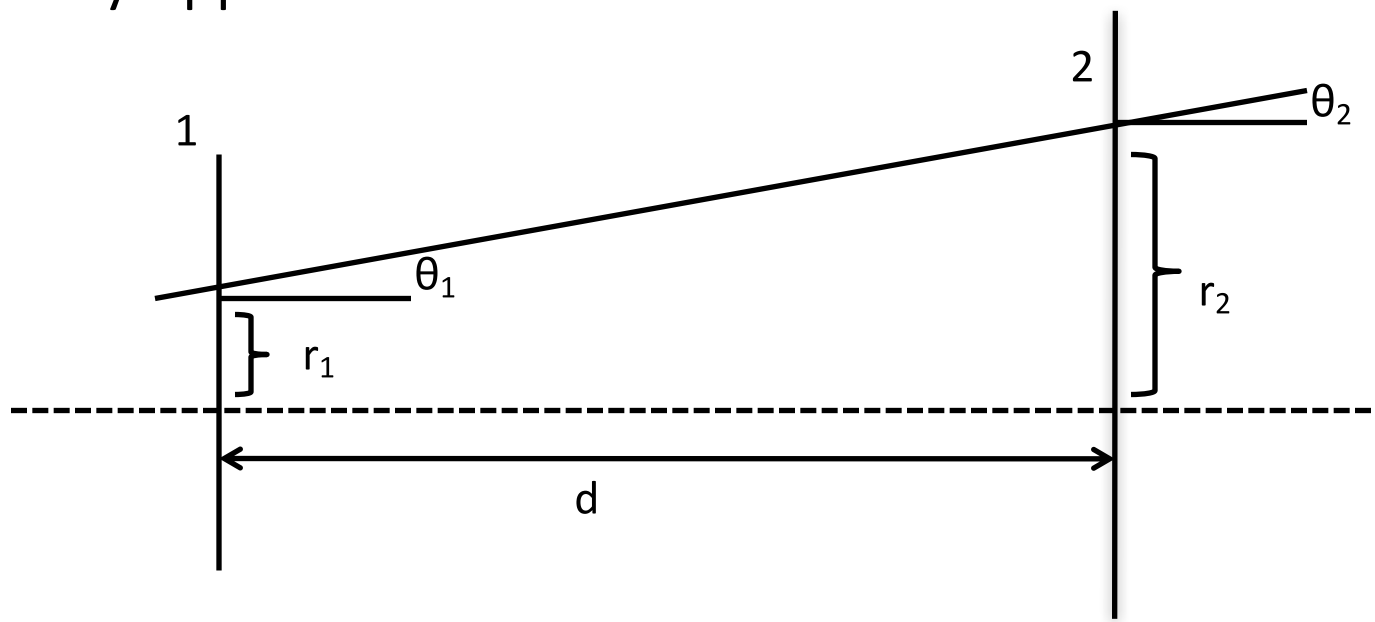
Ray Tracing

Paraxial Ray Approximation



$$r_2 = 1 \cdot r_1 + d \cdot r_1'$$
$$r_2' = 0 \cdot r_1 + 1 \cdot r_1'$$

$$\begin{bmatrix} r_2 \\ r_2' \end{bmatrix} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r_1 \\ r_1' \end{bmatrix}$$

General: $\begin{bmatrix} r_{out} \\ r'_{out} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} r_{in} \\ r'_{in} \end{bmatrix}$

$$\boxed{R_{out} = T\,R_{in}}$$

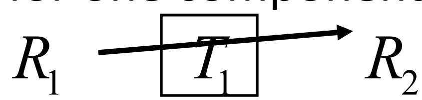
Matrix Multiplication:

$$c_{ij} = \sum_k a_{ik} b_{kj}$$

Successive Components Cascade:

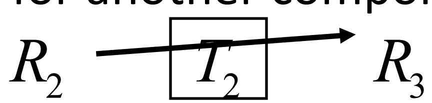
T_1 – “transmission” matrix for one component

$$R_2 = T_1 R_1$$



T_2 – “transmission” matrix for another component

$$R_3 = T_2 R_2$$

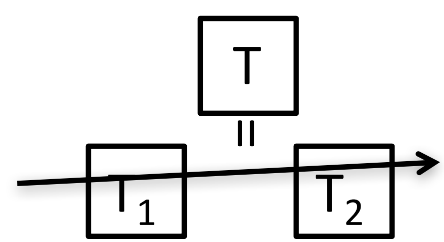


then $T_1 T_2$ represents the combination

$$R_3 = T_2 T_1 R_1$$

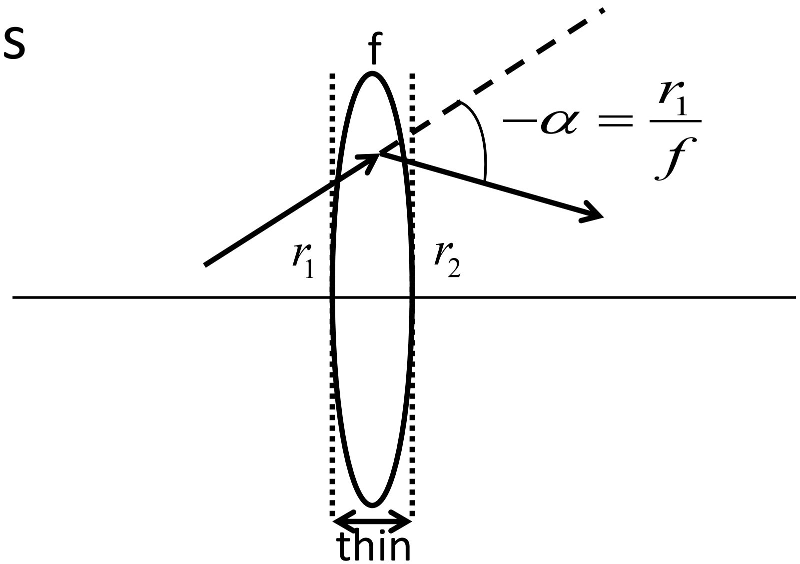
$$T = T_2 \times T_1$$

$$R_3 = T\,R_1$$



Common Ray Matrices

1. Lens



A lens bends the light towards the focal point by α regardless of angle of incidence

$f =$ focal length

$$r_2 = r_1$$

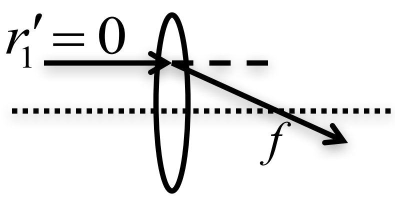
$$r_2' = -\frac{r_1}{f} + r_1'$$

$$T_{lens} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}$$

Det = AD – BC = **1**

<u>Special Cases</u>

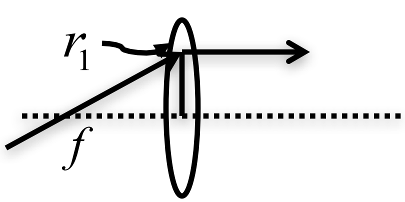
a)



$$r_2 = r_1$$

$$r_2' = -\frac{r_1}{f}$$

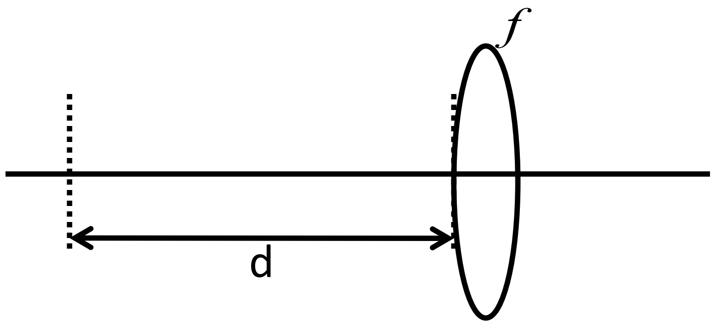
b)



$$r_2' = -\frac{r_1}{f} + r_1' = 0$$

$$r_1' = \frac{r_1}{f}$$

2) Free space propagation + Lens



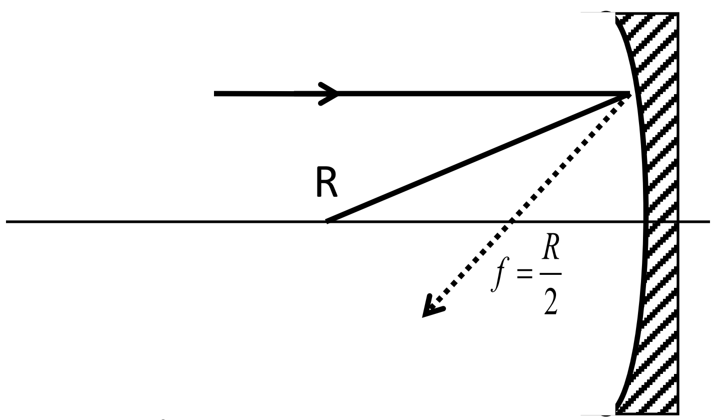
$$T_{lens \cdot d} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = (lens) \cdot d$$

$$T_{lens \cdot d} = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & d \\ -1/f & 1 - d/f \end{bmatrix}$$

Det = AD – BC = 1

$$C_{ij} = \sum_k a_{ik} \beta_{kj}$$

3) Mirror



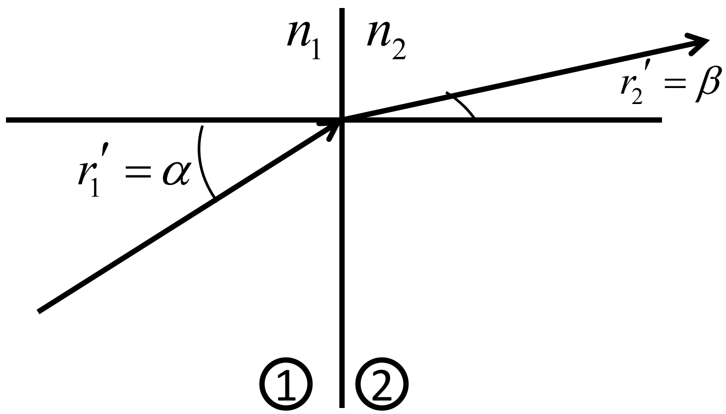
$$f = \frac{R}{2}$$

$$T_{mirror} = \begin{bmatrix} 1 & 0 \\ -2/R & 1 \end{bmatrix}$$

(directs ray toward the axis = lens with $f = \frac{R}{2}$)
Reverses direction but acts like a lens otherwise!

Example: Refraction of a Plane Surface

problem 2.2



$$r_1 = r_2$$

$$n_1 \sin\alpha = n_2 \sin\beta$$

small angles: paraxial approximation

$$n_1 \alpha \approx n_2 \beta$$

$$r_1' = \tan\alpha \approx \alpha$$

$$r_2' = \tan\beta \approx \beta$$

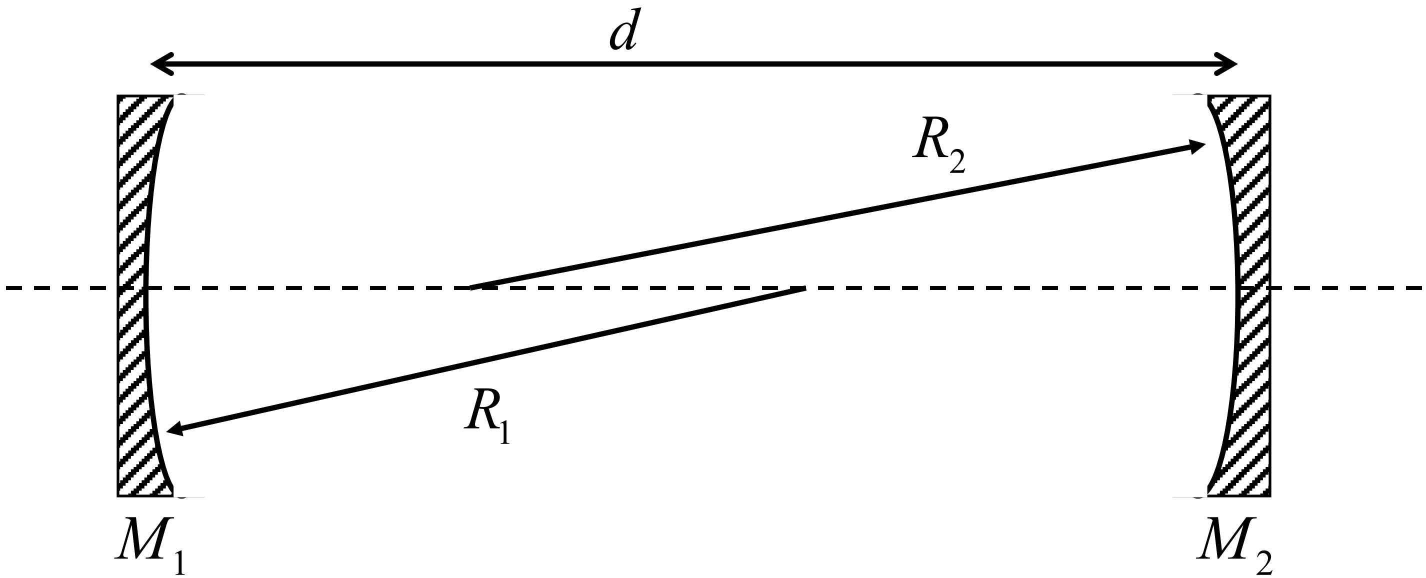
$$n_1 r_1' = n_2 r_2' \Rightarrow r_2' = \frac{n_1}{n_2} r_1'$$

$$r_2 = r_1 + 0 \cdot r_1'$$

$$r_2' = 0 \cdot r_1 + \frac{n_1}{n_2} \cdot r_1'$$

$$T = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}$$

Stability of Optical Cavities

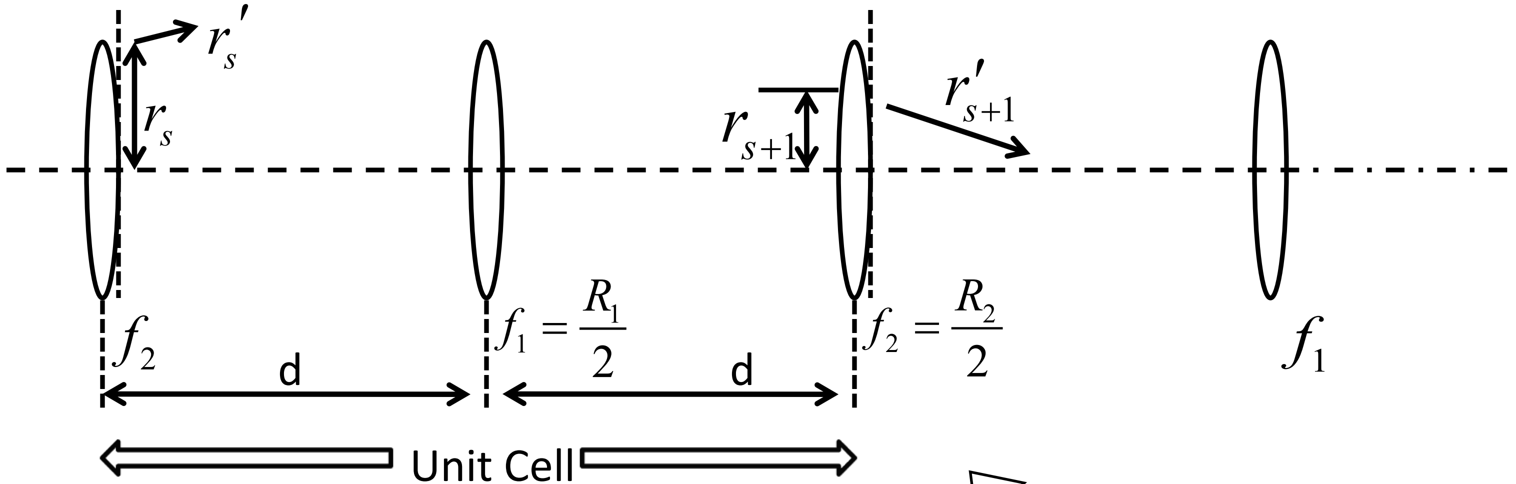


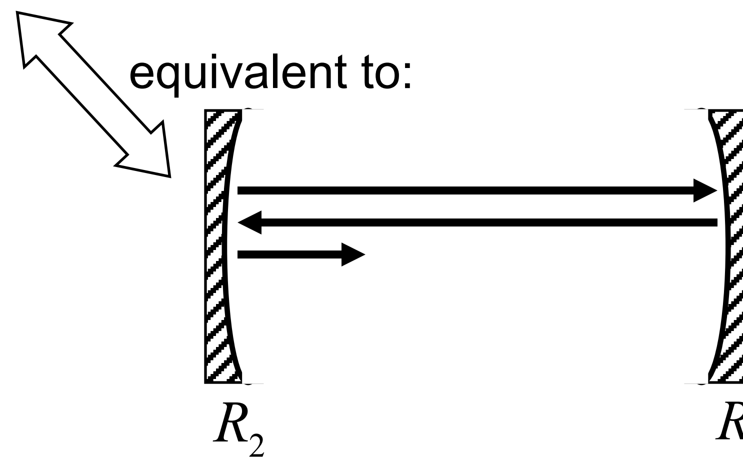
If there is r_{max} ⟶ stable

not ⟶ unstable

depends on alignment ⟶ conditionally stable

Propogation direction: →





$$r_{s+1} = Ar_s + Br_s'$$
$$r'_{s+1} = Cr_s + Dr_s'$$

unit cell:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & d \\ -1/f_2 & 1-d/f_2 \end{bmatrix} \begin{bmatrix} 1 & d \\ -1/f_1 & 1-d/f_1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-d/f_1 & d+d(1-d/f_1) \\ -1/f_2 - 1/f_1(1-d/f_2) & (1-d/f_2)(1-d/f_1)-d/f_2 \end{bmatrix}$$

General Consideration

$$r_{s+1} = Ar_s + Br'_s$$

$$r'_{s+1} = Cr_s + Dr'_s$$

$$r'_s = \frac{1}{B}(r_{s+1} - Ar_s) \overset{s \to s+1}{\Longrightarrow} r'_{s+1} = \frac{1}{B}(r_{s+2} - Ar_{s+1}) \qquad (1)$$

$$= Cr_s + Dr'_s$$

$$= Cr_s + \frac{D}{B}(r_{s+1} - Ar_s) \qquad (2)$$

Equation (1) & (2)

$$\frac{1}{B}(r_{s+2} - Ar_{s+1}) = Cr_s + \frac{D}{B}(r_{s+1} - Ar_s)$$

$$r_{s+2} - Ar_{s+1} = BCr_s + Dr_{s+1} - DAr_s$$

$$r_{s+2} - (A+D)r_{s+1} + (AD-BC)r_s = 0 \qquad AD - BC = 1$$

$$\boxed{r_{s+2} - (A+D)r_{s+1} + r_s = 0}$$

$$\frac{d^2r}{dt^2} + ar = 0$$ - Harmonic oscillator

$$\frac{dr}{dt} = \frac{\Delta r}{\Delta t} = \frac{r_{s+1} - r_s}{\Delta t} \qquad \frac{d^2r}{dt^2} = \frac{\Delta}{\Delta t}\left(\frac{\Delta r}{\Delta t}\right) = \frac{\frac{r_{s+2} - r_{s+1}}{\Delta t} - \frac{r_{s+1} - r_s}{\Delta t}}{\Delta t} = \frac{r_{s+2} - 2r_{s+1} + r_s}{(\Delta t)^2}$$

$$\Rightarrow r_{s+2} + br_{s+1} + r_s = 0$$ - Harmonic oscillator

$$r_s = ae^{jsq}$$

$$ae^{(s+2)jq} - (A+D)ae^{(s+1)jq} + ae^{jsq} = 0$$

$$e^{2jq} - (A+D)e^{jq} + 1 = 0$$

$$e^{jq} = \frac{A+D}{2} \pm \sqrt{\left(\frac{A+D}{2}\right)^2 - 1} = \frac{A+D}{2} \pm j\sqrt{1 - \left(\frac{A+D}{2}\right)^2}$$

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta = \frac{A+D}{2} \pm j\sqrt{1 - \left(\frac{A+D}{2}\right)^2}$$

$$\boxed{\cos\theta = \frac{A+D}{2}} \qquad \boxed{\sin\theta = \sqrt{1 - \cos^2\theta}}$$

Solutions:

$$r_s = r_0 e^{js\theta} + r_0^* e^{-js\theta}$$

or

$$r_s = r_{\max} \sin(s\theta + \alpha)$$