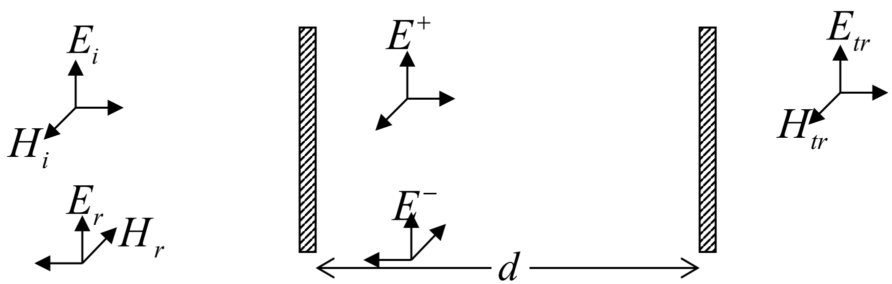
Resonant Optical Cavities



Simple cavity
(Fabry-Perot)

Resonance: round trip phase shift = (RTPS) $= 2kd = q \times 2\pi$

$$2kd = \frac{2\omega nd}{c} = \frac{4\pi d}{\lambda} = q \times 2\pi \qquad \Rightarrow d = \frac{q\lambda}{2} \qquad q \sim 10^6$$

$$\nu_q = q\frac{c}{2nd}$$

$$\nu_{q+1} - \nu_q = \frac{c}{2nd}$$ - free space range (FSR) $\sim 100 - 1000 MHz$

field traveling to the right:

$$E^+ = \sum E_N^+ = E_0\left\{1 + r_1 r_2 e^{-jk2d} + \left(r_1 r_2 e^{-jk2d}\right)^2 + \ldots\right\}$$

"0" Round Trip — "1" round trip — "2" round trips

$$\mathrm{E}^+ = \frac{\mathrm{E}_0}{1 - r_1 r_2 e^{-j2\theta}}$$

θ = kd

field traveling to the left:

$$E^- = r_2 e^{-j2\theta}\mathrm{E}^+ = \frac{\mathrm{E}_0 r_2 e^{-j2\theta}}{1 - r_1 r_2 e^{-j2\theta}}$$

$$I^+\left(z=0^+\right)=\frac{E_0^2}{2\eta}\left\{\frac{1}{1-r_1r_2e^{-2j\theta}-\left(r_1r_2e^{-2j\theta}\right)^*+|r_1r_2|^2}\right\}=\frac{I_0}{1-2|r_1r_2|\cos 2\theta+|r_1r_2|^2}$$

assume that r = |r| (no initial phase)

$$I^+\left(z=0^+\right)=\frac{I_0}{1-2|r_1r_2|\left[1-2\sin^2\theta\right]+|r_1r_2|^2}$$

$$\boxed{R_{1,2}=|r_{1,2}|^2}$$

$$\boxed{I^+\left(z=0^+\right)=\frac{\left(E_0\right)^2}{2\eta}\frac{1}{\left(1-\sqrt{R_1R_2}\right)^2+4\sqrt{R_1R_2}\sin^2\theta}}$$

Quality factor (Sharpness of the resonance)

$$Q = \frac{\nu_0}{\Delta \nu_{1/2}} = \frac{\omega_0}{\Delta \omega_{1/2}} = \frac{\lambda_0}{\Delta \lambda_{1/2}}$$

$$\frac{\mathrm{E_0}^2}{2\eta} = \left(1 - R_1\right) \times \frac{\mathrm{E}_{inc}^2}{2\eta}$$ - power transmission for M_1 mirror

$$\left(1 - R_2\right) \times I^+(z = 0^+)$$ - transmission for the cavity

$$\mathrm{I}_t = \left(1 - R_1\right)\mathrm{I}_{inc} \times \frac{\left(1 - R_2\right)}{\left(1 - \sqrt{R_1 R_2}\right)^2 + 4\sqrt{R_1 R_2}\sin^2\theta}$$

$$\boxed{T\left(\theta\right) = \frac{\mathrm{I}_{trans}}{\mathrm{I}_{inc}} = \frac{\left(1 - R_1\right)\left(1 - R_2\right)}{\left(1 - \sqrt{R_1 R_2}\right)^2 + 4\sqrt{R_1 R_2}\sin^2\theta}}$$

1) For resonance (ϴ=0) and R_2= R_2 = R; T=1 independent of R!
2) Since T=1 $(I_{inc} = I_{out}) = (1 - R_2) I^+$

$$\mathrm{I}^+ = \frac{1}{1 - R_2} \times I_{inc} \Rightarrow \text{For } R_2 = 99\% \qquad I_{inside} = 100 \cdot I_{outside}$$

$$\mathrm{T}_{1/2} = \frac{\mathrm{T}_{\max}}{2};\ T_{\max} \text{ at } \theta = 0 \ (\text{resonance!})$$

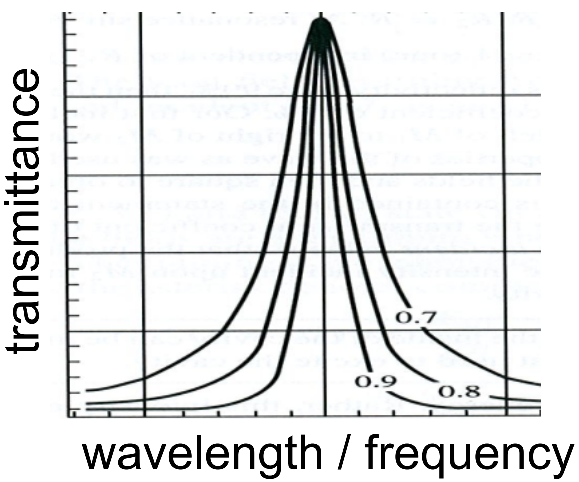
$$\mathrm{T}_{1/2} \text{ at } 4r_1r_2\sin^2\theta = (1 - r_1r_2)^2$$

$$\theta_{+,-} = \pm\frac{1}{2}\left(\frac{1 - r_1r_2}{\sqrt{r_1r_2}}\right)$$

$$kd \equiv \theta$$

$$\frac{2\pi\nu_{+,-}nd}{c} = \theta_{+,-} = \pm\frac{1}{2}\left(\frac{1 - r_1r_2}{\sqrt{r_1r_2}}\right);\quad \boxed{\Delta\nu_{1/2} = \frac{c}{2nd}\frac{1 - r_1r_2}{\pi\sqrt{r_1r_2}}}$$

$$Q = \frac{q(c/2nd)}{\Delta\nu_{1/2}} = \frac{2\pi nd}{\Delta\lambda_0}\frac{(R_1R_2)^{1/4}}{1 - (R_1R_2)^{1/2}}$$



$$\left(q = \frac{nd}{\lambda_0/2}\right) \quad \left(\begin{matrix} R:\ 99\% \\ Q \sim 10^9 \end{matrix}\right)$$

Finesse:

$$F = \frac{\text{free spectral range (FSR)}}{\text{full width at half maximum (FWHM)}} = \frac{c/2nd}{\Delta\nu_{1/2}}$$

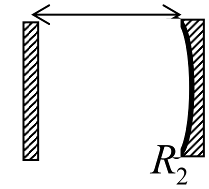
$$\boxed{F = \frac{\pi\left(R_1R_2\right)^{1/4}}{1-\left(R_1R_2\right)^{1/2}}}$$

$$\boxed{Q = \frac{2nd}{\lambda_0}F}$$

$F \sim 300$

d

Gaussian Modes



phase shift for $TEM_{m,p}$ mode: $\phi(d)-\phi(0)=kd-(1+m+p)\tan^{-1}\left(\frac{d}{z_0}\right)$

Resonance: $kd-(1+m+p)\tan^{-1}\left(\frac{d}{z_0}\right)=q\pi$ $\quad\left(z_0=\frac{\pi w_0^{\,2}}{\lambda}\right)$

Use: $z_0=(dR_2)^{\frac{1}{2}}\left(1-\frac{d}{R_2}\right)^{\frac{1}{2}}$

then after some algebra:

$$\nu_{m,p,q}=\frac{c}{2nd}\left\{q+\frac{1+m+p}{\pi}\tan^{-1}\left[\frac{(d/R_2)^{\frac{1}{2}}}{(1-d/R_2)^{\frac{1}{2}}}\right]\right\}$$

$$\nu_{m,p,q}=\frac{c}{2nd}\left[q+\frac{1+m+p}{\pi}\cos^{-1}\left(1-\frac{d}{R_2}\right)^{\frac{1}{2}}\right]$$

For M_1 with finite R_1:

$$\boxed{\nu_{m,p,q}=\frac{c}{2nd}\left[q+\frac{1+m+p}{\pi}\cos^{-1}\left(g_1g_2\right)^{\frac{1}{2}}\right]}\qquad g_{1,2}=1-\frac{d}{R_{1,2}}$$

Frequency degeneracy

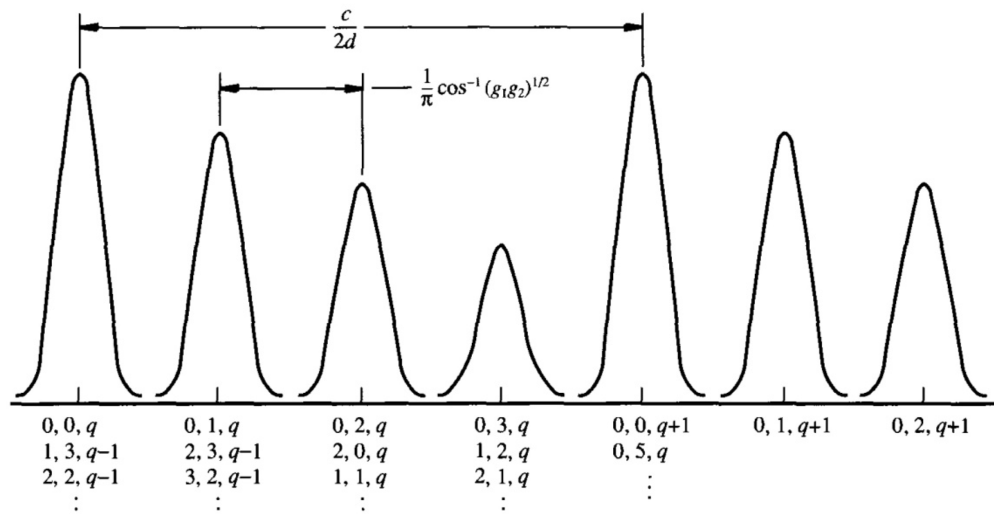


FIGURE 6.6. Frequency degeneracy in an optical cavity.

Photon Lifetime

The fraction of photons surviving 1 round trip: $S = R_1 R_2$ (survival factor)
The number of photons lost in 1 round trip: $\left[1-S\right]N_p$

$$\frac{dN_p}{dt} = -\frac{\left[1-S\right]N_p}{\tau_{RT}} \qquad \tau_{RT} = \frac{2nd}{c}$$

$$\frac{dN_p}{dt} = -\frac{N_p}{\tau_p} \qquad \boxed{\tau_p = \frac{\tau_{RT}}{1-S}} \text{ (photon lifetime)}$$

$$\mathrm{N}_p(t) = \mathrm{N}_p(0)\exp\left[-\frac{t}{\tau_p}\right]$$

$$\left[\tau_{RT} = \frac{2nd}{c} \qquad S = R_1 R_2 e^{-2\alpha d}\right] \text{(if there are losses)}$$

$$Q = 2\pi\frac{\text{energy stored in the system at resonance (W)}}{\text{energy lost in a cycle of oscillation}}$$

$$Q = \frac{2\pi}{\mathrm{T}}\mathbf{g}\frac{W}{\langle P\rangle} = \omega_0\frac{W}{\langle P\rangle} = \omega_0\frac{W}{-dW/dt} \qquad (<P> \text{- average power})$$

$$\frac{dW}{dt} = \frac{-\omega_0}{Q}W \Rightarrow \frac{d\left[h\nu N_p\right]}{dt} = \frac{\omega_0}{Q}\left[h\nu N_p\right] = -\frac{h\nu N_p}{\tau_p} \qquad \boxed{\tau_p = \frac{Q}{\omega_0}}$$

$$Q^{-1} = \frac{\Delta\omega_{1/2}}{\omega_0} \Rightarrow \boxed{\Delta\omega_{1/2}\tau_p = 1}$$

$$\tau_p = \frac{\tau_{RT}}{1-S} = \frac{2nd/c}{1-R_1R_2} \Rightarrow \Delta v_{1/2} = \frac{1}{2\pi\tau_p} = \frac{c\left(1-R_1R_2\right)}{\left(2nd\right)\cdot 2\pi}$$

$$Q = \frac{v_0}{\Delta v_{1/2}} = 2\pi v_0\left(\frac{2nd}{c}\right)\frac{1}{1-R_1R_2} = \boxed{Q = \frac{4\pi nd}{\lambda_0}\left(\frac{1}{1-R_1R_2}\right)}$$

$$F = \frac{FSR}{\Delta v_{1/2}} = \frac{c/\left(2nd\right)}{\Delta v_{1/2}} = \qquad \boxed{F = \frac{2\pi}{1-R_1R_2}}$$

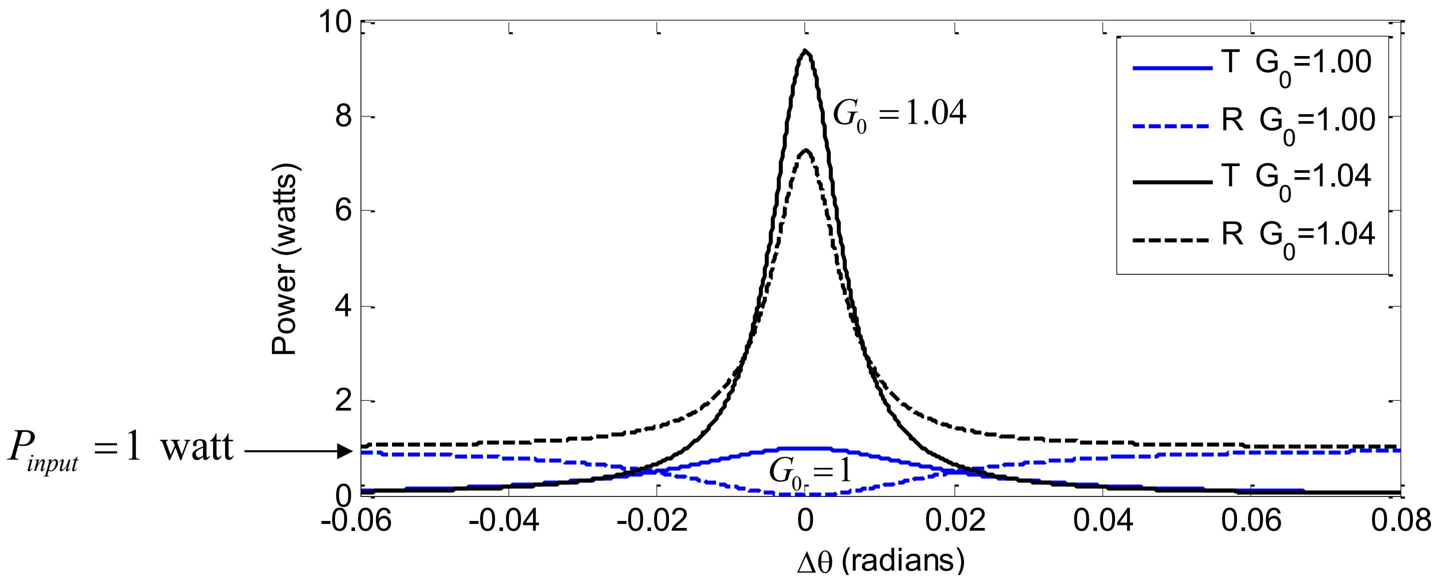
Cavity with Gain

A medium with a power gain G_0 between the two mirrors.
RT propagation: $1\cdot\exp[-jk\cdot 2d] \rightarrow G_0\cdot\exp[-jk\cdot 2d]$
Then a similar derivation yields:

$$T = \frac{G_0(1-R_1)(1-R_2)}{\left(1-G_0\sqrt{R_1R_2}\right)^2 + 4G_0\sqrt{R_1R_2}\sin^2\theta}$$

$$R_{net} = \left[\frac{E_r}{E_i}\right]^2 = \frac{\left(\sqrt{R_1}-\sqrt{R_2}\right)^2 + 4G_0\sqrt{R_1R_2}\sin^2\theta}{\left(1-G_0\sqrt{R_1R_2}\right)^2 + 4G_0\sqrt{R_1R_2}\sin^2\theta}$$

FWHM = $2\Delta\theta_{1/2} = \dfrac{1-G_0\sqrt{R_1R_2}}{4G_0^{1/2}\sqrt[4]{R_1R}}$



Both the reflected and transmitted powers are considerably larger than the incident power (1 watt)