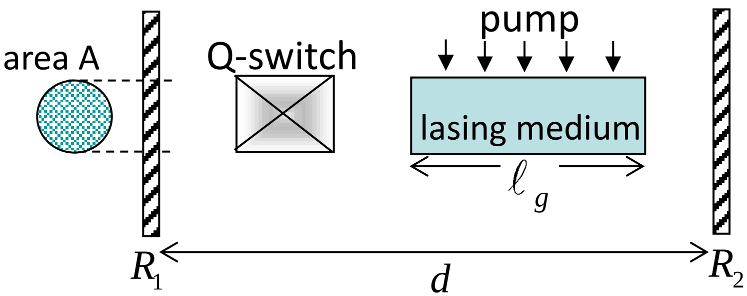
Q - Switched Lasers

Capable of generating 10's of MW peak power in 10 ns pulses

Principle of operation

1) Active medium is pumped while lasing is suppressed, Gain becomes very large.
2) Optical switch is opened, allowing optical power to build up from spontaneous emission.



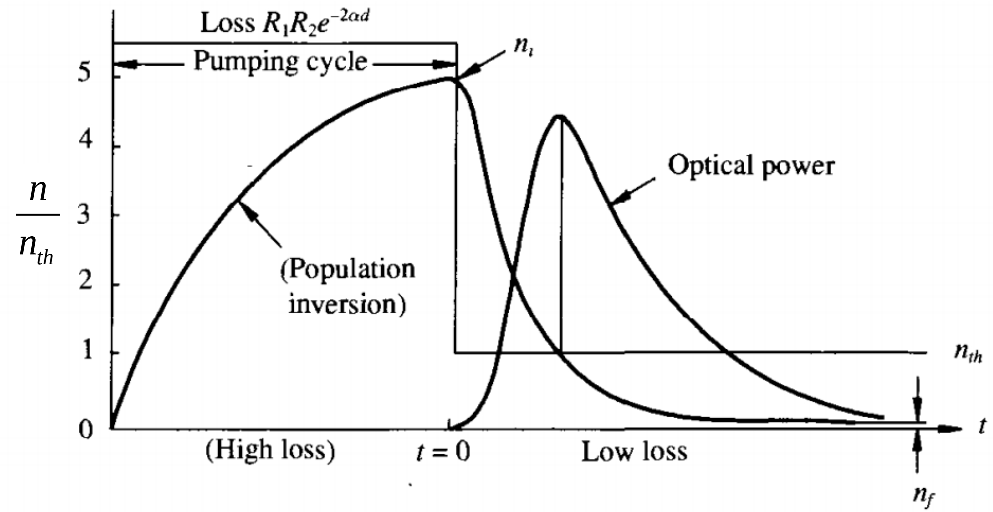


FIGURE 9.11. Guess at the sequence of events during a Q switch.

For net single-pass gain 5 [i.e. $\left(R_1R_2\right)^{\frac{1}{2}} e^{\gamma_0 \ell} = 5$], in 5 round trips, the photon flux would be amplified by $5^{10} \sim 10^7 \Rightarrow$ fast depleting of n_i after $t = 0$

$n \equiv \left(N_1 - N_2\right) A \, \ell_g$ - the total # of inverted atoms in the cavity interacting with an optical mode of cross-sectional area A

For a round trip, the number of photons, N_p, increased by $\mathrm{e}^{\underbrace{2(N_2-N_1)\sigma\ell_g}_{\gamma}}$ and decreased by $R_1R_2e^{-2\alpha d}$ (loss)$\Longrightarrow$

$$\frac{dN_p}{dt}=\left\{\frac{\left(R_1R_2e^{-2\alpha\ell}\right)\mathrm{e}^{2(N_2-N_1)\sigma\ell_g}-1}{\tau_{RT}}\right\}N_p$$

$$\left[N_2(t)-N_1(t)\right]\sigma\ell_g \equiv g(t)$$

- the line integrated gain

$$e^{-2g_{th}} \equiv R_1R_2e^{-2\alpha d}$$

- the threshold value of g

$$\underbrace{\tau_p}=\frac{\tau_{RT}}{1-R_1R_2e^{-2\alpha d}}=\frac{\tau_{RT}}{1-e^{-2g_{th}}}=\frac{\tau_{RT}}{1-\underbrace{S}}$$

photon lifetime for a passive cavity

fraction of photons surviving in a round-trip

$$\frac{dN_p}{dt}=\frac{\mathrm{e}^{2[g(t)-g_{th}]}-1}{1-e^{-2g_{th}}}\frac{N_p}{\tau_p}$$

Taylor series:

$$\mathrm{e}^x \approx 1+x+\ldots$$

$$\frac{dN_p}{dt}=\left[\frac{1+2\left[g(t)-g_{th}\right]-1}{1-\left(1-2g_{th}\right)}\right]\frac{N_p}{\tau_p}=\frac{N_p}{\tau_p}\left\{\frac{g(t)}{g_{th}}-1\right\}$$

$$\tau_p=\frac{\tau_{RT}}{1-e^{-2g_{th}}}\approx\frac{\tau_{RT}}{2g_{th}}\,(*)$$

$$g=\left(N_2-N_1\right)\sigma\ell_g$$

$$n=\left(N_2-N_1\right)A\,\ell_g$$

$$\boxed{\frac{dN_p}{dt}=\frac{N_p}{\tau_p}\left(\frac{n}{n_{th}}-1\right)}$$

- same as obtained previously

Timescale for the build up and decay of photons ~ a few $\tau_p (\square \ \tau_2$ or pump time)

$$\frac{dN_2}{dt} = -\frac{\sigma\left(I^+ + I^-\right)}{hv}\left(N_2 - N_1\right)$$

$$\frac{dN_1}{dt} = +\frac{\sigma\left(I^+ + I^-\right)}{hv}\left(N_2 - N_1\right)$$

subtract and $\times \ A\ell_g$

$$\frac{dn}{dt} = -2\left[\left(N_2 - N_1\right)\sigma\ell_g\right]\frac{\left(I^+ + I^-\right)A}{hv} \times \frac{\tau_{RT}}{2} \times \frac{2}{\tau_{RT}}$$

$$N_p = \frac{\left(I^+ + I^-\right)A}{hv} \times \frac{\tau_{RT}}{2} \qquad I = \frac{I^+ + I^-}{2}$$

$$g(t) = \left(N_2 - N_1\right)\sigma_{eq}$$

$$g = \left(N_2 - N_1\right)\sigma\ell_g$$

$$\frac{dn}{dt} = 4g(t)\frac{N_p}{\tau_{RT}} \qquad \frac{g(t)}{g_{th}} = \frac{n}{n_{th}}$$

$$\tau_{RT} \approx \tau_p (2g_{th}) - see(*)$$

$$\boxed{\begin{aligned} \frac{dn}{dt} &= -2\frac{n}{n_{th}}\frac{N_p}{\tau_p} \\ \frac{dN_p}{dt} &= \frac{N_p}{\tau_p}\left(\frac{n}{n_{th}} - 1\right) \end{aligned}}$$

if the photons increase by 1, then the inversion must decrease by 2

- same as we had except: no external pump (r), no relaxation (τ_2), and the factor of 2 $\left(\text{assumed that } N_2 \ \square \ N_1\right)$

$$T = \frac{t}{\tau_p}$$ - new variable [$T = \tau$ in previous notation]

$$\frac{dN_p}{dT} = N_p\left(\frac{n}{n_{th}} - 1\right) \qquad \frac{dn}{dT} = -2\frac{n}{n_{th}}N_p$$

$$\Rightarrow \frac{dN_p}{dn} = \frac{1}{2}\left(\frac{n_{th}}{n} - 1\right)$$

$$\Rightarrow \int_{N_{p,i}}^{N_p} dN_p = \frac{1}{2}\int_{n_i}^{n}\left(\frac{n_{th}}{n} - 1\right) dn$$

$$N_p - N_{p,i} = \frac{1}{2} n_{th} \ln\left(\frac{n}{n_i}\right) - \left(n - n_i\right)$$

long before the pulse, $N_{p,i} = 0$

$$N_p = \frac{1}{2}\left[n_{th} \ln\left(\frac{n}{n_i}\right) - \left(n - n_i\right)\right]$$

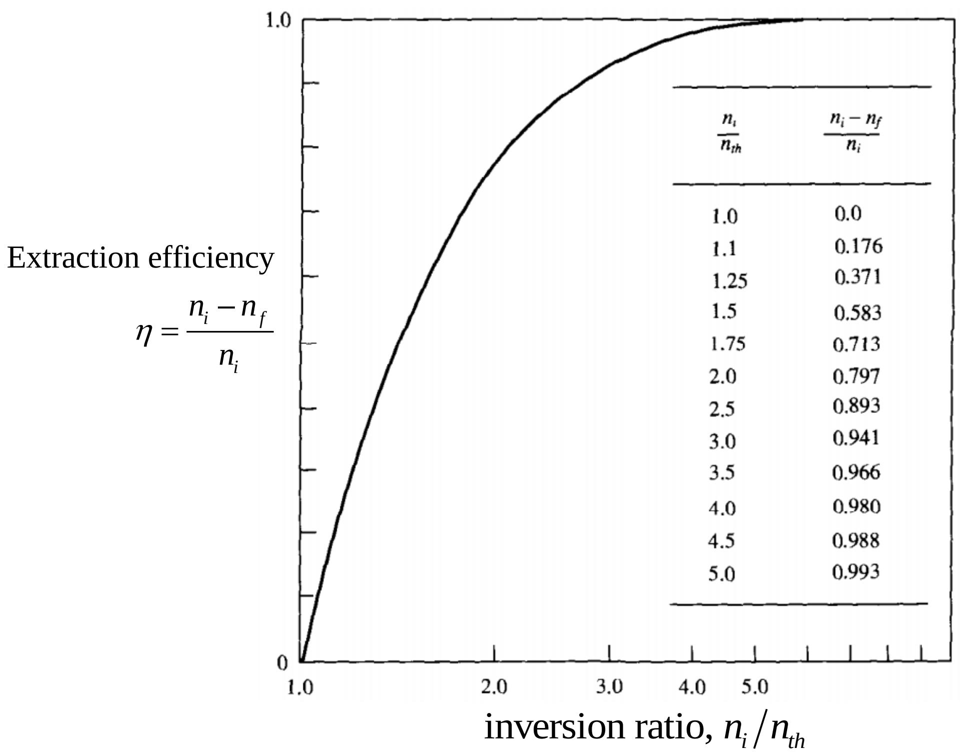
How much of the population inversion (stored energy) is in the medium?

For t ? pulse duration, $N_p = 0$ $n = n_f$

$$0 = \frac{1}{2}\left[n_{th} \ln\left(\frac{n_f}{n_i} \right) - \left(n_f - n_i \right) \right]$$

$$\frac{n_f}{n_i} = \exp\left[-\frac{n_i - n_f}{n_{th}} \right]$$ - transcendental equation

extraction efficiency $\frac{n_i - n_f}{n_i} \rightarrow 1$ when $\frac{n_i}{n_{th}}$ increases



$\frac{n_i}{n_{th}}$	$\frac{n_i - n_f}{n_i}$
1.0	0.0
1.1	0.176
1.25	0.371
1.5	0.583
1.75	0.713
2.0	0.797
2.5	0.893
3.0	0.941
3.5	0.966
4.0	0.980
4.5	0.988
5.0	0.993

FIGURE 9.13. The energy extraction efficiency for a Q switched pulse.

$$N_p = \frac{1}{2}\left[n_{th} \ln\left(\frac{n}{n_i} \right) - (n - n_i) \right]$$

$$P_{out} = \frac{N_p h\nu}{\tau_p} = \frac{h\nu}{2\tau_p}\left[n_{th} \ln\left(\frac{n}{n_i} \right) - (n - n_i) \right]$$

To find the peak power, we would usually let $\frac{\partial P}{\partial t} = 0$ But since n is a monotonic function of time, we can set, instead, $\frac{\partial P}{\partial n} = 0$

$$\frac{\partial P_{out}}{\partial n} = \frac{h\nu}{2\tau_p}\left[-1 + \frac{n_{th}}{n} \right] = 0$$

- see Fig on slide 1

$$\Rightarrow n = n_{th}$$

(makes sense, since gain = losses when $n = n_{th}$, so power neither increases nor decreases.)

$$P_{\max} = \frac{h\nu}{2\tau_p}\left[(n_i - n_{th}) - n_{th} \ln\left(\frac{n_i}{n_{th}} \right) \right]$$

for n_i □ n_{th}

$$P_{\max} = \frac{n_i h\nu}{2\tau_p}$$

$n_i/2$ = max # of photons
Build up time □ τ_p , so most of the photons are still present)
Rise time can be extremely short
Fall time $\sim \tau_p$

$$n_i = \left(N_{2i} - N_{1i}\right) A\ell_g \qquad n_f = \left(N_{2f} - N_{1f}\right) A\ell_g$$

$$\text{but } \left(N_{2i} - N_{2f}\right) A\ell_g = N_p = \left(N_{1f} - N_{1i}\right) A\ell_g$$

$$\Rightarrow n_i - n_f = \left\{\left(N_{2i} - N_{1f}\right) - \left(N_{2f} - N_{1f}\right)\right\} A\ell_g = 2N_p$$

$$N_p = \frac{n_i - n_f}{2}$$

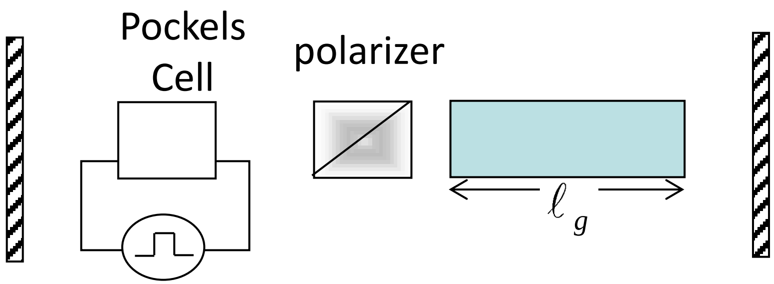
<u>Total energy in pulse</u> $= \boxed{\dfrac{n_i - n_f}{2} h\nu = W}$ $\qquad \boxed{\Delta t \,\square\, \dfrac{W}{P_{\max}}}$

Methods for Q - Switching

1) Rotating Mirror – Q is high when mirrors are alligned

2) Saturable absorber – in a cavity, Q is high after the absorber bleaches

3) Electro-optic Q - switch



Q is low when voltage is applied