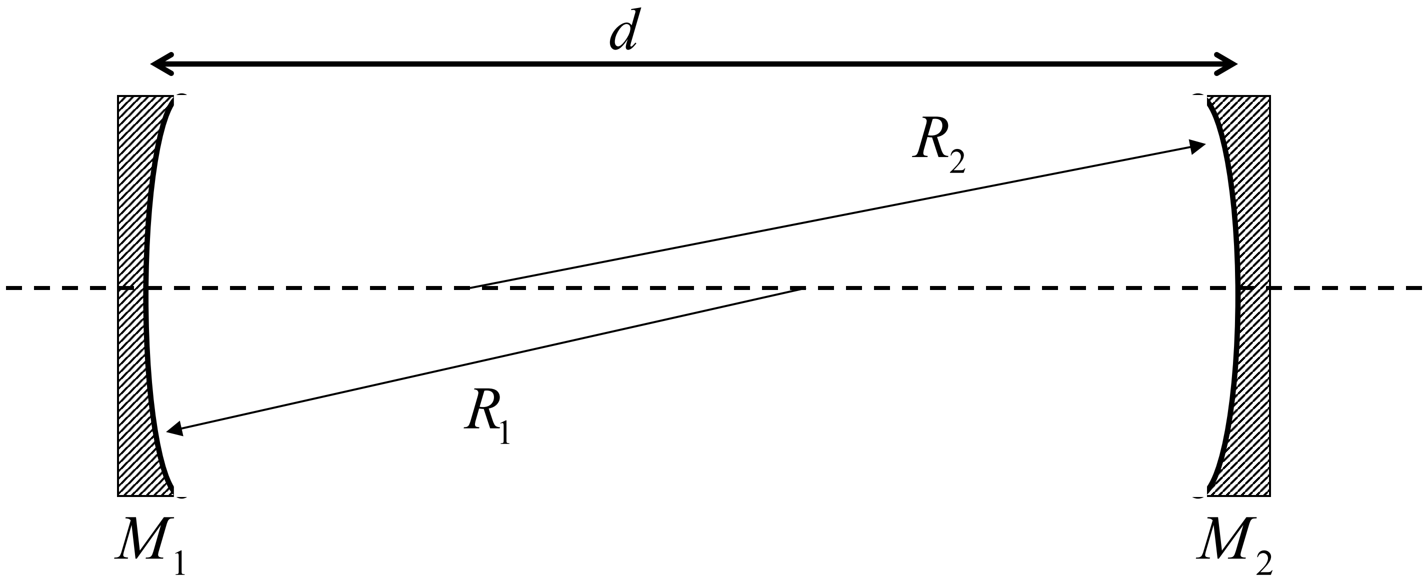
Recapitulation

Stability of Optical Cavities

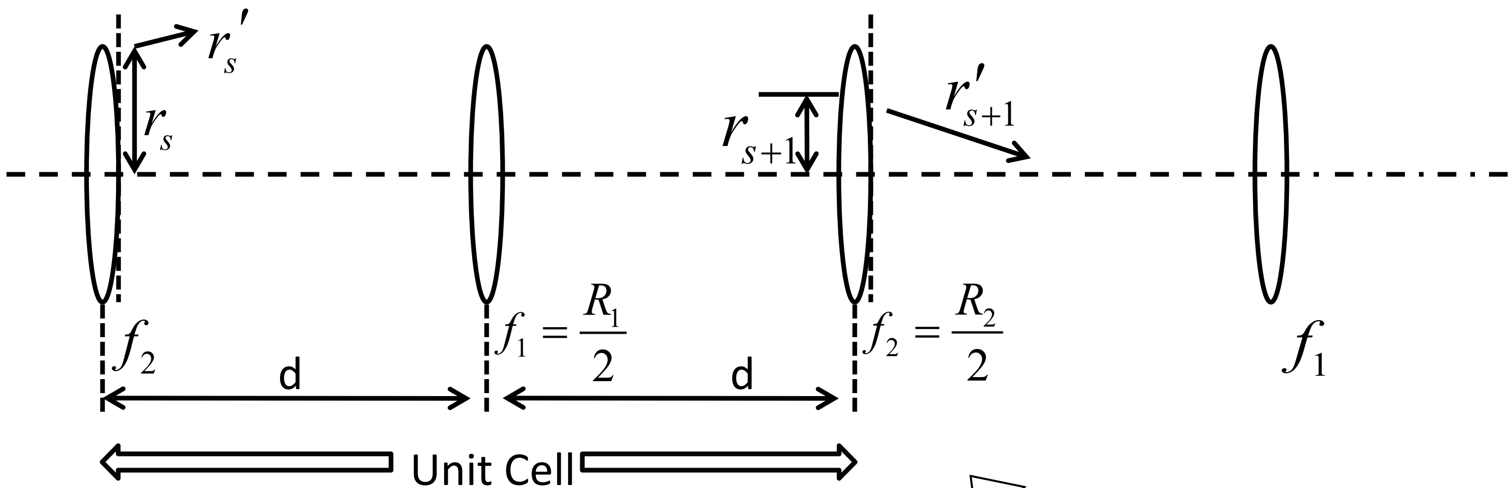


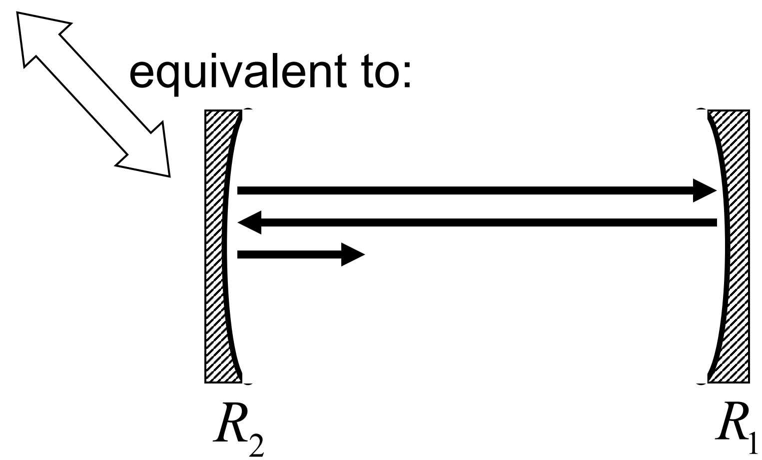
If there is r_{max} ⟶ stable

not ⟶ unstable

depends on alignment ⟶ conditionally stable

Propogation direction:





$$r_{s+1} = Ar_s + Br_s'$$
$$r_{s+1}' = Cr_s + Dr_s'$$

unit cell:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & d \\ -1/f_2 & 1 - d/f_2 \end{bmatrix} \begin{bmatrix} 1 & d \\ -1/f_1 & 1 - d/f_1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - d/f_1 & d + d(1 - d/f_1) \\ -1/f_2 - 1/f_1 (1 - d/f_2) & (1 - d/f_2)(1 - d/f_1) - d/f_2 \end{bmatrix}$$

$$\Rightarrow r_{s+2} + br_{s+1} + r_s = 0$$ - Harmonic oscillator

$$r_s = ae^{jsq}$$

$$ae^{(s+2)jq} - (A+D)ae^{(s+1)jq} + ae^{jsq} = 0$$

$$e^{2jq} - (A+D)e^{jq} + 1 = 0$$

$$e^{jq} = \frac{A+D}{2} \pm \sqrt{\left(\frac{A+D}{2}\right)^2 - 1} = \frac{A+D}{2} \pm j\sqrt{1 - \left(\frac{A+D}{2}\right)^2}$$

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta = \frac{A+D}{2} \pm j\sqrt{1 - \left(\frac{A+D}{2}\right)^2}$$

$$\boxed{\cos\theta = \frac{A+D}{2}} \qquad \boxed{\sin\theta = \sqrt{1-\cos^2\theta}}$$

Solutions:

$$r_s = r_0 e^{js\theta} + r_0^* e^{-js\theta}$$

or

$$r_s = r_{\max}\sin(s\theta + \alpha)$$

Stability Diagram

$$r_s = r_0 e^{js\theta} + r_0^* e^{-js\theta}$$

$$r_s = r_{\max} \sin(s\theta + \alpha)$$

$$\cos\theta = \frac{A+D}{2} \qquad \sin\theta = \sqrt{1-\cos^2\theta}$$

$$T_{cavity} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 - d/f_1 & d + d(1 - d/f_1) \\ -1/f_2 - 1/f_1 (1 - d/f_2) & (1 - d/f_2)(1 - d/f_1) - d/f_2 \end{bmatrix}$$

stable: $-1 \le \left(\cos\theta = \frac{\mathrm{A}+D}{2} \right) \le 1$

$$-1 \le \frac{1}{2}\left[\left(1 - \frac{d}{f_1}\right) + \left(1 - \frac{d}{f_2}\right)\left(1 - \frac{d}{f_1}\right) - \frac{d}{f_2} \right] \le 1$$

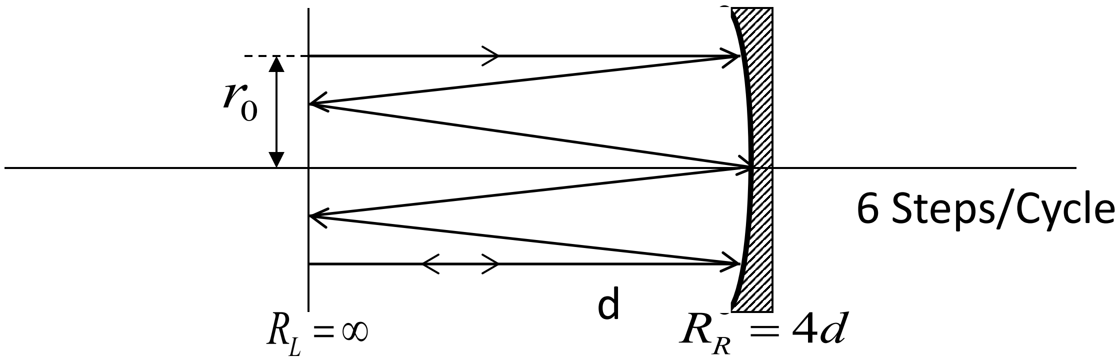
Algebra.......

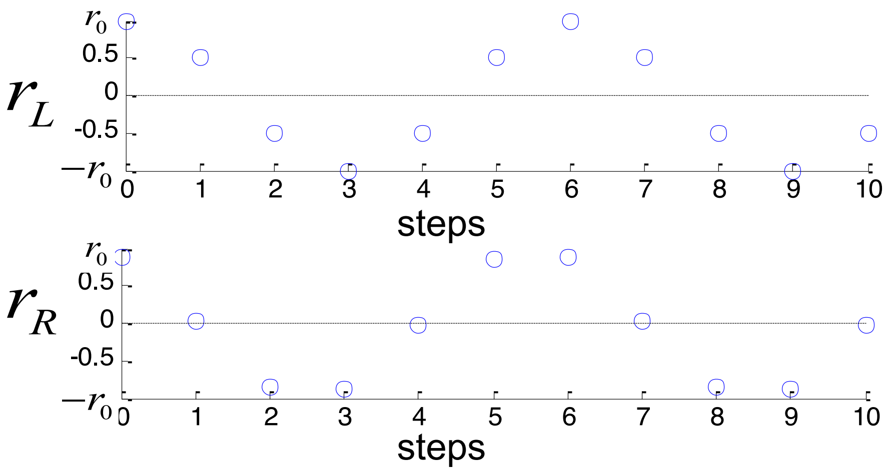
$$0 \le \left(1 - \frac{d}{R_1}\right)\left(1 - \frac{d}{R_2}\right) \le 1$$

$$g_1 \equiv 1 - \frac{d}{R_1} \qquad g_2 \equiv 1 - \frac{d}{R_2}$$

stable: $\boxed{0 \le g_1 g_2 \le 1}$

Example:





$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = (R_R)(2d) = \begin{bmatrix} 1 & 0 \\ -2/R_R & 1 \end{bmatrix} \begin{bmatrix} 1 & 2d \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2d \\ -2/R_R & 1 - 4d/R_R \end{bmatrix} = \begin{bmatrix} 1 & 2d \\ -1/2d & 0 \end{bmatrix}$$

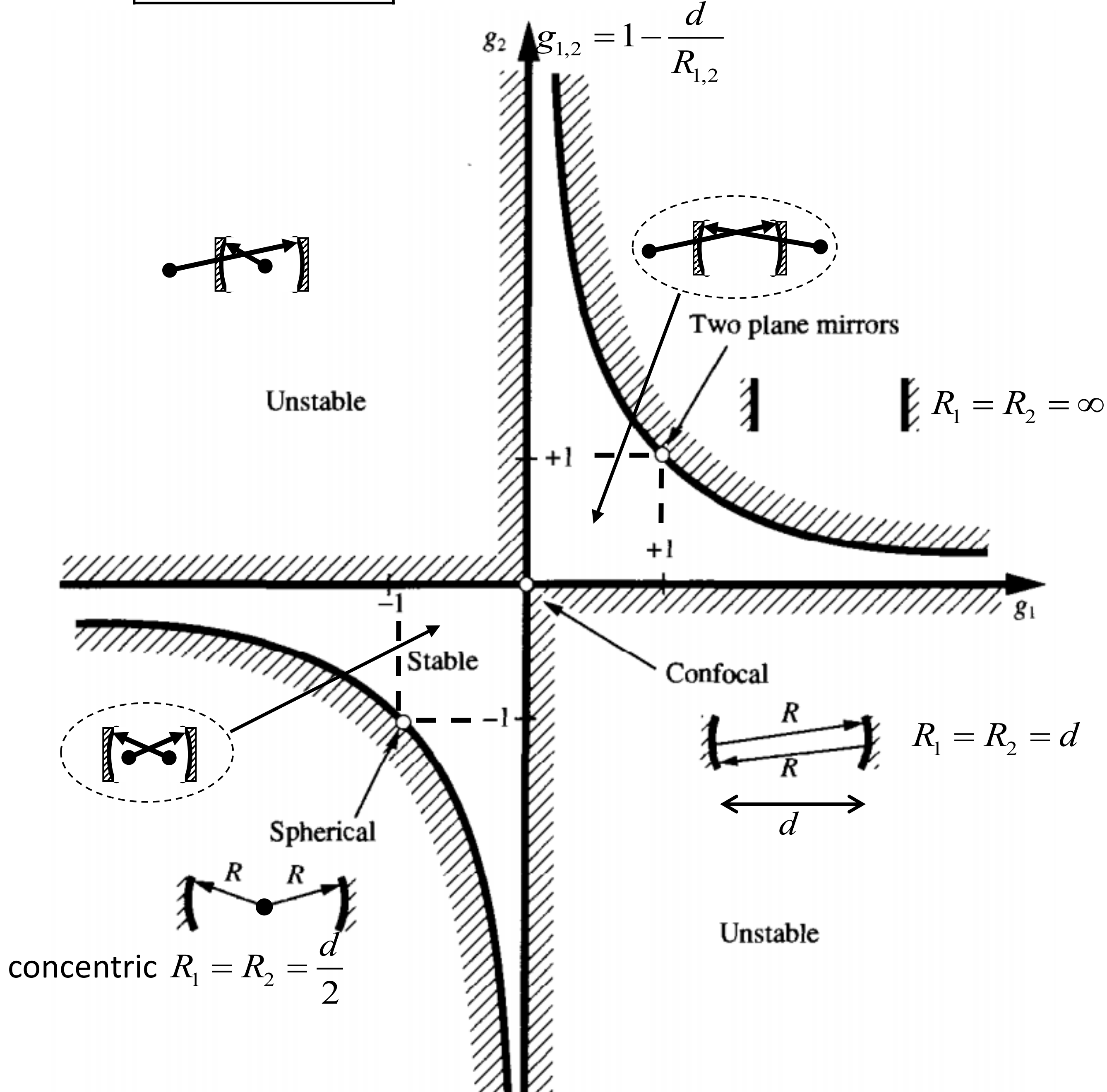
$$\cos\theta = \frac{A+D}{2} = \frac{1}{2} < 1$$

Stable!

$$g_L = 1 - \frac{d}{R_L} = 1 \qquad g_R = 1 - \frac{d}{R_R} = \frac{3}{4} \qquad g_1 g_2 < 1$$

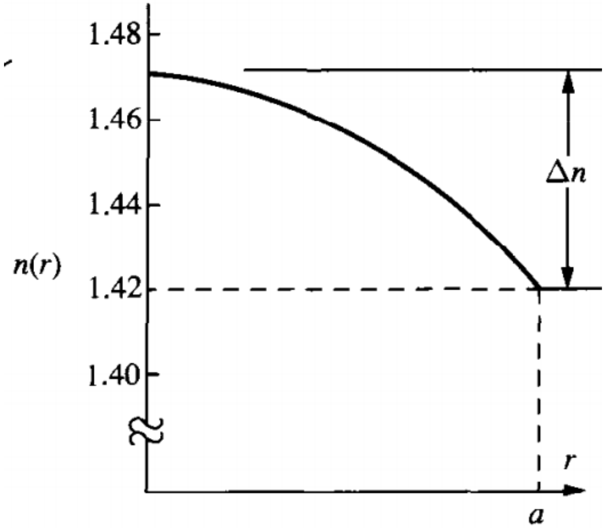
$$0 \le g_1 g_2 \le 1$$

$$g = 1 - \frac{d}{R}$$

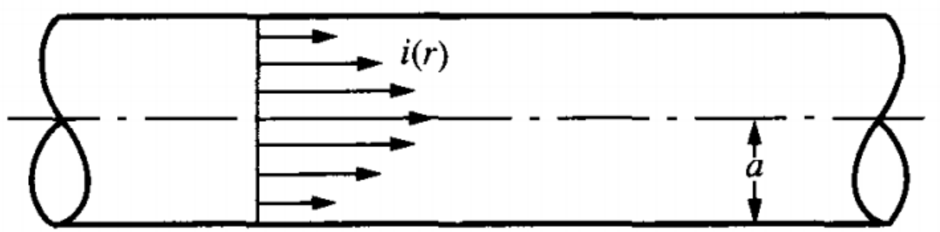


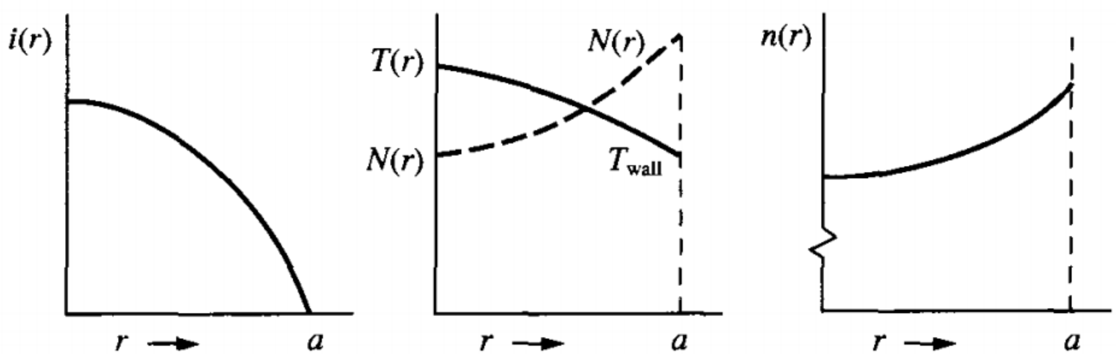
Continuous Lens – Like Media

(1) Fiber



(2) Gas Discharge





$$n-1=2\pi\alpha N(r)$$

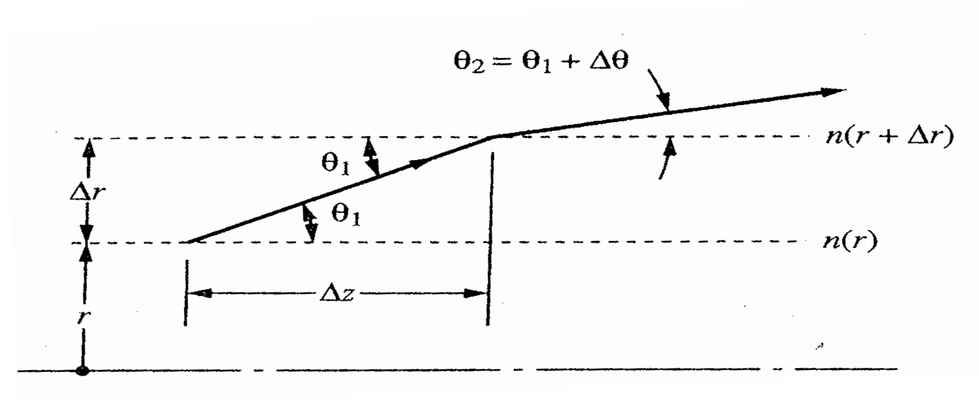
α – polarizability

$$(d=\alpha E)$$

$N(r)kT(r)=$ constant

$PV=NkT$

constant



Angles are Small: paraxial approximation

$$n(r)\cos\theta_1 = n(r+\Delta r)\cos(\theta_1 + \Delta\theta)$$

$$n(r)\cos\theta_1 = \left[n(r) + \frac{\partial n}{\partial r}\Delta r\right]\left[\cos\theta_1\cos\Delta\theta - \sin\theta_1\sin\Delta\theta\right] \qquad \theta_1 \to \theta$$

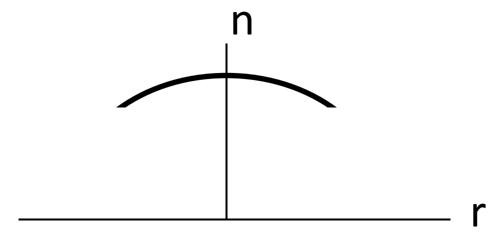
$$0 = \frac{\partial n}{\partial r}\Delta r\cos\theta - n(r)\sin\theta\Delta\theta$$

$$\frac{1}{n}\frac{\partial n}{\partial r} = \tan\theta\frac{\Delta\theta}{\Delta r}$$

$$\frac{1}{n}\frac{\partial n}{\partial r} = \frac{\Delta r}{\Delta z}\frac{\Delta\theta}{\Delta r} = \frac{\Delta\theta}{\Delta z}$$

$$\theta ;\ \tan\theta = \frac{\Delta r}{\Delta z} \Rightarrow \frac{\Delta\theta}{\Delta z} \to \frac{d^2 r}{dz^2} \qquad \boxed{\frac{1}{n(r)}\frac{\partial n}{\partial r} = \frac{d^2 r}{dz^2}}$$

a thin positive lens: (fiber!)



$$n(r) = n_0\left(1 - \frac{r^2}{2l^2}\right)$$

$$\Delta n = n_0 - n(a) = \frac{a^2}{2l^2} n_0 \qquad l = a\sqrt{\frac{n_0}{2\Delta n}} = 50\mu m\sqrt{\frac{1.47}{0.1}} \simeq 200\mu m$$

$$\frac{d^2 r}{dz^2} + \frac{r}{l^2} = 0$$

$$r = A_1 \cos\frac{z}{l} + A_2 \sin\frac{z}{l} \qquad r' = \tan\theta = \frac{dr}{dz} = -\frac{A_1}{l}\sin\frac{z}{l} + \frac{A_2}{l}\cos\frac{z}{l}$$

$$r(z=0) = r_1 \qquad \left(\frac{dr}{dz}\right)_{z=0} = r_1' \Rightarrow \begin{pmatrix} A_1 = r_1 \\ A_2 = r_1' l \end{pmatrix}$$

$$r = r_1 \cos\frac{z}{l} + r_1' l \sin\frac{z}{l} \qquad r' = -\frac{r_1}{l}\sin\frac{z}{l} + r_1'\cos\frac{z}{l}$$

$$\mathrm{T} = \begin{pmatrix} \cos\frac{d}{l} & l\sin\frac{d}{l} \\ \frac{-1}{l}\sin\frac{d}{l} & \cos\frac{d}{l} \end{pmatrix} \qquad \lim_{l\to\infty} T = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$$

cf. $T = \begin{pmatrix} 1 & 0 \\ \frac{-1}{f} & 0 \end{pmatrix}$ or $T = \begin{pmatrix} 1 & d \\ \frac{-1}{f} & \frac{d}{f} \end{pmatrix}$

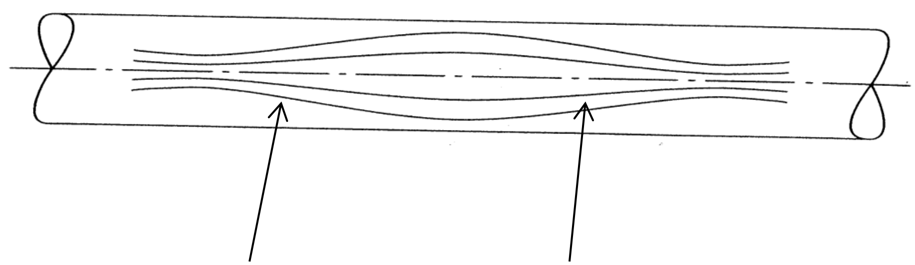
$$\Rightarrow \frac{1}{f} = \frac{1}{l}\sin\frac{d}{l} \qquad f = l\left(\sin\frac{d}{l}\right)^{-1}$$

If $n(r) = n_0\left(1 + \frac{r^2}{2L^2}\right)$ - discharge

$l \rightarrow jL$ to set $\begin{cases} \cos\frac{d}{l} \Rightarrow \cosh\frac{d}{L} \\ \sin\frac{d}{l} \Rightarrow \sinh\frac{d}{L} \end{cases}$

$$T = \begin{pmatrix} \cosh\frac{d}{L} & L\sinh\frac{d}{L} \\ -\frac{1}{L}\sinh\frac{d}{L} & \cosh\frac{d}{L} \end{pmatrix}$$ for $$n(r) = n_0\left(1 + \frac{r^2}{2L^2}\right)$$

For lens $f = l\left(\sin\frac{d}{l}\right)^{-1}$



beam undulating

$\frac{d}{l} > \pi$ $\frac{d}{l} < \pi$

negative lens positive (converging) lens