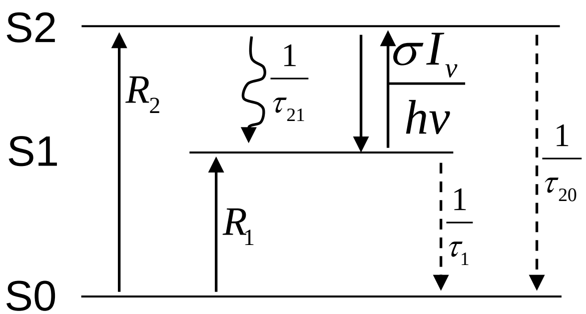
Power in Laser Oscillation

pump power -> laser power

Consider 3-level system and use rate equations (see section 8.3)



$$\frac{g_2}{g_1} = 1$$

use gain coefficient: $\gamma(\nu) = (N_2 - N_1)\sigma(\nu)$

and solution for $(N_2 - N_1)$

$$(N_2 - N_1) = \frac{\gamma(\nu)}{\sigma(\nu)} = \frac{R_2\tau_2\left(1 - \frac{\tau_1}{\tau_{21}}\right) R_1\tau_1}{1 + \left(\tau_1 + \tau_2 - \frac{\tau_1\tau_2}{\tau_{21}}\right)\frac{\sigma(\nu)I_\nu}{h\nu}} \qquad (8.3.11c)$$

For $\tau_{20}^{-1} << \tau_{21}^{-1}$, $\tau_2 = \tau_{21}$ $\Rightarrow$

$$(N_2 - N_1) = \frac{\gamma(\nu)}{\sigma(\nu)} = \frac{R_2\tau_2\left(1 - \frac{\tau_1}{\tau_2}\right) - R_1\tau_1}{1 + \tau_2\frac{\sigma(\nu)I_\nu}{h\nu}}$$

$$(N_2 - N_1) = \frac{\gamma(\nu)}{\sigma(\nu)} = \frac{R_2\left[1 - \frac{\tau_1}{\tau_2}\left(1 + \frac{R_1}{R_2}\right)\right]}{\frac{1}{\tau_2} + \frac{\sigma(\nu)I_\nu}{h\nu}} = \frac{R}{\frac{1}{\tau_2} + \frac{\sigma(\nu)I_\nu}{h\nu}}$$

effective pump rate (→ R)

total decay rate (→ $\frac{1}{\tau_2}$)

$$(N_2 - N_1)\left(\frac{1}{\tau_2} + \frac{\sigma(\nu)I_\nu}{h\nu}\right) = R \qquad (1)$$

$$\left(\frac{\sigma(\nu)}{h\nu}\right)I_\nu = \left(\frac{R}{N_2 - N_1} - \frac{1}{\tau_2}\right)$$

Below threshold $I_\nu = 0 \qquad \Rightarrow N_2 - N_1 = R\tau_2$

Continue to increase pump until threshold, N_t

$$N_t = \left(N_2 - \frac{g_2}{g_1}N_1\right)_{threshold} = \frac{\gamma_{th}(\nu)}{\sigma(\nu)}$$

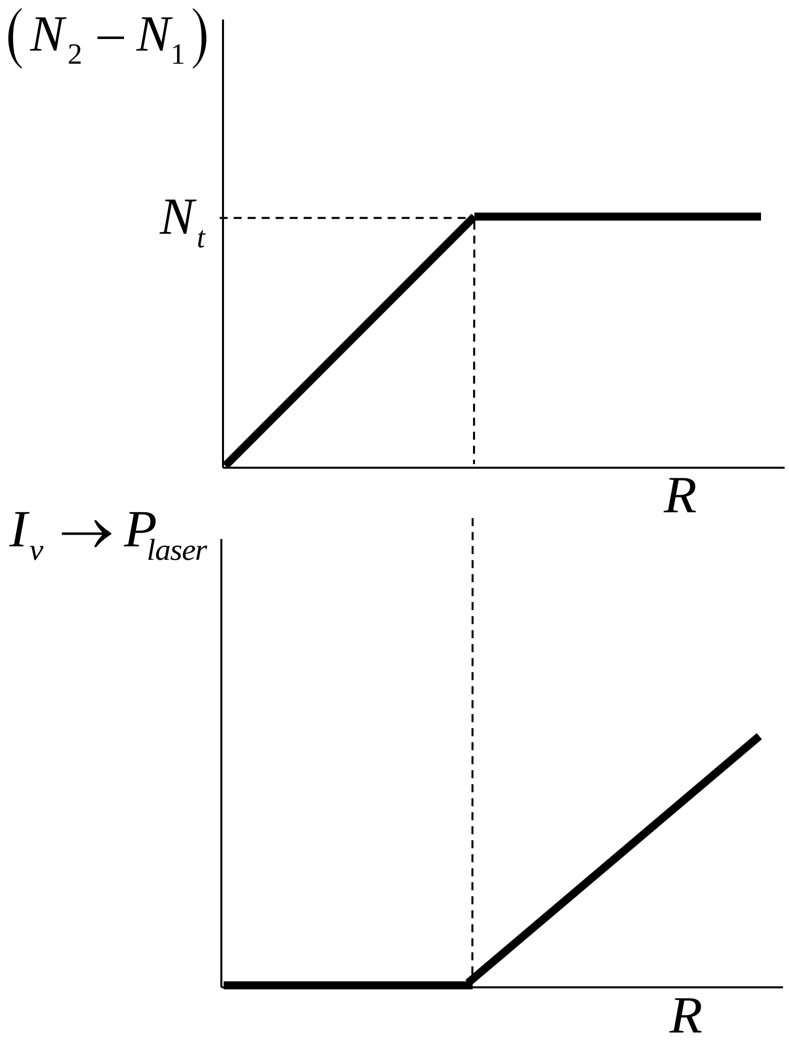
$$\gamma_{th} = -\frac{1}{2\ell}\ln R_1 R_2 = \frac{1}{\frac{c}{n}\tau_p}$$

(see section 8.8)

↑ reflectance

$$\sigma = A_{21}\frac{\lambda^2}{8\pi}g(\nu)$$

Increasing R beyond N_t/τ_2 cannot increase $(N_2 - N_1)$



$$N_t = \frac{R}{\frac{1}{\tau_2} - \frac{\sigma I_v}{hv}}$$

see (1)

- Describes how I_v increases with R

$$\frac{\sigma I_v}{hv} N_t = R - \frac{N_t}{\tau_2} \quad \text{for } R \geq N_t/\tau_2$$

$$P = (N_t V_g)\left(\frac{\sigma I_v}{hv}\right) hv$$

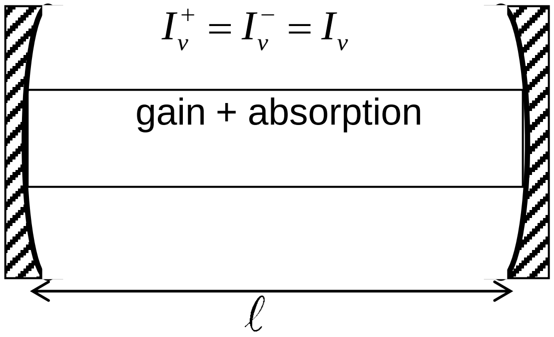
- $N_t V_g$: total # of excess Excited atoms
- $\frac{\sigma I_v}{hv}$: Rate of stimulated emission per atom
- hv: energy per photon

$$\boxed{\frac{P}{V_g hv} = R - \frac{N_t}{\tau_2}}$$

- $\frac{P}{V_g hv}$: photon emission rate density
- R: pumping rate density
- $\frac{N_t}{\tau_2}$: spontaneous emission rate density

Optimum Output Coupling for Optical Resonator

Homogeneously broadened. Standing wave.



$$\gamma = \frac{\gamma_0}{1 + \left(I_\nu^+ + I_\nu^-\right)/I_S}$$

Actually, there is interference between the fields associated with I_ν^+ and I_ν^-, which leads to a spatial variation of the saturation. This is complicated, and ignored here.

Optimum coupling: if transmission is too large, losses exceed gain – no generation. If transmission is too small – little goes out.

$$I_\nu = \frac{I_S}{2}\left(\frac{\gamma_0}{\gamma} - 1\right) = \frac{I_S}{2}\left(\frac{\gamma_0}{\gamma_t} - 1\right)$$

clamped at γ_t

$$e^{2\gamma_t \ell} \underbrace{R_1 R_2 e^{-2\alpha\ell}} = 1$$ - Threshold condition

fraction retained after round trip in the cavity

$$R_1 R_2 e^{-2\alpha\ell} = 1 - L = S$$ (L-fraction lost in round trip)

$$L = 1 - R_1 R_2 e^{-2\alpha\ell}$$

$$\left(L << 1\right) \Rightarrow 1 - L = e^{-2\gamma_t \ell} \simeq 1 - 2\gamma_t \ell$$ or $$2\gamma_t \ell = -\ln(1-L) \simeq L$$

$$I_v = \frac{I_S}{2}\left(\frac{\gamma_0}{\gamma_t} - 1\right)$$

$$2\gamma_t \ell \simeq L$$

$$I_v = \frac{I_S}{2}\left(\frac{2\gamma_0 \ell}{L} - 1\right)$$

$$g_0 \simeq \gamma_0 \ell$$

small signal gain/pass

$$I_v = \frac{I_S}{2}\left(\frac{2g_0}{\underbrace{L}_{\text{total losses}}} - 1\right)$$

I_v: intensity circulated Inside cavity

In the text

$$L' \to L$$

sum of all other losses (abs.) → L'; output mirror transmission → T

$$L = L' + T$$

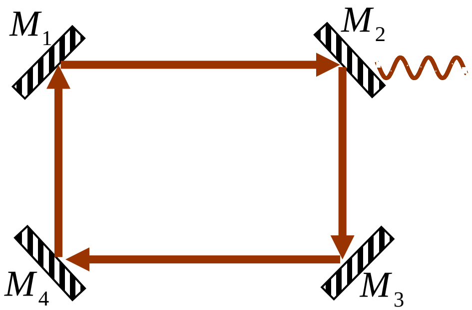
$$L' = L - T \qquad T_2 = 1 - R_2$$ - transmission through M_2

$$L = 1 - R_1 R_2 e^{-2\alpha\ell}$$

$$I_{out} = I_v T = \frac{I_S}{2}\left(\frac{2g_0}{L' + T} - 1\right)T$$

- standing wave

$T = T_2$ if M_2 provides transmission for ring lasers:



(for ring cavity)

$$I_{out} = I_S\left(\frac{g_0}{L'+T}-1\right)T$$

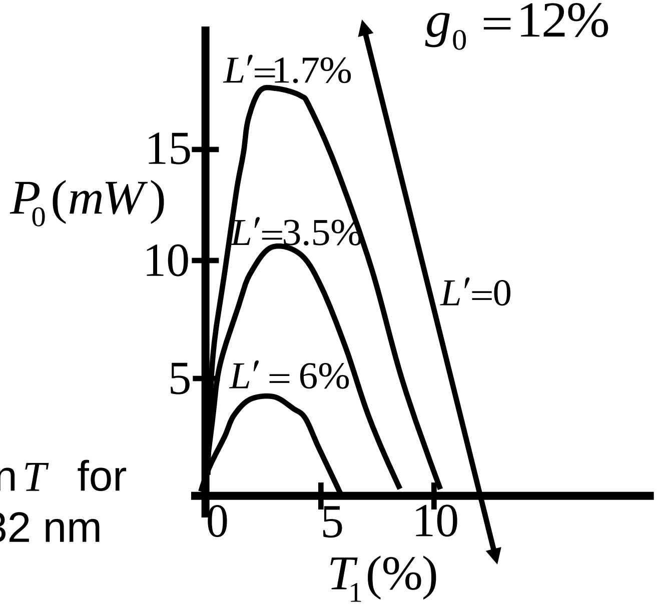
$T = T_2$ if M_2 provides transmission

Find T_{opt}:

$$\left[T\times\left(\frac{g_0}{L'+T}-1\right)\right]' = \frac{g_0}{L'+T}-1-T\frac{g_0}{(L'+T)^2} = \frac{g_0(L'+T)-g_0T}{(L'+T)^2}-1=0$$

$$g_0L' = (L'+T)^2$$

$$T_{opt} = \sqrt{g_0L'}-L'$$



Useful power output P_0 vs mirror transmission T for various values of internal loss L' in He-Ne 632 nm laser.