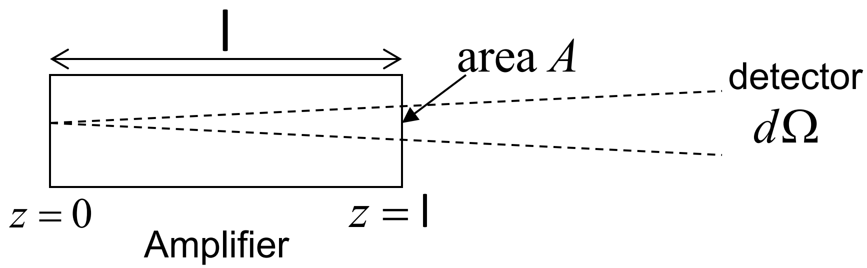
Amplified Spontaneous Emission (ASE)

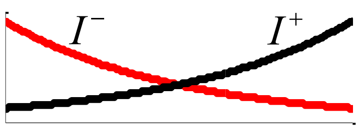
We'll show that ASE: i) limits the gain of an amplifier
ii) creates background radiation (broad-band)



Notation:

$$I_v = \int I(v)dv$$

$$\rho_v = \int \rho(v)dv$$



(Spontaneous emission is continuously added to I^+ along z and, simultaneously, stimulated emission amplifies the power from previous lengths)

$$\frac{dI^+(z)}{dz} = \gamma(v)I^+(z) + \text{spont. noise}$$

$$\frac{d}{dz}\Big[\underbrace{I^+(v,z)dv}_{\text{total intensity in band } dv}\Big] = \underbrace{\gamma_0(v)}_{\text{small-signal assumption}}\Big[I^+(v,z)dv\Big] + hvA_{21}N_2 g(v)dv\underbrace{\frac{d\Omega}{4\pi}}_{\text{fraction that reaches detector}} \qquad (1)$$

(spectral intensity: $I^+(v,z)$)

$$I^+(v,0)=0$$

$$I^+(v,l)=\frac{hvA_{21}N_2g(v)}{\gamma_0(v)}\left[\exp\left(\gamma_0(v)l\right)-1\right]\frac{d\Omega}{4\pi}$$

$$G_0(v)=\exp\left(\gamma_0(v)l\right) \qquad \gamma_0(v)=A_{21}\frac{\lambda_0^2}{8\pi n^2}g(v)\left(N_2-\frac{g_2}{g_1}N_1\right)$$

$$I^+(v,l)=\frac{8\pi hvn^2}{\lambda_0^2}\frac{N_2}{N_2-\frac{g_2}{g_1}N_1}\left[G_0(v)-1\right]\frac{d\Omega}{4\pi}$$

$$\boxed{I^+(v,l)=\frac{8\pi hv^3n^2}{c^2}\frac{N_2}{N_2-\frac{g_2}{g_1}N_1}\left[G_0(v)-1\right]\frac{d\Omega}{4\pi}}$$

(valid for both amplifier, $G_0>1$ and attenuator, $G_0<1$)

ASE intensity

(1) Optically Thin Medium

recall: $\gamma_0 = A_{21} \frac{\lambda^2}{8\pi n^2} g(\nu)\left(N_2 - \frac{g_2}{g_1} N_1 \right)$

$$G_0(\nu) - 1 = \exp\left(\gamma_0(\nu) l\right) - 1 \; ; \;\; \gamma_0 l$$

$$I^+(\nu, l) = h\nu A_{21} N_2 l\, g(\nu) \frac{d\Omega}{4\pi} = h\nu A_{21} N_2 g(\nu) \frac{d\Omega}{4\pi} \frac{V}{A}$$

$$N_2 \frac{V}{A} = N_2 l \; = \text{\# of atoms in length } l \big/ \text{unit area}$$

Same result as 1-st term in (1) is ignored!

No amplification, just random emission accumulated along the length.

$N_2 < \frac{g_2}{g_1} N_1$ - attenuation

(2) Thermal Population

use $\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-h\nu/kT}$

$$\boxed{I^+(\nu, l) = \frac{8\pi h \nu^3 n^2}{c^2} \frac{N_2}{N_2 - \frac{g_2}{g_1} N_1} \left[G_0(\nu) - 1 \right] \frac{d\Omega}{4\pi}}$$

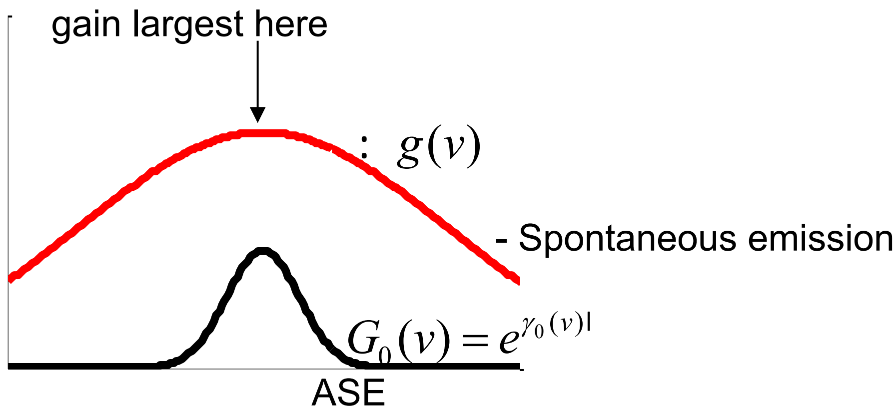
$$I^+(\nu, l) = \underbrace{\left[\frac{8\pi h \nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1} \right]}_{\text{Planck formula}} \frac{d\Omega}{4\pi} \underbrace{\left(1 - e^{-|\gamma_0(\nu)| l} \right)}_{\text{Amount which is not absorbed by the blackbody (i.e. available to the outside world)}}$$

Kirchoff's radiation law:

A body can emit only as much blackbody radiation at a frequency ν as it can absorb.

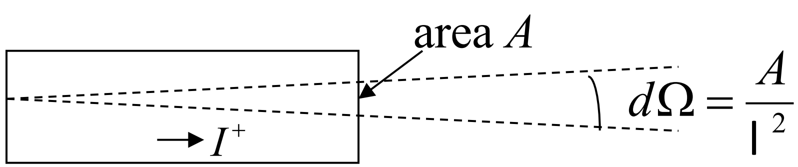
Spectral narrowing of amplified spontaneous emission

Before saturation the ASE tends to spectrally narrow! (at low intensity)



At saturation, linewidth broadens again, amplification center decreases, wings catchup

- Find maximum gain of an amplifier imposed by ASE:



If the ASE saturates the population inversion, that inversion energy cannot be extracted by an externally injected coherent laser signal.

recall gain saturation for homogeneous line:

$$\gamma(\nu) = \frac{\gamma_0(\nu)}{1+\frac{\tau_2}{h\nu}\sigma(\nu)I_\nu^+} \qquad (\text{for } \tau_1 << \tau_2)$$

When the gain is sufficiently high, the ASE depletes N_2!

$$\frac{I_\nu^+}{I_S} = \frac{\tau_2}{h\nu}\sigma(\nu)I_\nu^+ \gg 1$$

Where I_ν is the ASE intensity

The ASE is broadband!

$$I^+(\nu) = \frac{8\pi h\nu^3}{c^2}\frac{N_2}{N_2 - \frac{g_2}{g_1}N_1}\left[e^{\gamma_0(\nu)\ell} - 1\right]\frac{d\Omega}{4\pi}$$

$$\sigma(\nu) = A_{21}\frac{\lambda_0^2}{8\pi n^2}g(\nu)$$

Use $I_\nu^+ \simeq I(\nu_0)\Delta\nu$, where $\Delta\nu \sim \frac{1}{g(\nu_0)}$

$$g(\nu) = \frac{(\Delta\nu/2\pi)}{(\nu-\nu_0)^2 + (\Delta\nu/2)^2}$$

Then

$$\frac{\tau_2}{h\nu}\sigma(\nu_0)I_\nu^+ = \frac{\tau_2}{h\nu}\left[A_{21}\frac{\lambda_0^2}{8\pi n^2}g(\nu_0)\right]\frac{8\pi h\nu_0^3}{c^2}\frac{N_2}{N_2 - \frac{g_2}{g_1}N_1}\left[e^{\gamma_0(\nu)\ell} - 1\right]\frac{d\Omega}{4\pi}\Delta\nu \simeq 1 \qquad (2)$$

assumptions:

$$g(\nu)\Delta\nu \sim 1$$

$$\tau_2 A_{21} \sim 1 \qquad (\tau_2 \sim \tau_{21})$$

$$\frac{N_2}{N_2 - \frac{g_2}{g_1} N_1} \sim 1 \qquad (N_2 >> N_1 \text{ - ideal for lasing})$$

$$\frac{\tau_2}{h\nu}\left[A_{21}\frac{\lambda_0^2}{8\pi n^2} g(\nu_0)\right]\frac{8\pi h\nu_0^3}{c^2}\frac{N_2}{N_2 - \frac{g_2}{g_1}N_1} \simeq \tau_{21} A_{21} g(\nu_0)\frac{N_2}{N_2 - \frac{g_2}{g_1}N_1} \sim \frac{1}{\Delta\nu}$$

use (2): $$\frac{\tau_2}{h\nu}\left[A_{21}\frac{\lambda_0^2}{8\pi n^2} g(\nu_0)\right]\frac{8\pi h\nu_0^3}{c^2}\frac{N_2}{N_2 - \frac{g_2}{g_1}N_1}\left[e^{\gamma_0(\nu)\ell} - 1\right]\frac{d\Omega}{4\pi}\Delta\nu \simeq 1$$

$$\left[e^{\gamma_0(\nu)\ell} - 1\right]\frac{d\Omega}{4\pi} \simeq 1$$

$$\left[G_0(\nu_0) - 1\right]\frac{A}{4\pi\ell^2} \simeq 1$$

$$\boxed{G_{0_{\max}}(\nu_0) \sim 1 + \frac{4\pi\ell^2}{A}}$$

ASE limitation on gain

The limitation is on the small-signal gain only!
It does not limit the amount of energy that can be extracted.

We can extract as much power as the effective pump rate can supply, provided that the ASE does not use it first!

Therefore, for many high-energy applications (laser fusion) one prefers a low gain but high energy storage capability in the amplifiers.