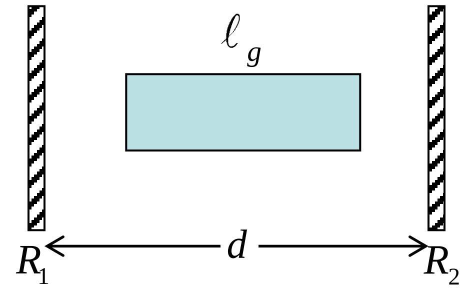
Laser Dynamics

Steady state operation is only one mode of operation

- Relaxation oscillation effects
- Gain switching
- Q - switching



$(\ell_g \equiv \ell)$

N_p - total # of photons in cavity

$n = \left(N_2 - \frac{g_2}{g_1} N_1 \right) V$ - population inversion (total, not density)

$\tau_p = \frac{2n_r \, d/c}{(1 - R_1 R_2)}$ - photon lifetime in passive cavity

$\gamma \propto n$

$$\frac{dN_p}{dt} = \frac{\overbrace{G^2 R_1 R_2}^{\text{returns after round trip}} - 1}{2n_r \, d/c} N_p + \underbrace{N_2 c \sigma_{SE}}_{\text{spontaneous emission}}$$

$(R_1 R_2 \simeq 1)$

$$\frac{G^2 R_1 R_2 - 1}{2n_r \, d/c} = \frac{e^{2\gamma\ell} R_1 R_2 - 1}{2n_r \, d/c} \simeq \frac{R_1 R_2 - 1}{2n_r \, d/c} + \frac{2\gamma\ell}{2n_r \, d/c} = -\frac{1}{\tau_p} + \gamma \frac{c}{n_r} \frac{\ell}{d}$$

$$\Rightarrow \boxed{\frac{dN_p}{dt} = \gamma \frac{c}{n_r} \frac{\ell}{d} N_p - \frac{N_p}{\tau_p}} + N_2 c \sigma_{SE}$$

$$\frac{dN_p}{dt} = \underbrace{N_p\left(\frac{\gamma c}{n_r}\right)\left(\frac{\ell}{d}\right)}_{\text{gain}} - \underbrace{\frac{N_p}{\tau_p}}_{\text{losses}} + \underbrace{N_2 c \sigma_{SE}}_{\text{spontaneous emission (small)}}$$

$\tau = \frac{t}{\tau_p}$ generation occurs at $\gamma = \gamma_{th}$; when $\frac{dN_p}{d\tau} = 0$ -steady state

$$\frac{dN_p}{d\tau} = \tau_p \frac{dN_p}{dt} = \tau_p N_p\left[\frac{c\gamma\ell}{n_r d} - \frac{1}{\tau_p}\right] = N_p\left[\frac{\gamma}{\left(\frac{n_r d}{c\ell\tau_p}\right)} - 1\right]$$

since $\gamma \propto n$ $\left[\gamma = \left(N_2 - \frac{g_2}{g_1}N_1\right)\sigma\right]$

$$\frac{dN_p}{d\tau} = \tau_p \frac{dN_p}{dt} = N_p\left[\frac{\gamma}{\gamma_{th}} - 1\right] \Rightarrow \boxed{\frac{dN_p}{d\tau} = N_p\left[\frac{n}{n_{th}} - 1\right]}$$

$$\frac{dN_2}{dt} = R_2 - \frac{N_2}{\tau_2} - \frac{\sigma I}{h\nu}\left(N_2 - \frac{g_2}{g_1}N_1\right)$$

$$n = \left(N_2 - \frac{g_2}{g_1}N_1\right)V \simeq N_2 V \qquad \left(N_2 \gg \frac{g_2}{g_1}N_1\right),\ (\tau_2 \gg \tau_1)$$

$$\frac{dn}{dt} = r - \frac{n}{\tau_2} - \frac{\sigma I\ n}{h\nu} = r - \frac{n}{\tau_2} - \frac{I}{I_s}\frac{n}{\tau_2}$$

$$I_s = \frac{h\nu}{\sigma\tau_2}, \quad \tau = \frac{t}{\tau_p}, \quad r \equiv RV$$

$$\tau_p \frac{dn}{dt} = r\tau_p - n\frac{\tau_p}{\tau_2} - \frac{I}{I_s}\frac{\tau_p}{\tau_2}n$$

$$I \propto \frac{N_p}{\tau_p} \Rightarrow I\tau_p \propto N_p, \quad I_s\tau_2 \propto n_{th}$$

when $I \to I_s$

$$n \to n_{th} \Rightarrow n_{th} \propto \gamma_{th} \propto I_s$$

$$\frac{dn}{d\tau} = r\tau_p - n\frac{\tau_p}{\tau_2} - N_p\frac{n}{n_{th}}$$

$$\frac{I}{I_s}\frac{\tau_p}{\tau_2} = \frac{N_p}{n_{th}}$$

$$\boxed{\frac{dn}{d\tau} = -N_p\frac{n}{n_{th}} - n\frac{\tau_p}{\tau_2} + r\tau_p}$$

$$\frac{dN_p}{d\tau} = N_p\left(\frac{n}{n_{th}} - 1\right) \qquad (1)$$

$$\frac{dn}{d\tau} = r\tau_p - N_p\frac{n}{n_{th}} - n\frac{\tau_p}{\tau_2} \qquad (2)$$

At equilibrium (steady state): $n_e = n_{th}$

$$\left(\frac{\sigma I_\nu}{h\nu} = \frac{R}{N_2 - N_1} - \frac{1}{\tau_2}\right)$$

$$N_{p,e} = \left(r - {n_{th}}/{\tau_2}\right)\tau_p \qquad (r > r_{th})$$

At threshold ($N_p = 0$): $$r_{th} = \frac{n_{th}}{\tau_2}$$

$$N_{p,e} = \left(r - r_{th}\right)\tau_p \qquad \left(r \simeq r_{th}\right)$$

How do n, N_p approach $n_e, N_{p,e}$ after a small change to n or N_p? <u>Dynamics?</u>

Define: $$n = n_{th} + n_1(t)$$
$$N_p = N_{p,e} + N_{p,1}(t); r = r_0 + r_1(t)$$

$$\frac{dn}{d\tau} = \frac{dn_1}{d\tau} = \left(r_0 + r_1(t)\right)\tau_p - \left(N_{p,e} + N_{p,1}(\tau)\right)\left(\frac{n_{th} + n_1(t)}{n_{th}}\right) - \left(n_{th} + n_1(t)\right)\frac{\tau_p}{\tau_2}$$

$$\simeq r_0\tau_p + r_1(t)\tau_p - \left[N_{p,e} + N_{p,1}(\tau) + N_{p,e}\frac{n_1(t)}{n_{th}}\right] - \frac{n_{th}\tau_p}{\tau_2} - \frac{n_1(t)\tau_p}{\tau_2}$$

to 1st order only, small perturbations

but $$r_0\tau_p - \frac{n_{th}\tau_0}{\tau_2} - N_{p,e} = 0 \qquad (*)$$ see (2) $\left(n \simeq n_{th}\right)$

$$\frac{dn_1}{d\tau} = r_1(t)\tau_p - N_{p,1} - N_{p,e}\frac{n_1(t)}{n_{th}} - n_1(t)\frac{\tau_p}{\tau_2}$$

$$\frac{dn_1}{d\tau} = r_1(t)\tau_p - N_{p,1} - \left[\frac{N_{p,e}}{n_{th}} + \frac{\tau_p}{\tau_2}\right]n_1(t)$$

but $$\frac{N_{p,e}}{n_{th}} + \frac{\tau_p}{\tau_2} = \frac{r_0\tau_p}{n_{th}}$$

see (*)

$$\frac{dn_1}{d\tau} = r_1(t)\tau_p - N_{p,1} - \frac{r_0\tau_p n_1(t)}{n_{th}} \qquad (3)$$

Similarly for $\frac{dN_p}{d\tau}$

$$\frac{dN_p}{d\tau} = N_p\left(\frac{n}{n_{th}} - 1\right) = \frac{dN_{p,1}}{d\tau} = \left[N_{p,e} + N_{p,1}(t)\right]\left[\frac{n_{th} + n_1(t)}{n_{th}} - 1\right]$$

$$\frac{dN_p}{d\tau} = N_{p,e}\frac{n_1(t)}{n_{th}} = \left(r_0 - \frac{n_{th}}{\tau_2}\right)\tau_p\frac{n_1(t)}{n_{th}}$$ see (*)

$$\frac{dN_{p,1}}{d\tau}=\left[\frac{r_0}{n_{th}}-\frac{1}{\tau_2}\right]\tau_p n_1(t)$$

$$\boxed{\frac{dN_{p,1}}{d\tau}=\left[\frac{r_0}{r_{th}}-1\right]\frac{\tau_p}{\tau_2}n_1(t) \qquad (4)}$$

$r_{th}=\frac{n_{th}}{\tau_2}$ see (2)

To find solution for constant r (or for stepwise increase $\_\_\lceil r_0$; r1=0)

From (4),(3):

$$\boxed{\frac{dn_1}{d\tau}=r_1(t)\tau_p-N_{p,1}-\frac{r_0\tau_p n_1(t)}{n_{th}} \qquad (3)}$$

$$\frac{d^2N_{p,1}}{d\tau^2}=\left[\frac{r_0}{r_{th}}-1\right]\frac{\tau_p}{\tau_2}\frac{dn_1(t)}{d\tau}$$

$$=\left[\frac{r_0}{r_{th}}-1\right]\frac{\tau_p}{\tau_2}\left(-N_{p,1}(t)-\frac{r_0\tau_p}{n_{th}}n_1(t)\right)=\left[\frac{r_0}{r_{th}}-1\right]\frac{\tau_p}{\tau_2}\left(-N_{p,1}(t)\right)-\frac{r_0\tau_p}{n_{th}}\frac{dN_{p,1}}{d\tau}$$

$\left[\frac{r_0}{r_{th}}-1\right]\frac{\tau_p}{\tau_2}n_1=\frac{dN_{p,1}}{d\tau}$ from (4)

$$\frac{d^2N_{p,1}}{d\tau^2}+\frac{r_0\tau_p}{n_{th}}\frac{dN_{p,1}}{d\tau}+\left[\frac{r_0}{r_{th}}-1\right]\frac{\tau_p}{\tau_2}N_{p,1}(t)=0$$

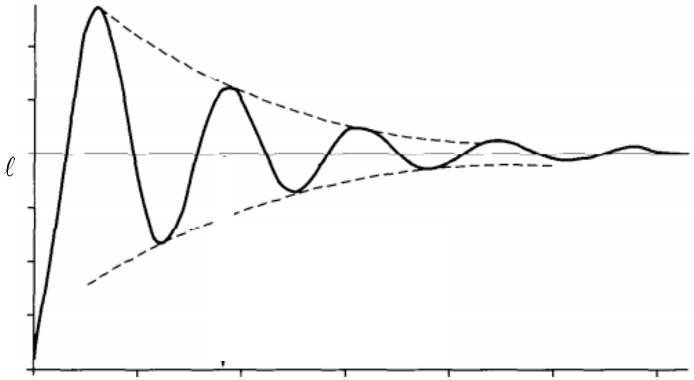
rescale $t=\tau\tau_p$

$$\frac{d^2N_{p,1}}{dt^2}+\frac{r_0}{n_{th}}\frac{dN_{p,1}}{dt}+\left[\frac{r_0}{r_{th}}-1\right]\frac{1}{\tau_2\tau_p}N_{p,1}(t)=0$$

Solution: damped harmonic oscillator

$$\frac{d^2 N_{p,1}}{dt^2} + \frac{r_0}{n_{th}} \frac{dN_{p,1}}{dt} + \left[\frac{r_0}{r_{th}} - 1\right] \frac{1}{\tau_2 \tau_p} N_{p,1}(t) = 0$$

Solution: damped harmonic oscillator



$$N_{p,1} \propto \exp(-\alpha \pm j\omega_m)t$$

$$(-\alpha \pm j\omega_m)^2 + \frac{r_0}{n_{th}}(-\alpha \pm j\omega_m) + \left(\frac{r_0}{r_{th}} - 1\right)\frac{1}{\tau_p \tau_2} = 0$$

real part: $\alpha^2 - \omega_m^2 + \frac{r_0}{n_{th}}(-\alpha) + \left(\frac{r_0}{r_{th}} - 1\right)\frac{1}{\tau_p \tau_2} = 0$ $\quad (n_{th} = r_{th}\tau_2)$

imag. part: $\mp 2\alpha\omega_m \pm \frac{r_0}{n_{th}}\omega_m = 0 \Rightarrow \quad \alpha = \frac{r_0}{2n_{th}}$

$$\frac{r_0^2}{4n_{th}^2} - \omega_m^2 - \frac{r_0}{n_{th}} \overbrace{\frac{r_0}{2n_{th}}}^{\alpha} + \left(\frac{r_0}{r_{th}} - 1\right)\frac{1}{\tau_p \tau_2} = 0$$

$$\boxed{\omega_m = \sqrt{\left(\frac{r_0}{r_{th}} - 1\right)\frac{1}{\tau_p \tau_2} - \left(\frac{r_0}{2n_{th}}\right)^2} = \frac{1}{\tau_2}\sqrt{\left(\frac{r_0}{r_{th}} - 1\right)\frac{\tau_2}{\tau_p} - \left(\frac{r_0}{2r_{th}}\right)^2}} \quad (n_{th} = r_{th}\tau_2)$$

This resonant response (ringing) of the cavity limits the rate at which the system can respond to perturbations. Very important in diode lasers, for example.

Now treat the case of sinusoidal modulation of the pump

$$\frac{d^2 N_{p,1}}{d\tau^2} = \left(\frac{r_0}{r_{th}} - 1\right)\frac{\tau_p}{\tau_2}\underbrace{\left[r_1(t)\tau_p - N_{p,1}(t) - \frac{r_0 \tau_p n_1(t)}{n_{th}}\right]}_{\frac{dn_1}{dt}\ (3)}$$

see (3),(4)

$$\frac{d^2 N_{p,1}}{d\tau^2} = \left(\frac{r_0}{r_{th}} - 1\right)\frac{\tau_p^2}{\tau_2} r_1(t) - \left(\frac{r_0}{r_{th}} - 1\right)\frac{\tau_p}{\tau_2}\left(-N_{p,1}(t)\right) - \frac{r_0 \tau_p}{n_{th}}\frac{dN_{p,1}}{d\tau}$$

used (4): $\left(\frac{r_0}{r_{th}} - 1\right)\frac{\tau_p}{\tau_2} n_1 = \frac{dN_{p,1}}{d\tau}$

$$\frac{d^2 N_{p,1}}{d\tau^2} + \frac{r_0 \tau_p}{n_{th}}\frac{dN_{p,1}}{d\tau} + \left(\frac{r_0}{r_{th}} - 1\right)\frac{\tau_p}{\tau_2} N_{p,1}(t) = \left(\frac{r_0}{r_{th}} - 1\right)\frac{\tau_p^2}{\tau_2} r_1(t)$$

rescale $t = \tau\tau_p$

$$\boxed{\frac{d^2 N_{p,1}}{dt^2} + \frac{r_0}{n_{th}}\frac{dN_{p,1}}{dt} + \left(\frac{r_0}{r_{th}} - 1\right)\frac{1}{\tau_p \tau_2} N_{p,1}(t) = \left(\frac{r_0}{r_{th}} - 1\right)\frac{1}{\tau_2} r_1(t)}$$

for $r_1(t) = r_1 \exp\left(j\omega t \right)$

$N_{p,1}(t) = N_{p,1} \exp j\left(\omega t + \theta\right)$ $\quad N_{p,1}$ is real

$$\left[\left(j\omega \right)^2 + \frac{r_0}{n_{th}} \left(j\omega \right) + \left(\frac{r_0}{r_{th}} - 1 \right) \frac{1}{\tau_2 \tau_p} \right] N_{p,1}\, e^{j\theta} = \left(\frac{r_0}{r_{th}} - 1 \right) \frac{r_1}{\tau_2}$$

$$\left| N_{p,1}\, e^{i\theta} \right| = \boxed{ N_{p,1} = \frac{ \left(\frac{r_0}{r_{th}} - 1 \right) \frac{r_1}{\tau_2} }{ \sqrt{ \left[\left(\frac{r_0}{r_{th}} - 1 \right) \frac{1}{\tau_2 \tau_p} - \omega^2 \right]^2 + \frac{r_0^2 \omega^2}{r_{th}^2 \tau_2^2} } } }$$

let $\omega_0 = \left(\frac{r_0}{r_{th}} - 1 \right)^{1/2} \frac{1}{\sqrt{\tau_2 \tau_p}}$ $\quad \Rightarrow \quad$ resonant response at $\omega \simeq \omega_0$

$$\boxed{ N_{p,1} = \frac{ \omega_0^2 r_1 \tau_p }{ \sqrt{ \left(\omega_0^2 - \omega^2 \right)^2 + \frac{r_0^2 \omega^2}{r_{th}^2 \tau_2^2} } } }$$