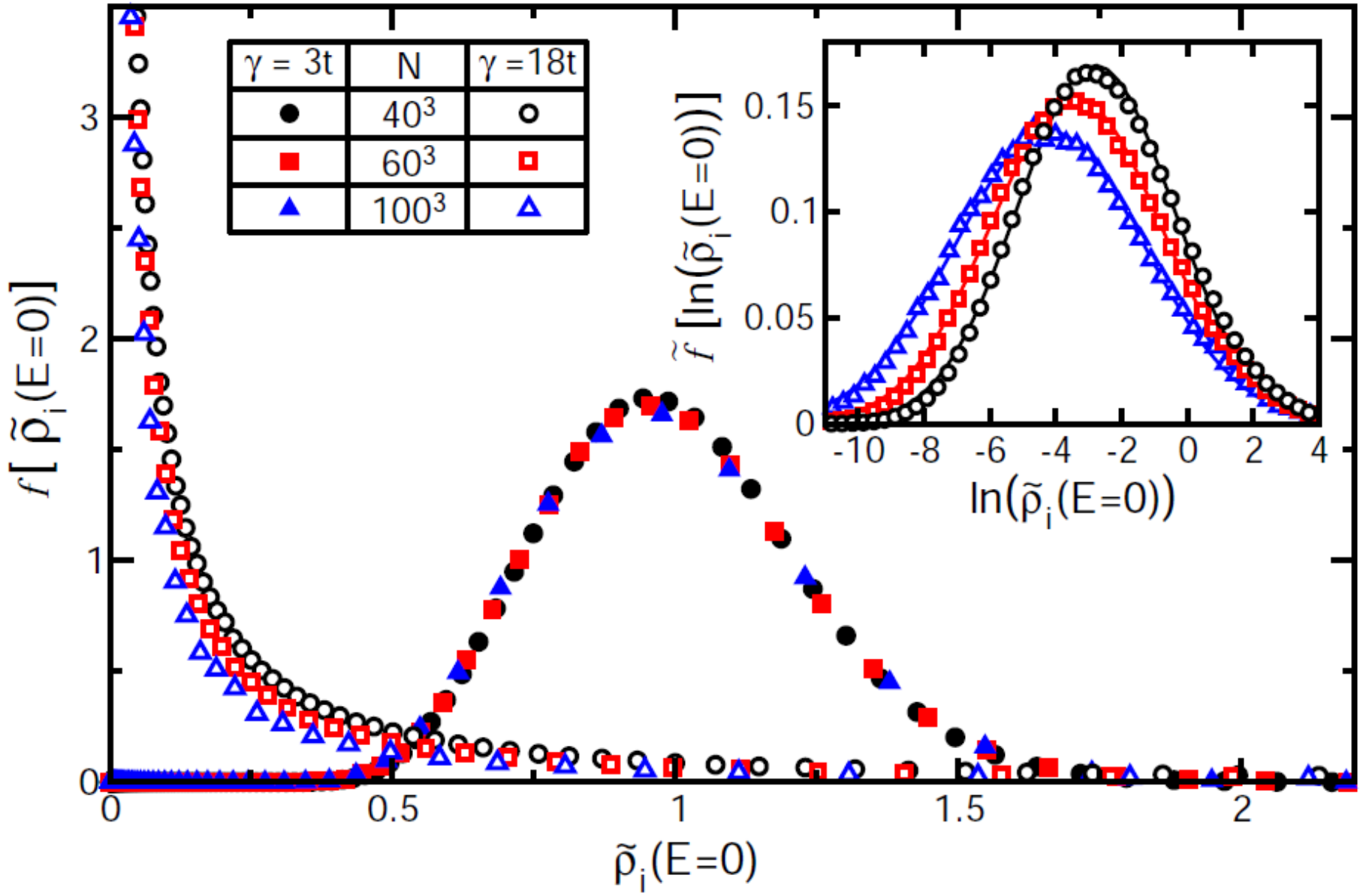
$$P_q = \int d^d r |\psi(\mathrm{r})|^{2\mathrm{q}}$$ Inverse participation ratio

$$\langle P_q \rangle \sim \begin{cases} L^0 & \text{Insulator} \\ L^{-\tau_q} & \text{Critical} \\ L^{-d(q-1)} & \text{Metal} \end{cases}$$

$$\tau_q = \underbrace{d(q-1)}_{\text{normal}} + \underbrace{\Delta_q}_{\text{anomalous}} \equiv D_q(q-1)$$

Local density of states

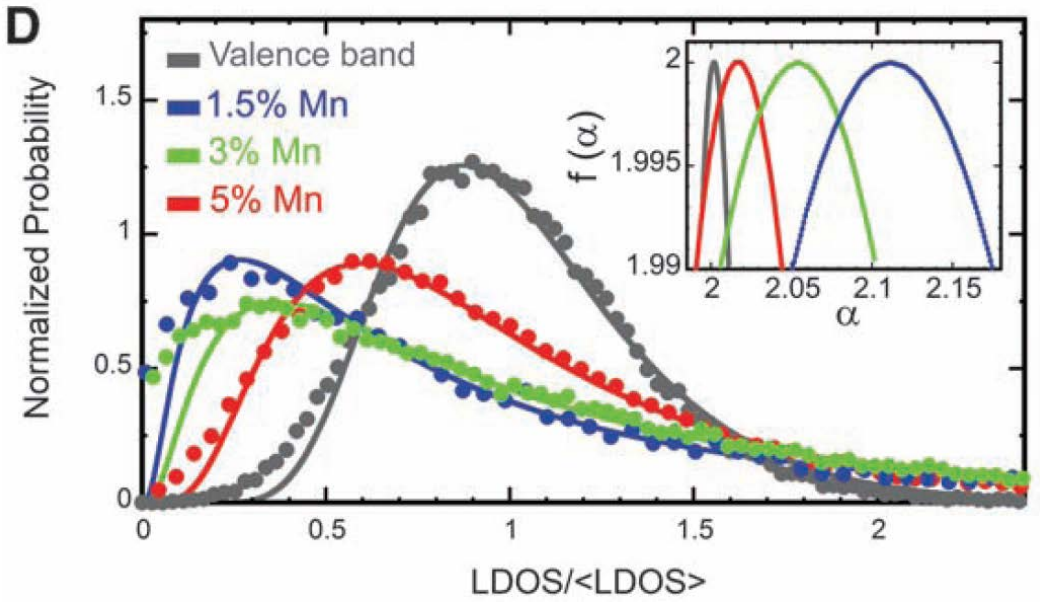
Local DoS

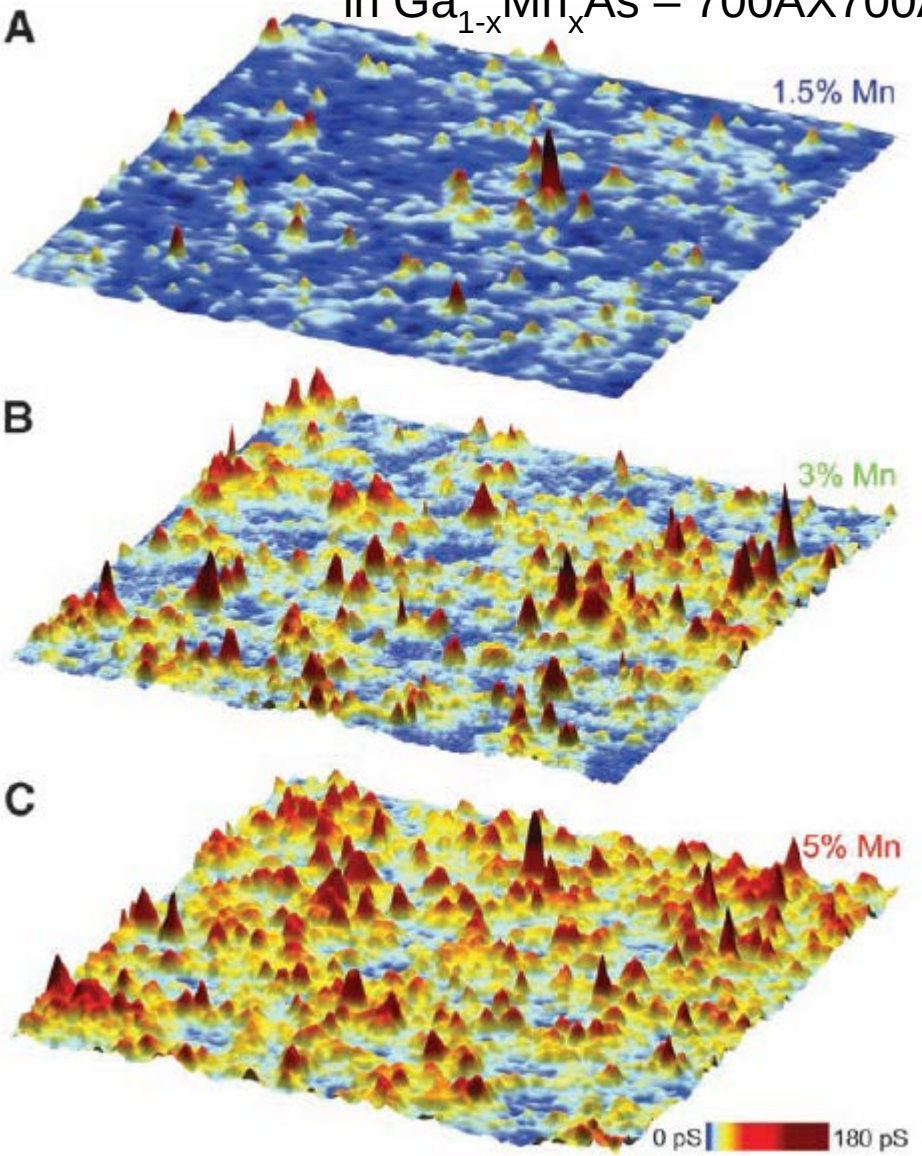


G.Schubert et al Phys. Rev. B **81**, 155106 (2010)

Anderson localization - multifractility

Spatial variation of the local dos in $Ga_{1-x}Mn_xAs$ – 700ÅX700Å





Richardella et al, Science **327** 665

Anderson localization - multifractility

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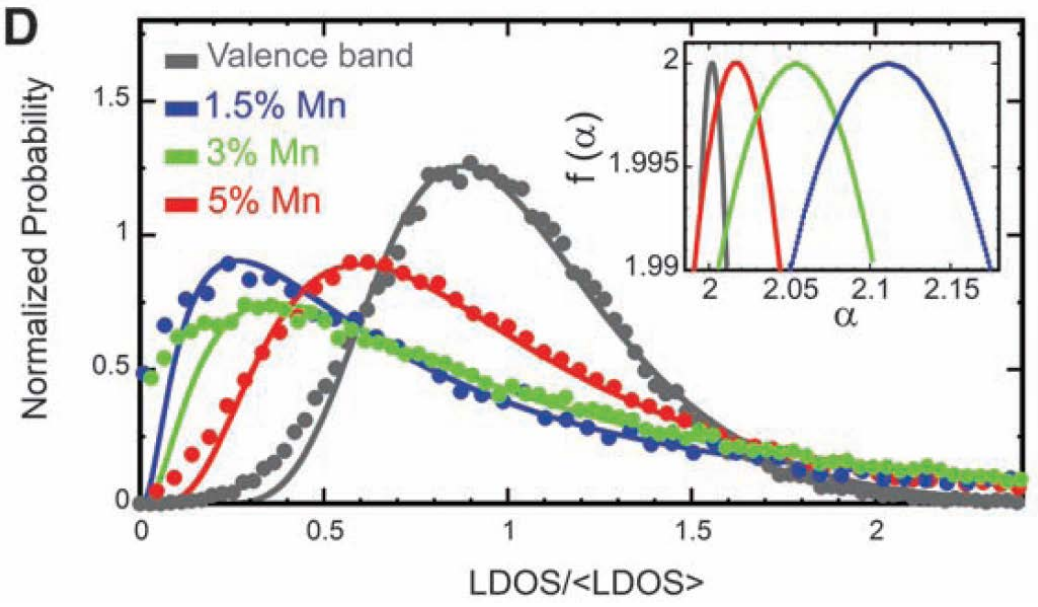
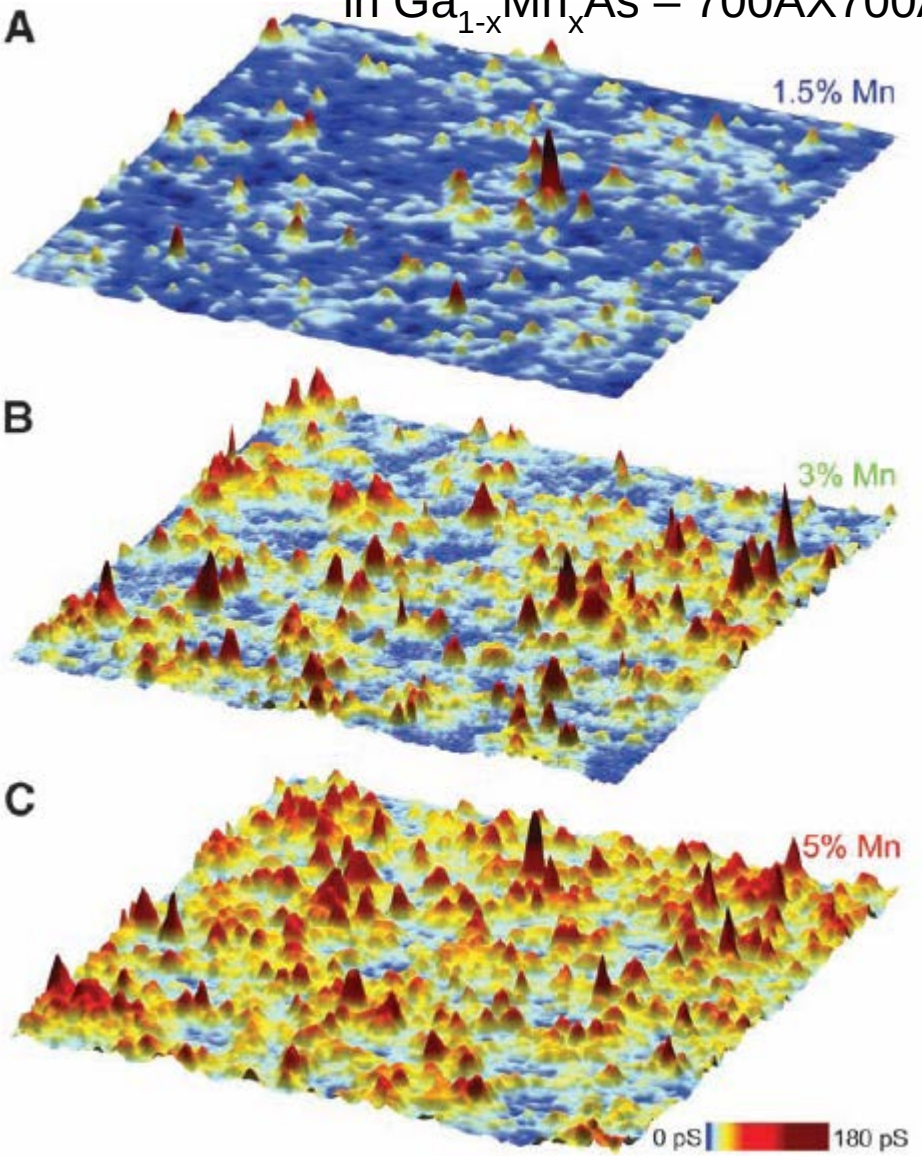




Rethink disorder averaging

Richardella et al, Science

Theoretical approaches

- Exact Diagonalization
- Kernel Polynomial Expansion
- Transfer matrix method

Need large lattices for any reasonable accuracy

Almost impossible to incorporate interaction

- Scaling and RG based methods: Perturbative and weak disorder

Extending DMFT: Typical medium theory

Initial Hybridization

Given a hybridization function;
Solve N impurity problems - e_i

{G_{ii}(w)
}

Construct a Typical Green's function

$$\rho_{typ}(\omega) = \exp\left[\int d\varepsilon_j P(\varepsilon_j) \ln \rho_j(\omega)\right].$$

$$G_{typ}(\omega) = \int_{-\infty}^{\infty} d\omega' \frac{\rho_{typ}(\omega')}{\omega - \omega'}.$$

Obtain a typical medium self-energy

$$G_{typ}(\omega) = \frac{1}{N_s}\sum_{\boldsymbol{k}} \frac{1}{\omega - \varepsilon_{\boldsymbol{k}} + \mu - \Sigma_{TMT}(\omega)}$$

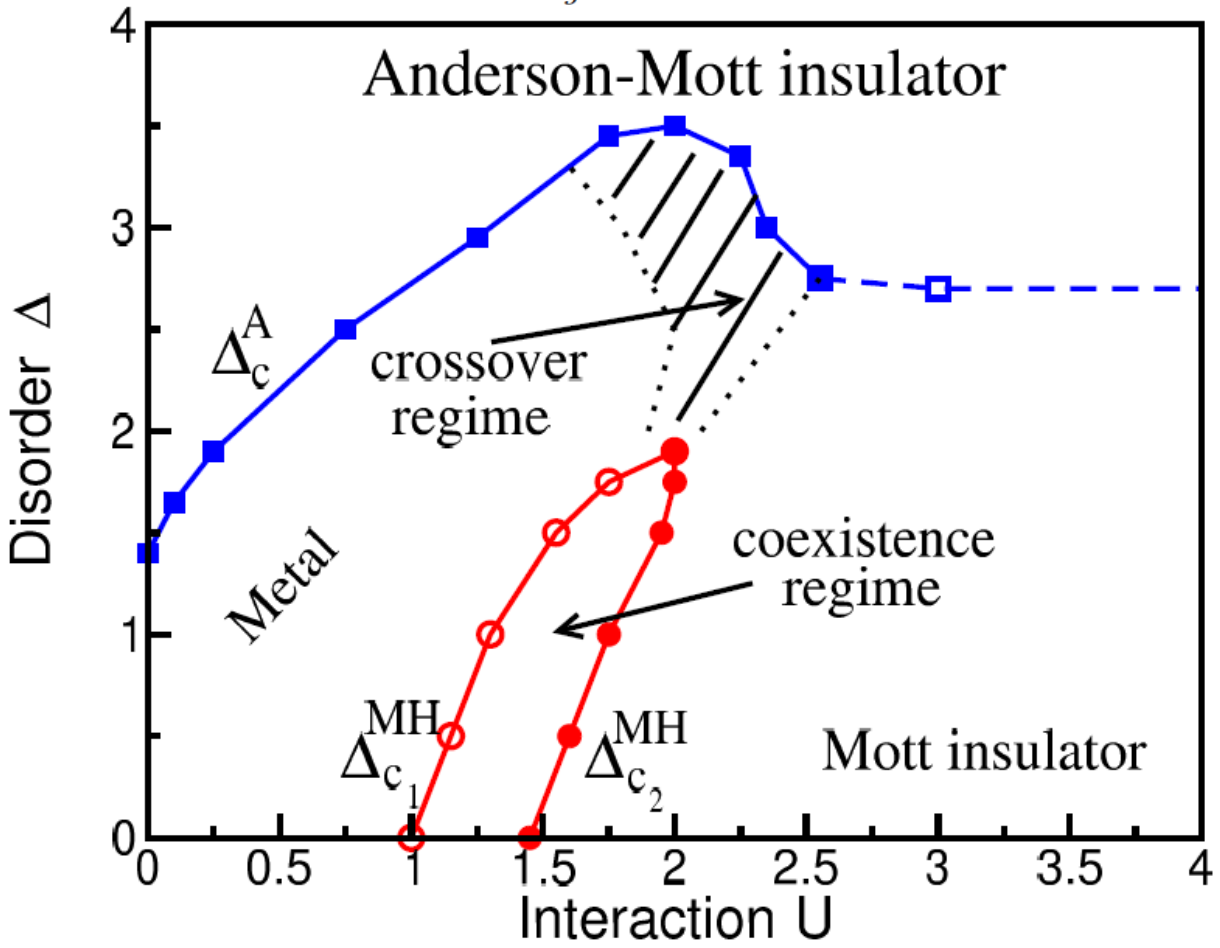
Get a new typical medium hybridization

$$G_{typ}(\omega) = \frac{1}{\omega + \mu - \Delta_{typ}(\omega) - \Sigma_{TMT}(\omega)}$$

E. Miranda, V. Dobrosavljevic in *Conductor Insulator Quantum Phase Transitions,* Oxford Univ Press (2013).

Anderson-Hubbard model

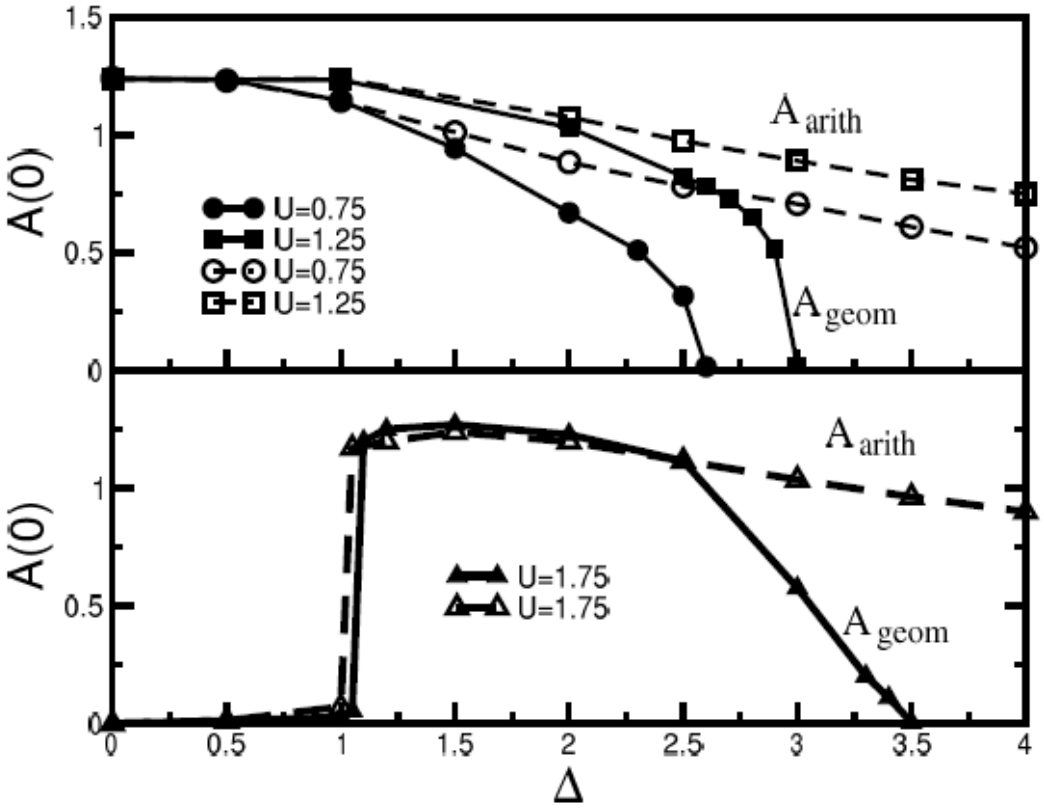
Hamiltonian: $H = \sum_{ij\sigma} t_{ij\sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + \sum_{i\sigma} \epsilon_i \, n_{i\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$



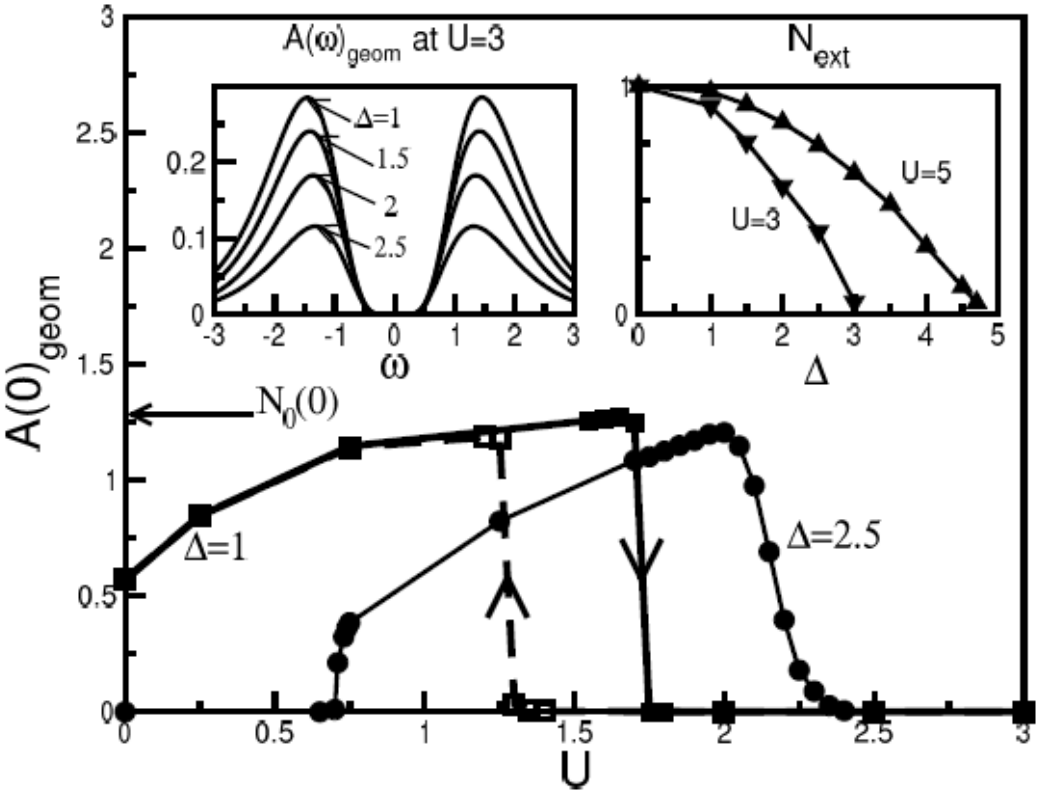
Byczuk, Hofstetter and Vollhardt *50 yrs of AL* World Scientific (2010)

Interactions+Disorder:

Co-operation or Competition



Fixed interactions: Disorder decreases Metallicity.



Fixed disorder: Interactions screen disorder Initially; Larger U leads to MIT.

Extending TMT to clusters

- Typical medium dynamical cluster approximation

K-dependent typical density of states

$$\rho^c_{typ}(K,\omega) = \overbrace{\exp\left(\frac{1}{N_c}\sum_{i=1}^{N_c}\left\langle \ln \rho^c_i(\omega,V)\right\rangle\right)}^{\text{local TDOS}} \underbrace{\left\langle \frac{\rho^c(K,\omega,V)}{\frac{1}{N_c}\sum_i \rho^c_i(\omega,V)}\right\rangle}_{\text{nonlocal}}.$$

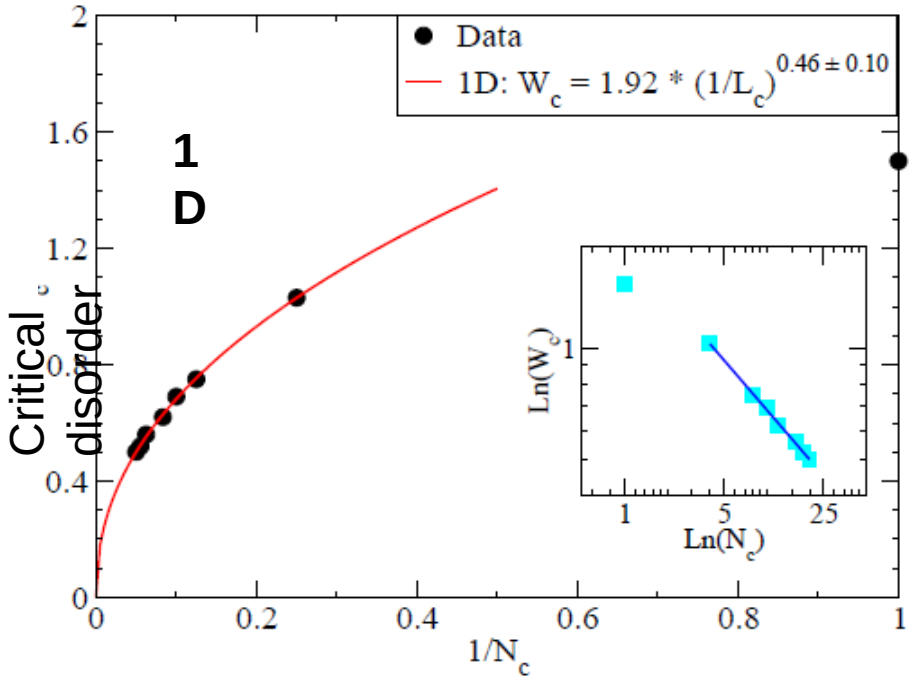
K-dependent coarse-grained Green's function

$$\overline{G}(K,\omega) = \int \frac{N^c_0(K,\epsilon)d\epsilon}{\left[G^c_{typ}(K,\omega)\right]^{-1} + \Gamma(K,\omega) - \epsilon + \overline{\epsilon}(K) + \mu}$$

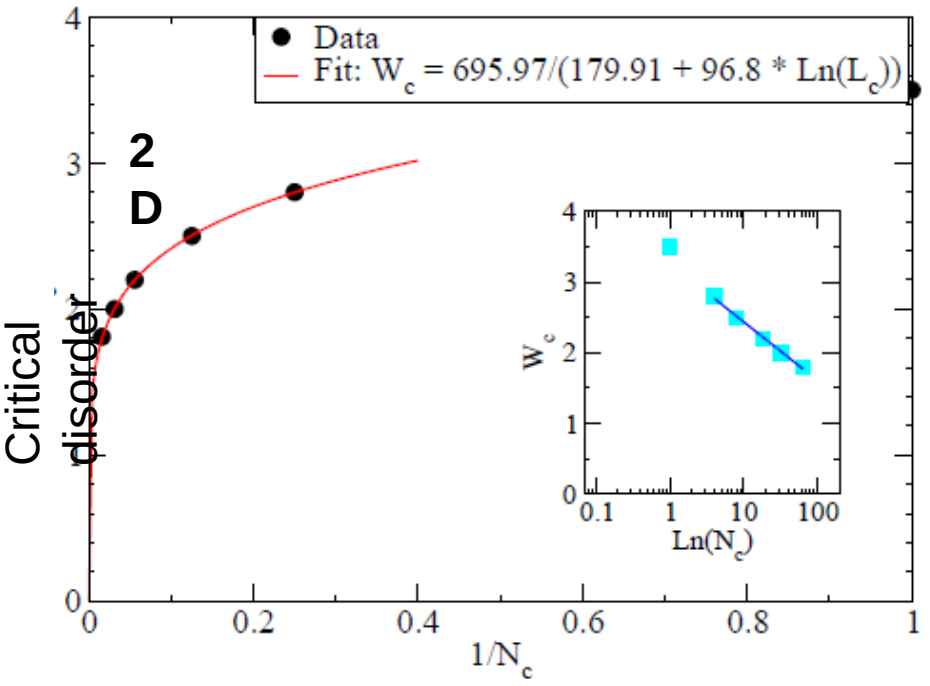
Implies a K-dependent hybridization function $\Gamma(K,\omega)$.

C. E. Ekuma et al Phys. Rev. B **89**, 081107(R) –February 2014

Benchmarks for 1D and 2D



Inverse cluster size

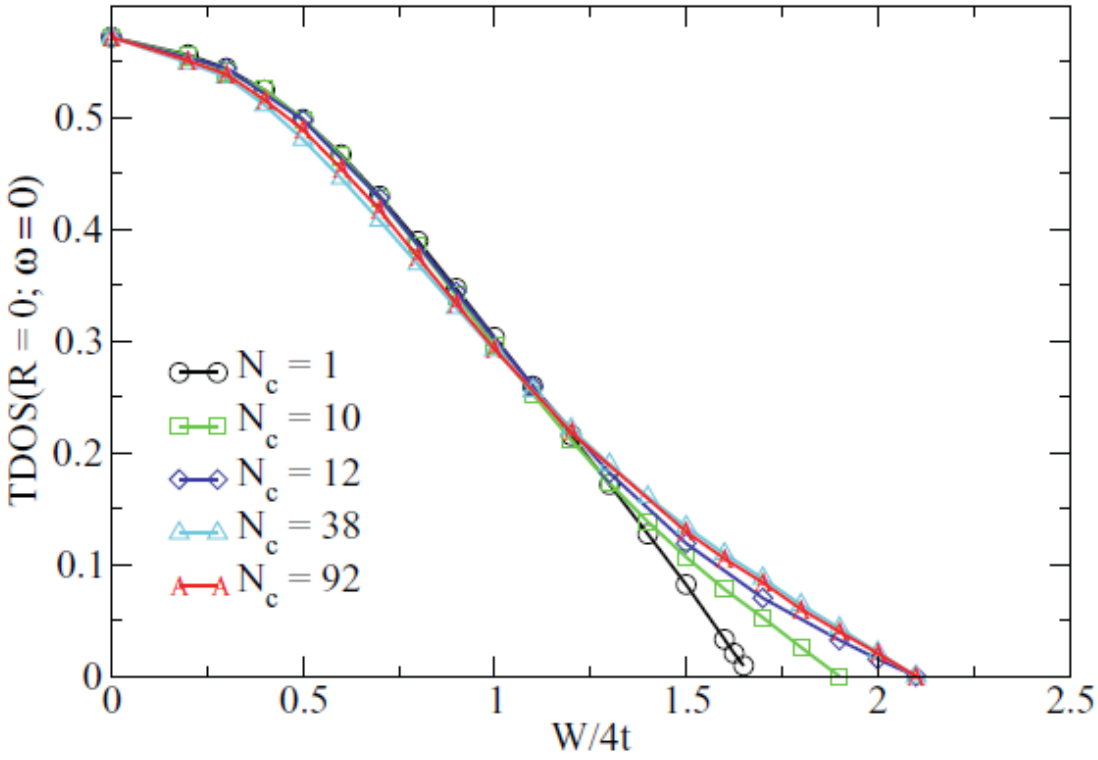


Inverse cluster size

C. E. Ekuma et al, cond-mat 1306.5712

Benchmark for 3D

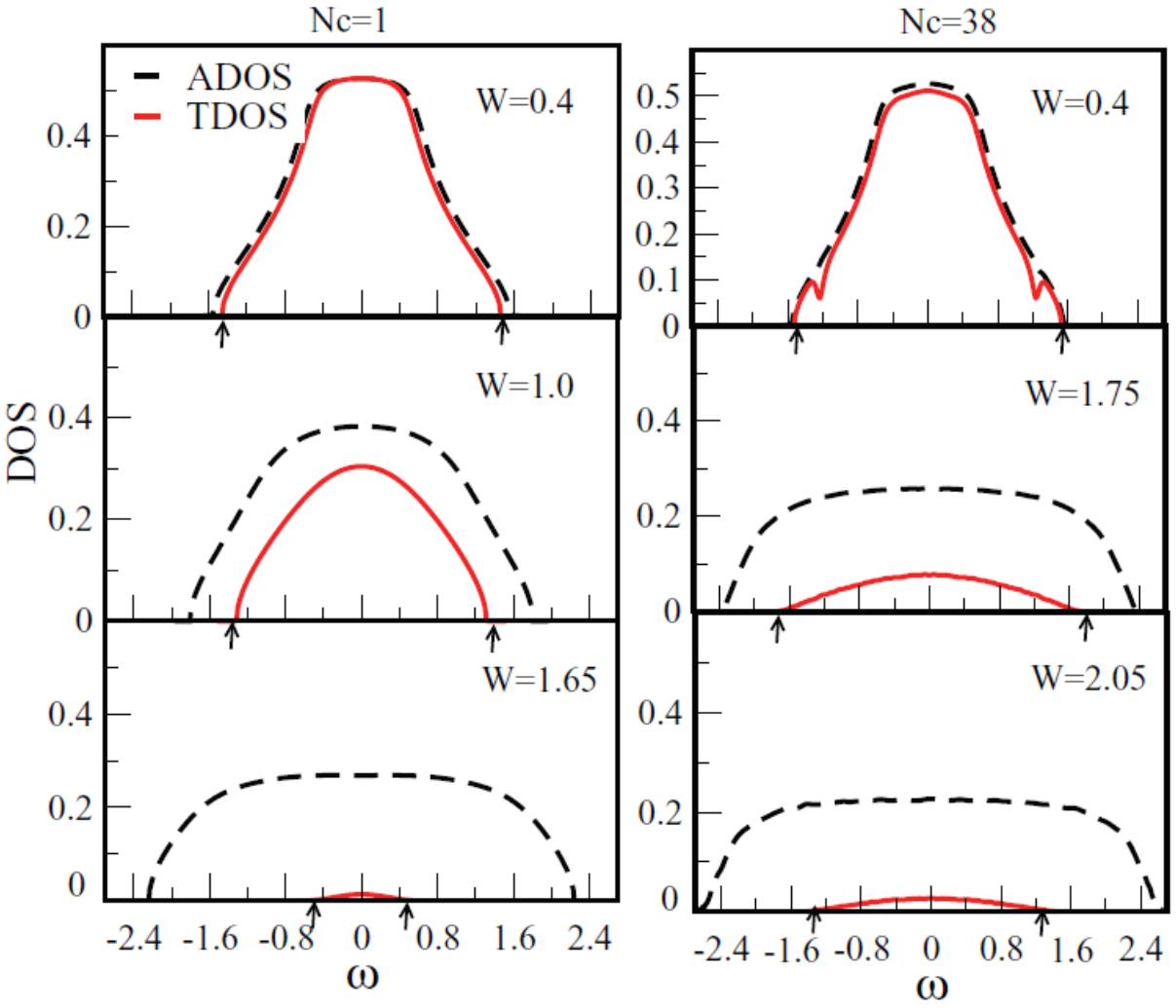
- Nc=1 corresponds to TMT
  - Does not get the re-entrance of mobility edge
  - Critical disorder underestimated
  - Exponent=1
- TMDCA (3D)
  - Rentrant mobility edge
  - Critical disorder ~ 2.1
  - Exponent ~ 1.67
  - Rapid convergence with Nc.



C. E. Ekuma et al Phys. Rev. B **89**, 081107(R) –February 2014

TMDCA – Density of states (U=0)

ADOS: Arithmetically averaged DoS TDOS: Typical average of DoS



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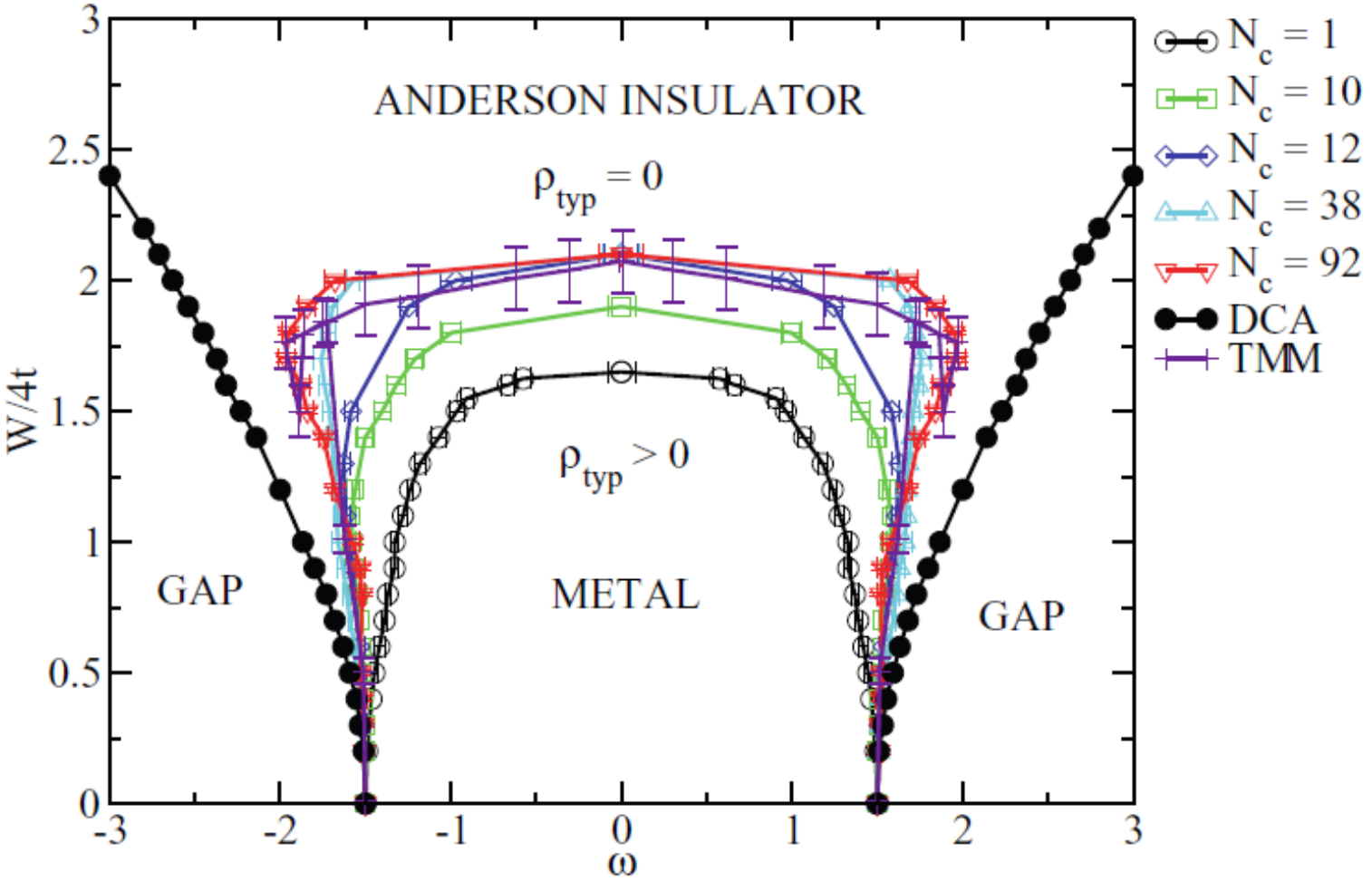
Mobility edge

Rentrance behaviour



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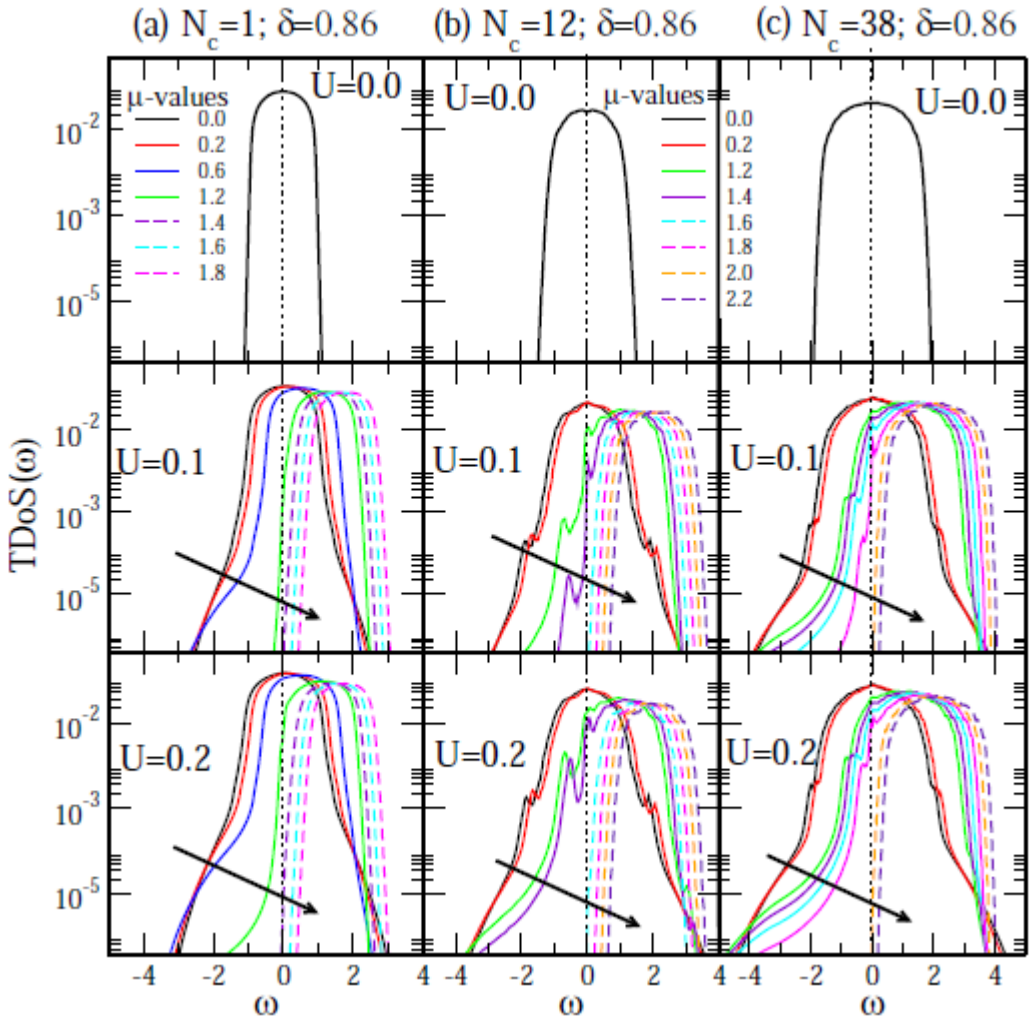
Weak Interactions + Strong disorder in 3D

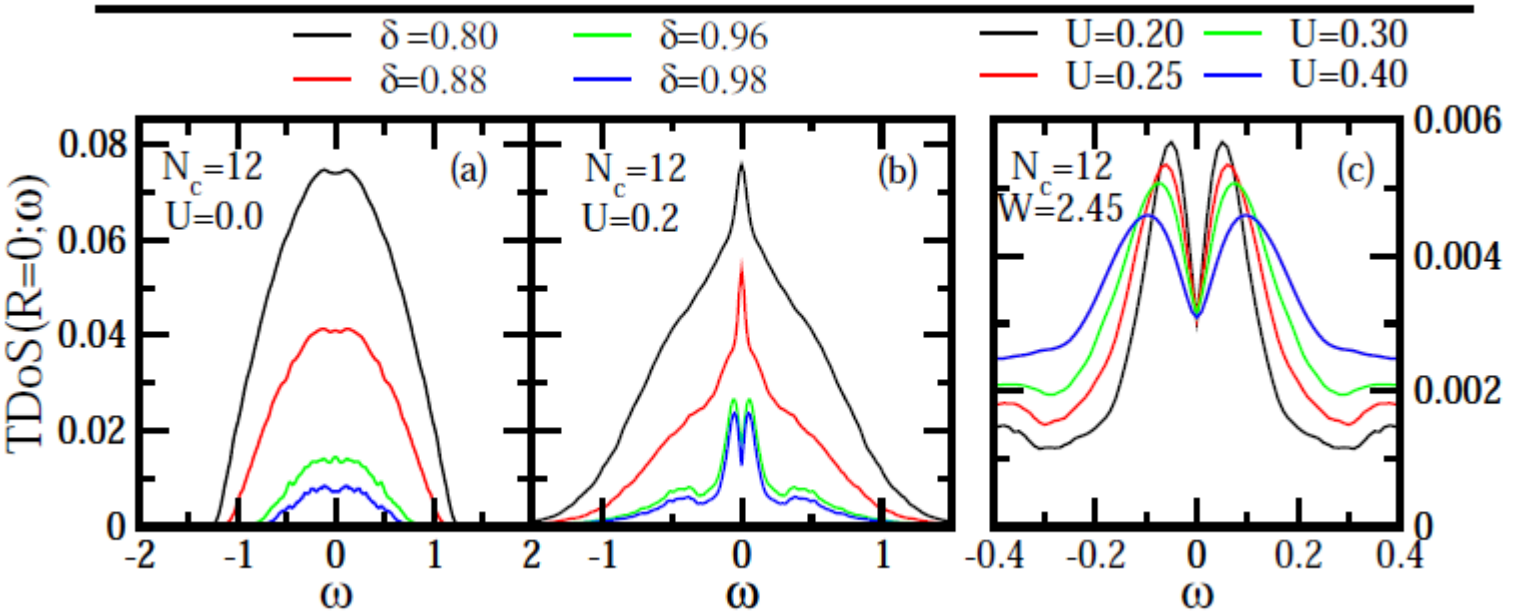
Mobility edge survives in the presence of interactions.



C.Ekuma et al ArXiv: 1503.00025

Interactions+Disorder in 3D

Pseudogap at intermediate disorder (W ~ W_c).



C.Ekuma et al ArXiv: 1503.00025

Thank you

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