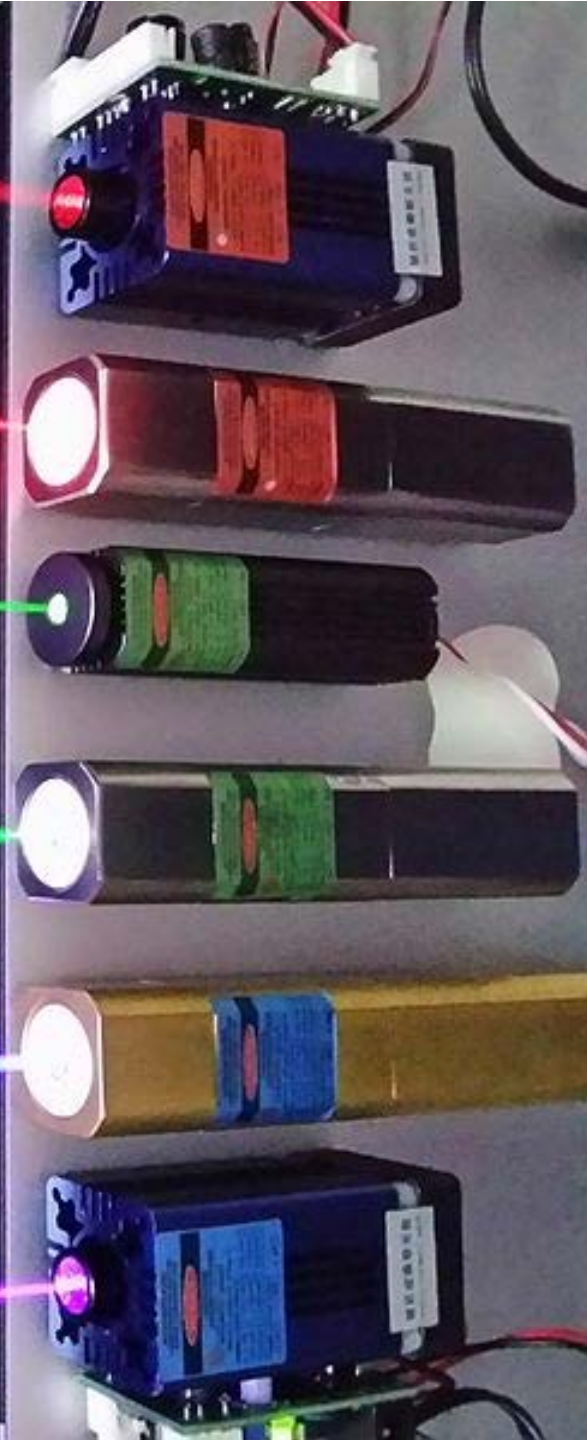


# Fiber Optic Communications

## Lecture 4: Lasers

- Laser Principles
- Laser Transitions
- General Rate Equations



# LASER

Three essential parts of a laser:

## 1. Active/gain medium

Solid, liquid, gas

Three-level, four-level, or more

## 2. Optical feedback (optical cavities)

## 3. Pumping

Optical pumping

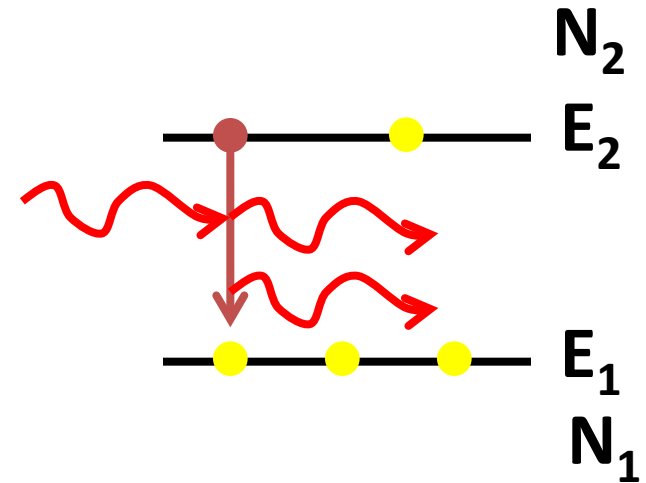
Electrical pumping

Electrical discharge

Current injection in semiconductors

Pumping by chemical reaction

Kinetic energy of charges particles



# Amplifier Pumping

- Like other amplifiers, lasers require an external source of power to provide the energy required to augment the input signal.
- This “pump” excites the electrons in the atoms, causing them to move from lower to higher atomic energy levels.
- In order to achieve amplification, the pump **must** provide population inversion
- Pumps can be electrical, optical, or chemical



# Theory of Laser Amplification

- A two level system has electron populations  $N_1$  and  $N_2$ . Define  $N = N_2 - N_1$
- If  $N > 0$  ( $N_2 > N_1$ ), **population inversion** exists and the medium can act as an *amplifier*
- If  $N < 0$  ( $N_2 < N_1$ ), the medium acts as an attenuator and the photon-flux decreases



# Gain Coefficient

Photon flux increase:

$$\phi(z) + d\phi(z)$$

$$d\phi = NW_i dz$$

$$\frac{d\phi(z)}{dz} = \gamma(\nu) \phi(z)$$

Where gain coefficient:

$$\gamma(\nu) = N\sigma(\nu) = N \frac{\lambda^2}{8\pi t_{sp}} g(\nu)$$

Optical Intensity:

$$I(z) = h\nu\phi(z)$$

$$I(z) = I(0) \exp[\gamma(\nu) z]$$

# Gain and Loss

Gain coefficient

$$\gamma(\nu) = N\sigma(\nu) = N\frac{\lambda^2}{8\pi t_{sp}} g(\nu).$$

In thermal equilibrium,  $N_1 \gg N_2$  (media are normally lossy)

Net loss of photons if  $N_1 > N_2$  (medium is lossy,  $\gamma > 0$ )

Net gain of photons if  $N_1 < N_2$  (medium is amplifying,  $\alpha < 0$ )

Population inversion if  $N_1 < N_2$

Active medium, gain medium:  $\alpha < 0$ ,  $\gamma = -\alpha$

$$\frac{N_2}{N_1} = e^{-(E_2 - E_1)/kT}$$

# Gain Coefficient

- The gain in the photon-flux density per unit length of the medium:

$$\gamma(\nu) = N\sigma(\nu) = N\frac{\lambda^2}{8\pi t_{sp}}g(\nu).$$

- The overall gain of the laser amplifier is defined as the ratio of the photon-flux density at the output to the photon-flux density at the input such that

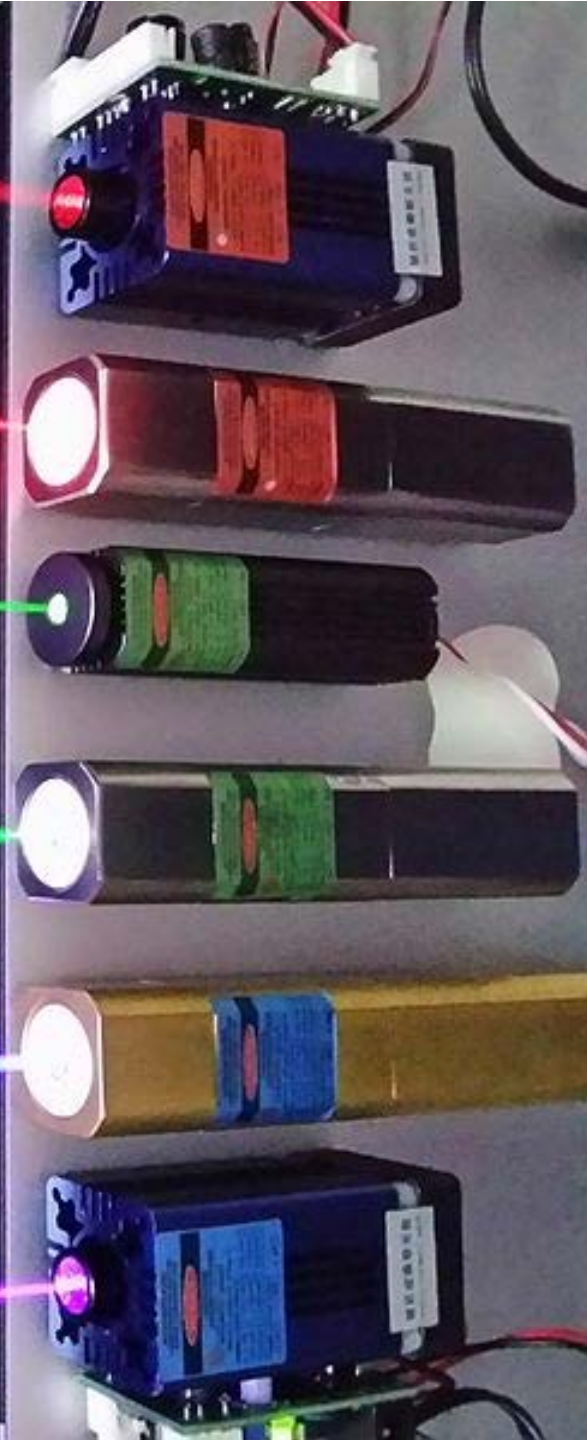
$$G(\nu) = \exp[\gamma(\nu)d].$$



# Fiber Optic Communications

## Lecture 4: Lasers

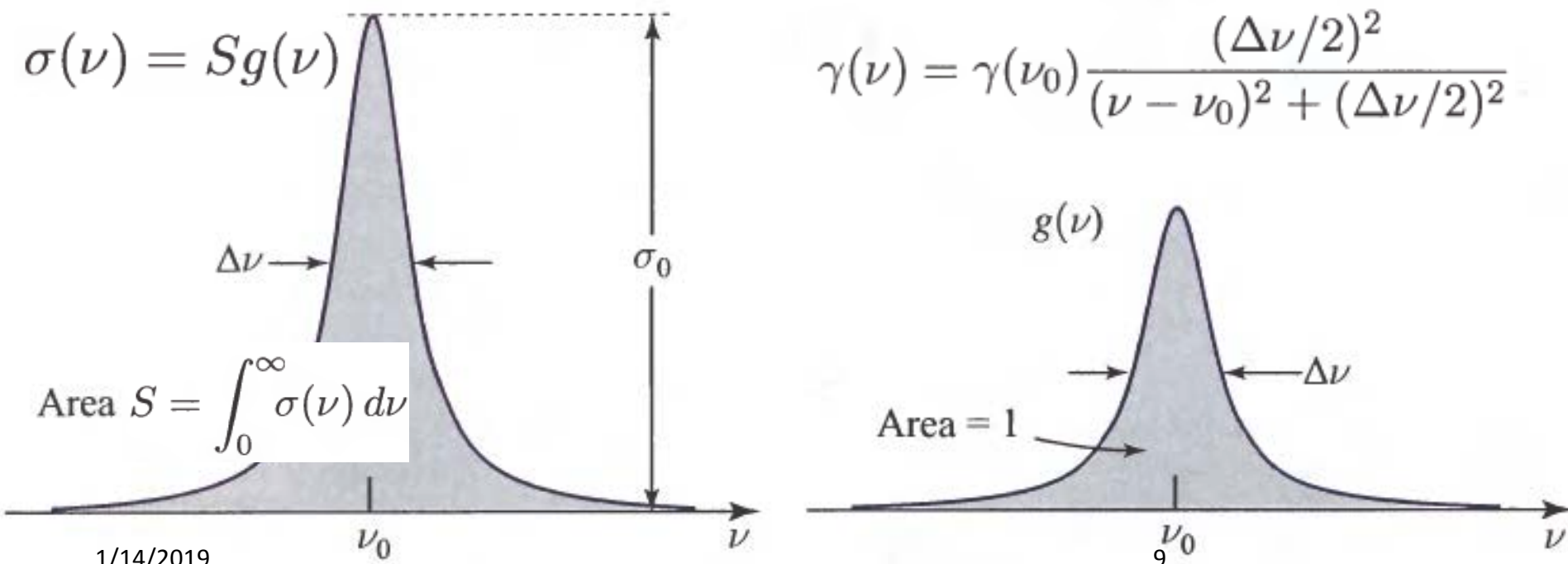
- Laser Principles
- Laser Transitions
- General Rate Equations





# Lineshape Function

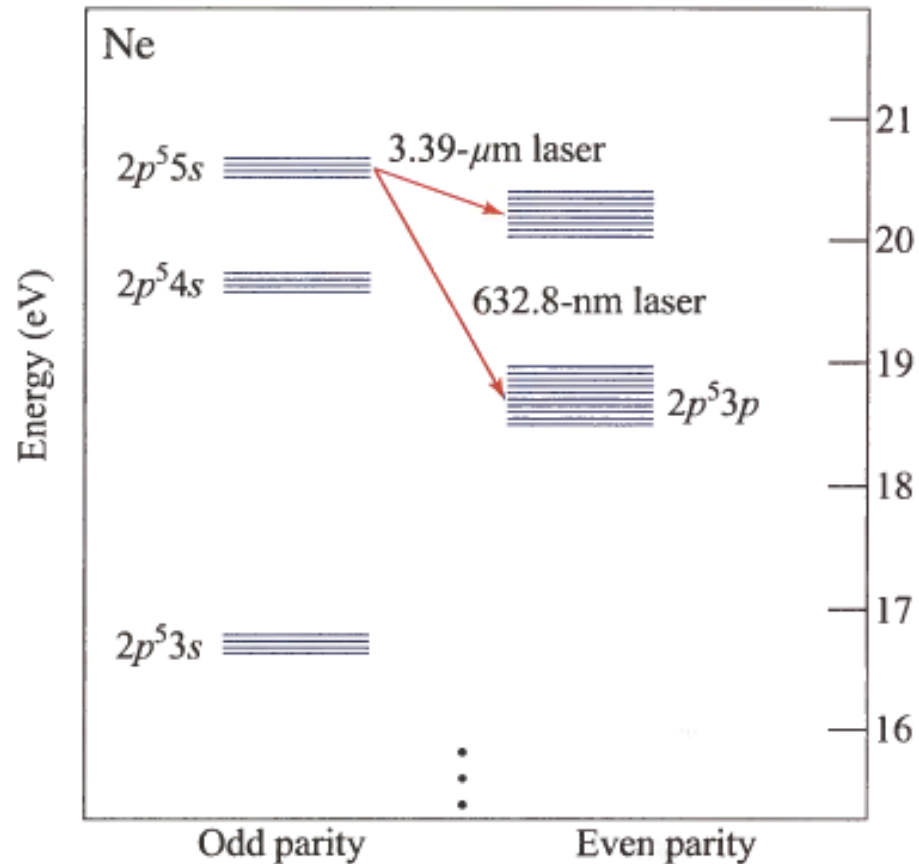
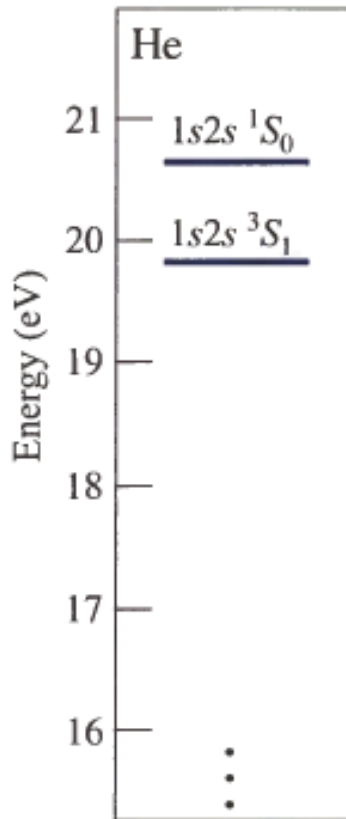
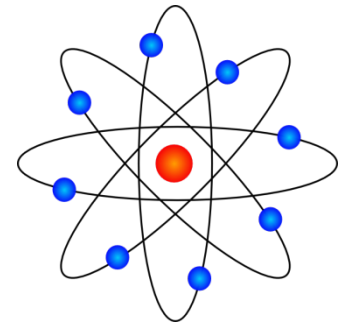
- Transition cross-section characterizes the interaction of the atom with the radiation. This is known as the **transition strength** or **oscillator strength**
- Normalized **lineshape function**  $g(\nu)$
- **Linewidth**  $\Delta\nu$  is the Full Width at Half Max (FWHM)



# Atoms

Discrete Energy values

$$E_n = -\frac{M_r Z^2 e^4}{(4\pi\epsilon_0)^2 2\hbar^2} \frac{1}{n^2}, \quad n = 1, 2, 3, \dots,$$



# Energy Levels

Atoms:

Nucleus of charge  $+Ze$

$Z$  – atomic number

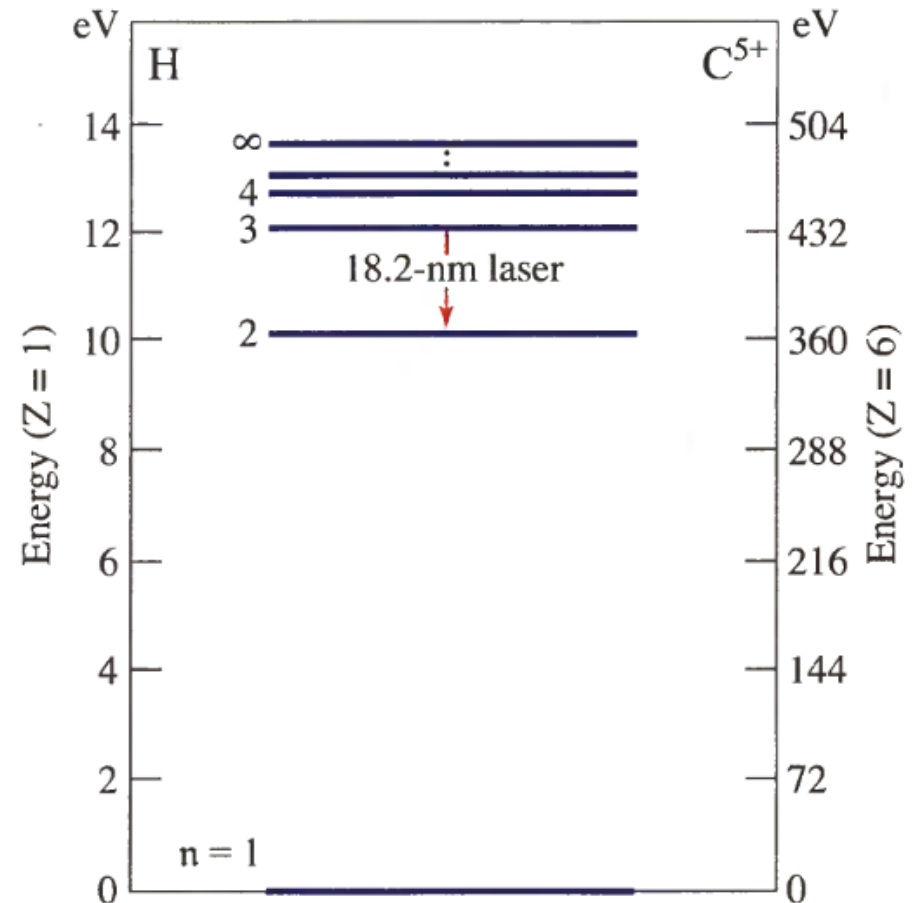
Discrete energy levels

Transitions

**Thermal equilibrium:**

Random transitions –

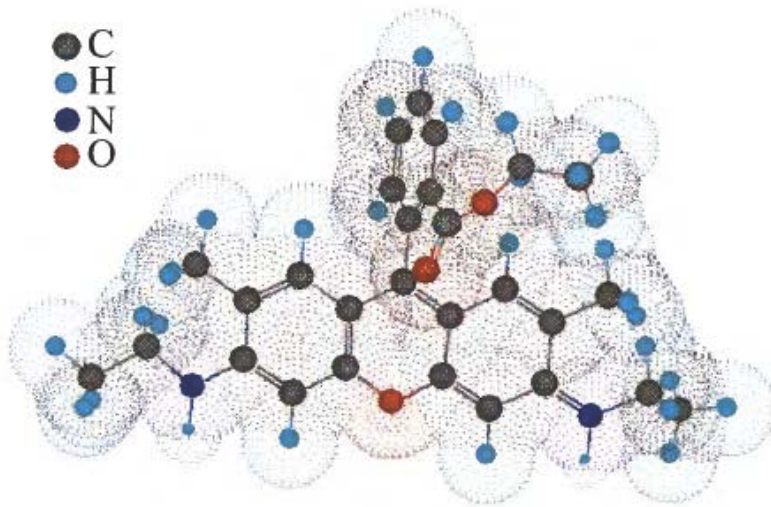
Temperature is the principal determinant of behaviour and the fluctuations in energy-level occupancy



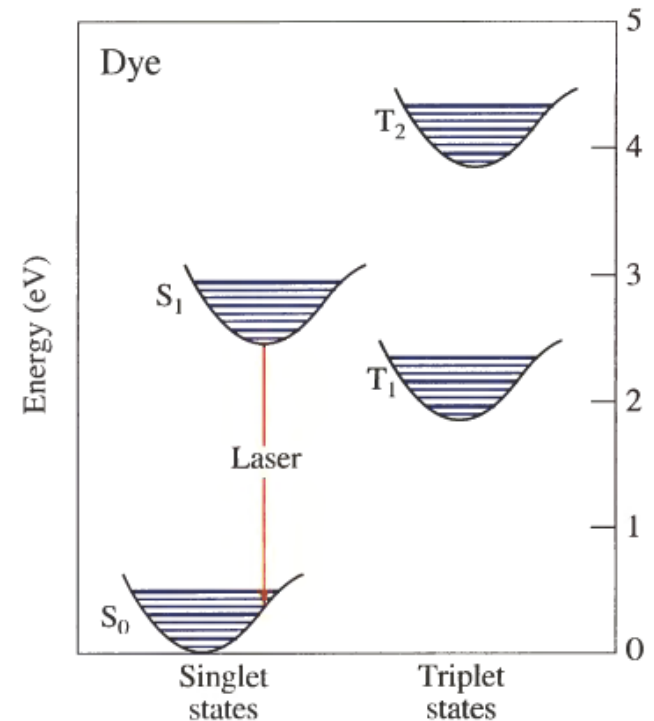


# Solids

- Atoms or molecules can form a crystal lattice
- Energy Levels are determined by the atomic orbitals plus the neighboring lattice atoms (tight-binding picture)
- Energy Bands – When many atoms come into close proximity

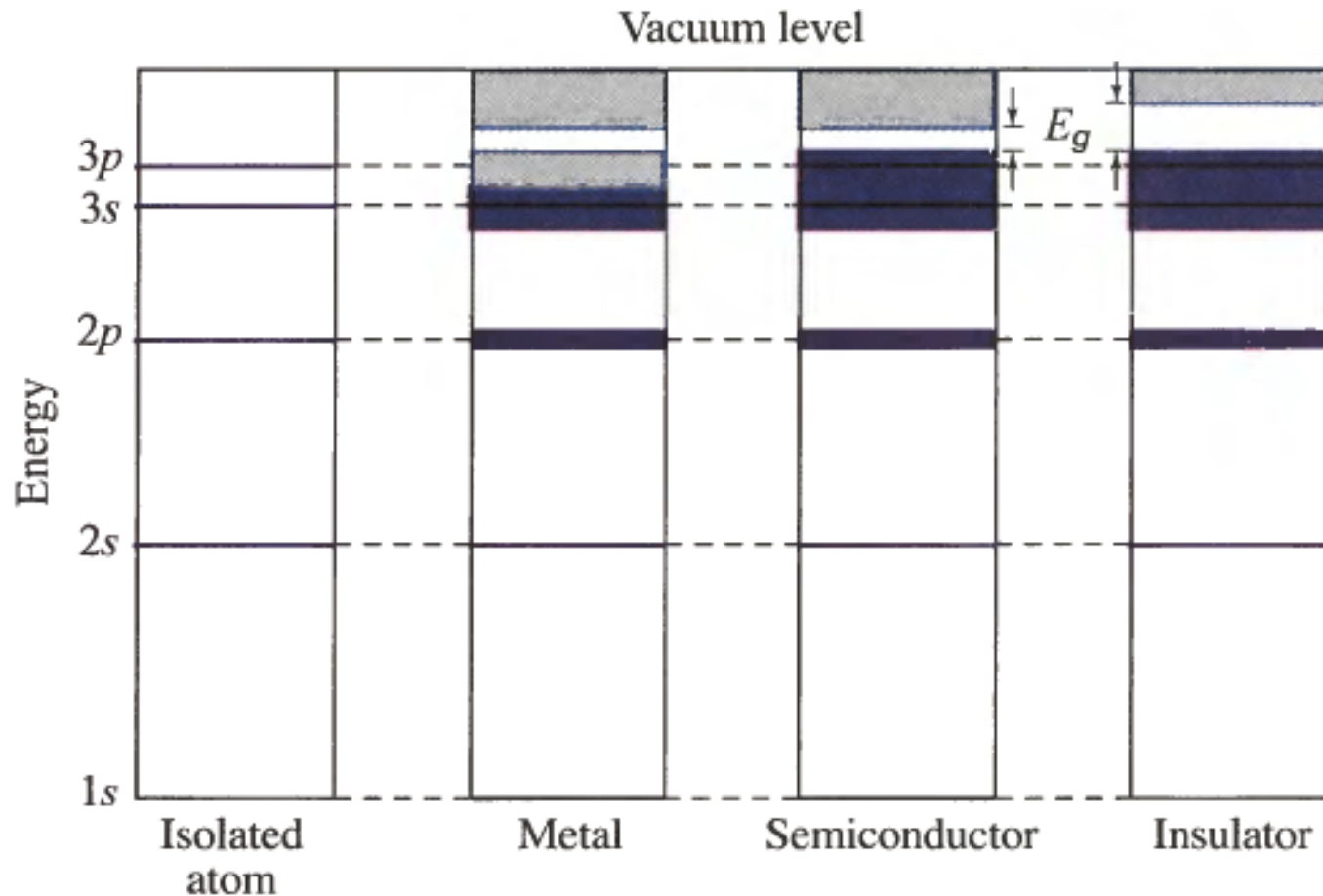


Rhodamine-6G



# Energy Bands

Recall the definitions of conduction band, valence bands, forbidden bands, and bandgap energy



# Examples

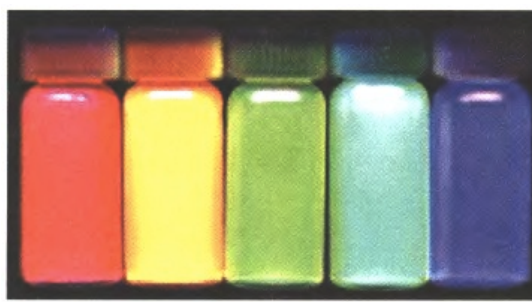
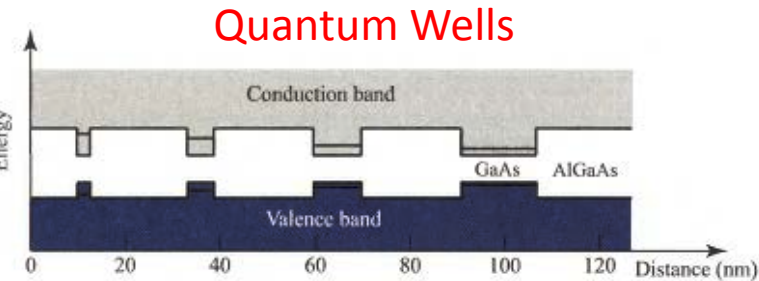
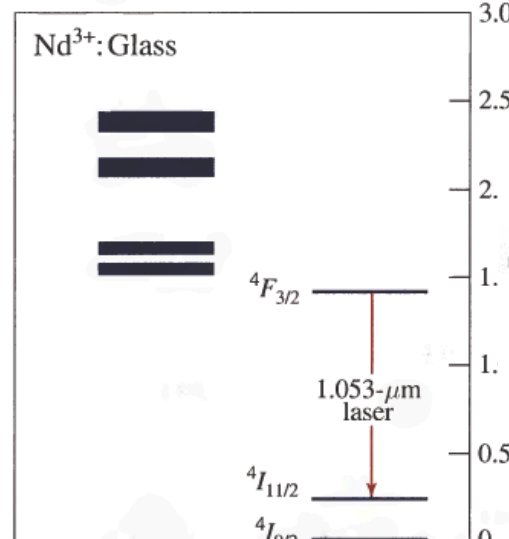
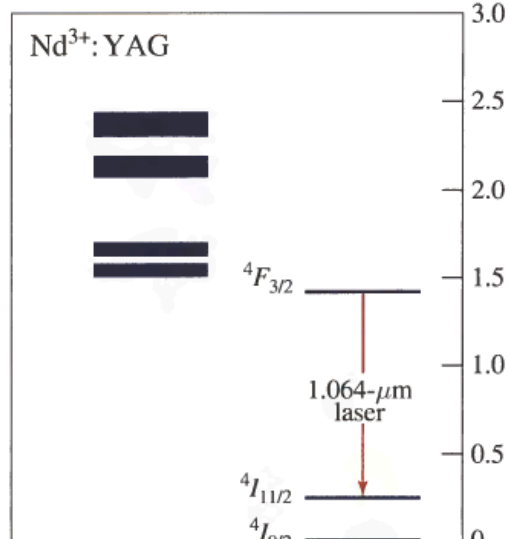
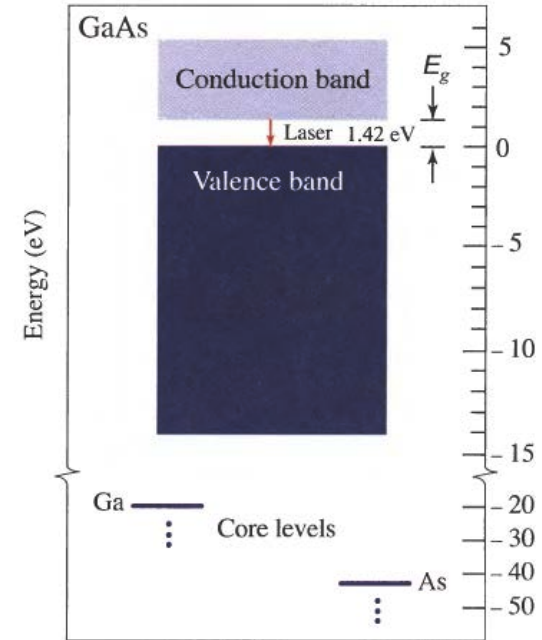
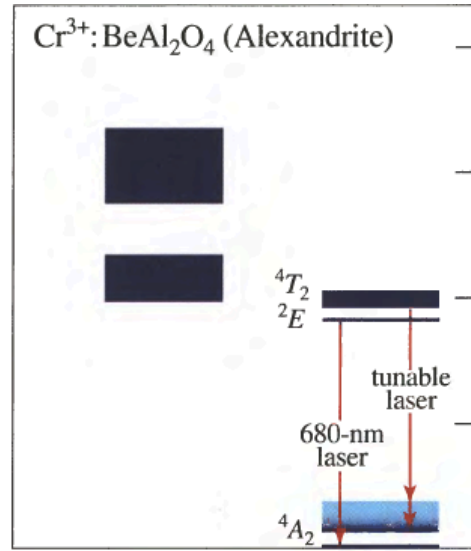
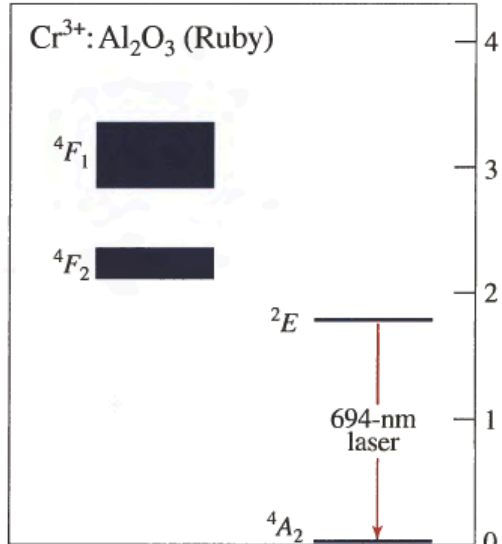


Photo Courtesy Dong-Kyun Seo,  
Arizona State University

## Doped Dielectrics

## Semiconductors



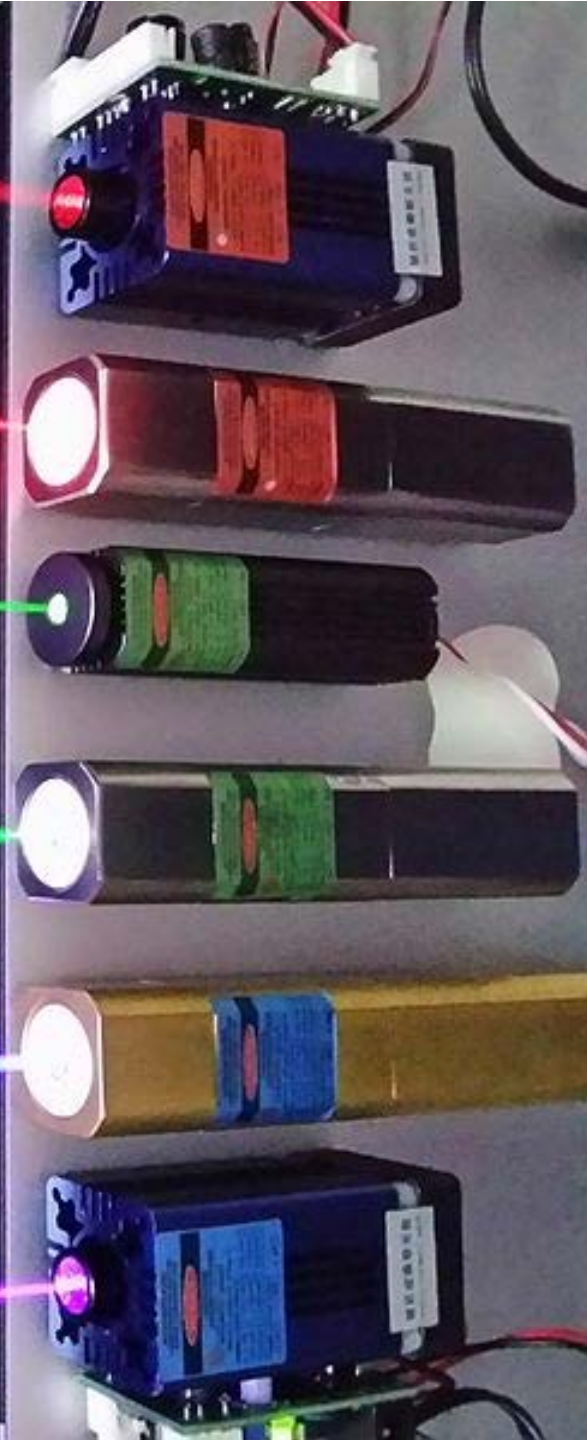
Quantum wires, quantum dots, etc...



# Fiber Optic Communications

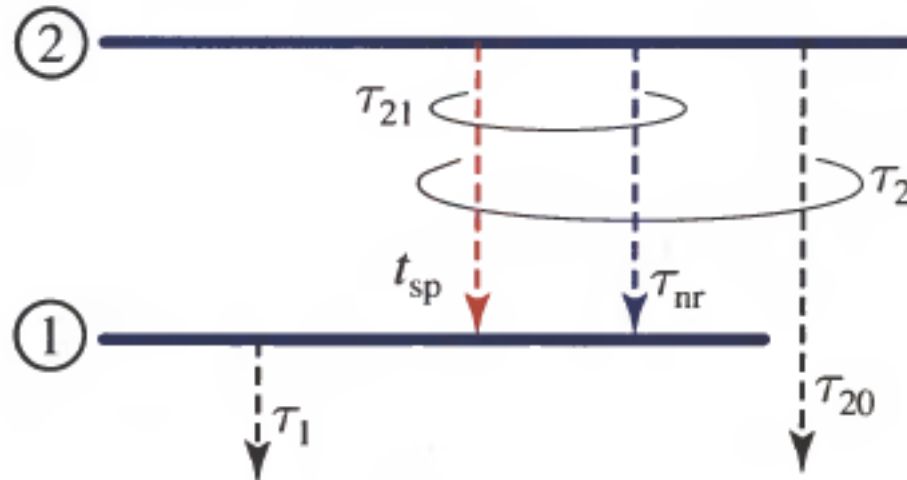
## Lecture 4: Lasers

- Laser Principles
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- General Rate Equations



# Rate Equations

- **Rate Equations** describe the rates of change in population densities  $N_1$  &  $N_2$  as the result of pumping, radiative, and non-radiative transitions



Electrons in an unpumped system will ultimately decay to lower energy levels unless there is a pump pushing them back up to higher energies



# Rate equations of isolated two-level system pumped by a light source

Isolated two-level system:

$$N_1 + N_2 = N_t$$

Interaction with light

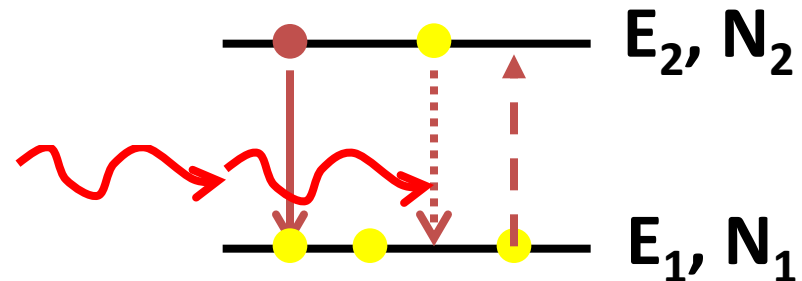
$$\frac{dN_2}{dt} = -\frac{N_2}{t_{sp}} + \frac{\bar{n} N_1}{t_{sp}} - \frac{\bar{n} N_2}{t_{sp}}$$

Define (population inversion)

$$N(t) = N_2(t) - N_1(t)$$

$$N_1(t) = \frac{1}{2} (N_t - N(t))$$

$$N_2(t) = \frac{1}{2} (N_t + N(t))$$





Differential equation in  $\Delta N(t)$

$$\frac{dN(t)}{dt} = -\left(\frac{1}{t_{sp}} + \frac{2n}{t_{sp}}\right)N(t) - \frac{N_t}{t_{sp}}$$

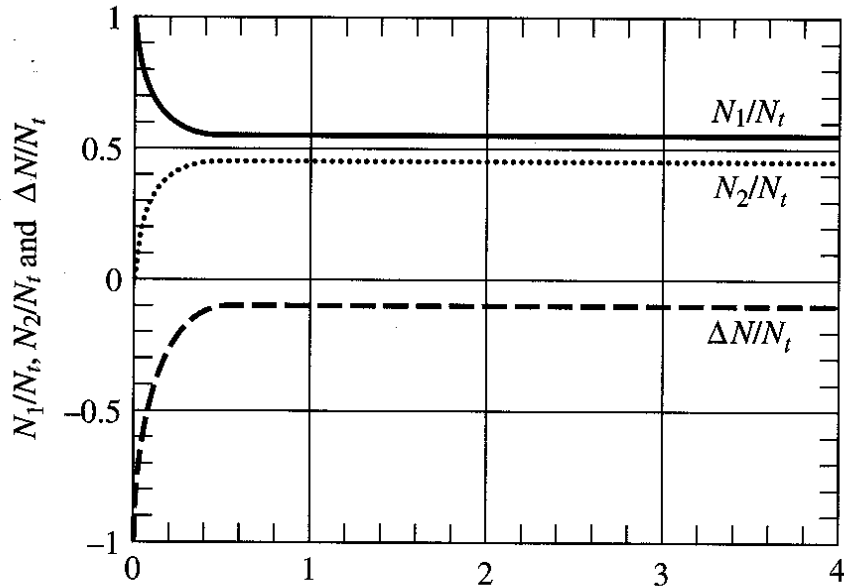
Solution

$$N(t) = \left[N(0) + \frac{N_t}{1+2n}\right]e^{-\left(\frac{1+2n}{t_{sp}}\right)t} - \frac{N_t}{1+2n}$$

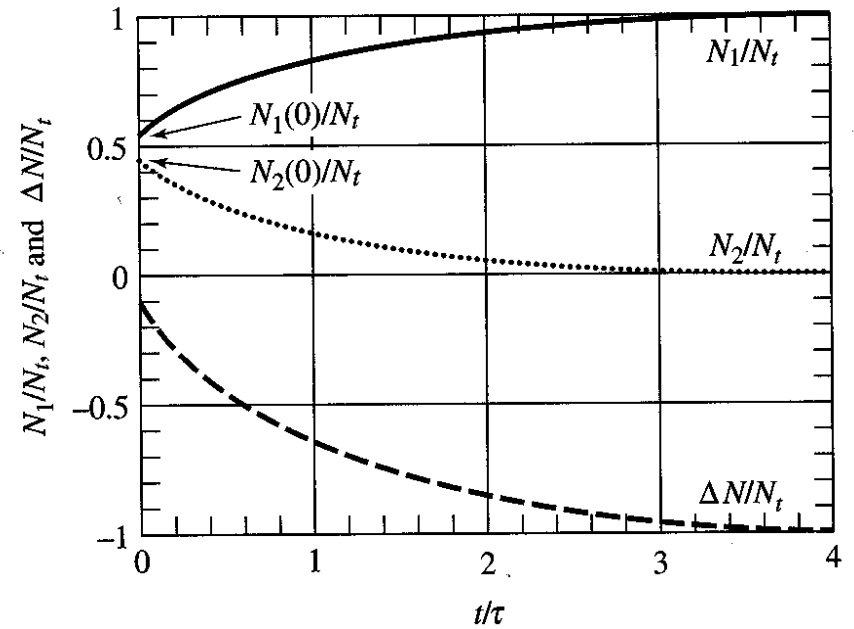
No pumping:

$$N_2(t) = N_2(0)e^{-t/t_{sp}}, N_1(t) = N_t - N_2(t)$$

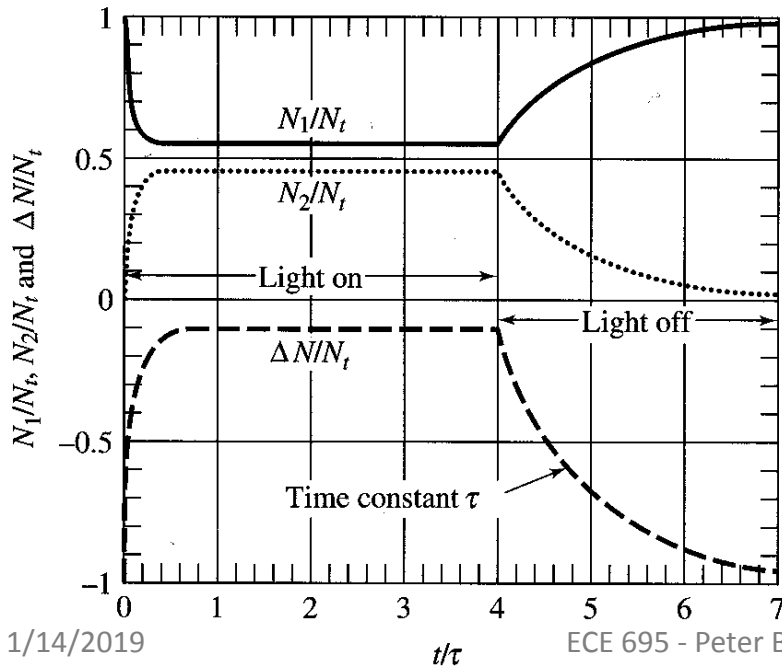
Light on



Light off



Light on and then off



No two-level system can be pumped into population inversion

Differential equation in  $\Delta N(t)$

$$\frac{dN(t)}{dt} = -\left(\frac{1}{t_{sp}} + \frac{2n}{t_{sp}}\right)N(t) - \frac{N_t}{t_{sp}}$$

Solution

$$N(t) = \left[N(0) + \frac{N_t}{1+2n}\right]e^{-\left(\frac{1+2n}{t_{sp}}\right)t} - \frac{N_t}{1+2n}$$

No pumping:

$$N_2(t) = N_2(0)e^{-t/t_{sp}}, N_1(t) = N_t - N_2(t)$$

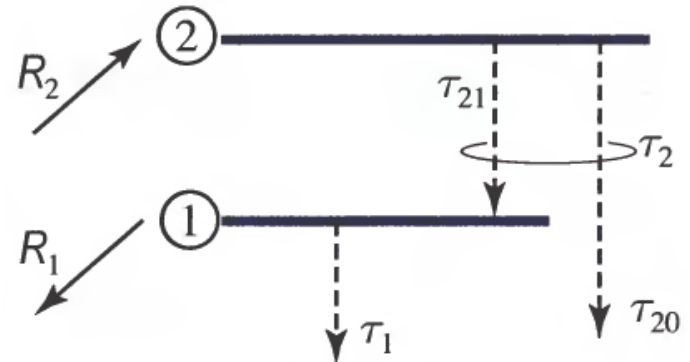
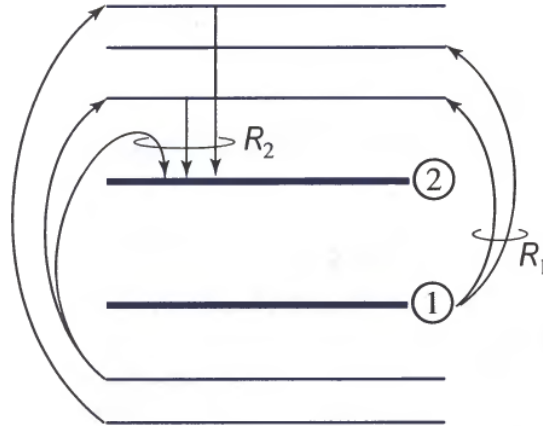
# Pumping Schemes

- The object is to make use of an excitation process that increases the number of atoms populated in level 2 while decreasing the number populated in level 1
- It is impossible to achieve population inversion in a 2-level system
- We should examine 3- and 4-level systems



# Rate Equations (No Amplifier Radiation)

- Rates of change in population densities arising from pumping and decay:



$$\frac{dN_2}{dt} = R_2 - \frac{N_2}{\tau_2} \quad \frac{dN_1}{dt} = -R_1 - \frac{N_1}{\tau_1} + \frac{N_2}{\tau_{21}}$$

Under Steady-State conditions, these equations can be solved, and the population difference can be determined

$$N_0 = R_2 \tau_2 \left( 1 - \frac{\tau_1}{\tau_{21}} \right) + R_1 \tau_1 ,$$

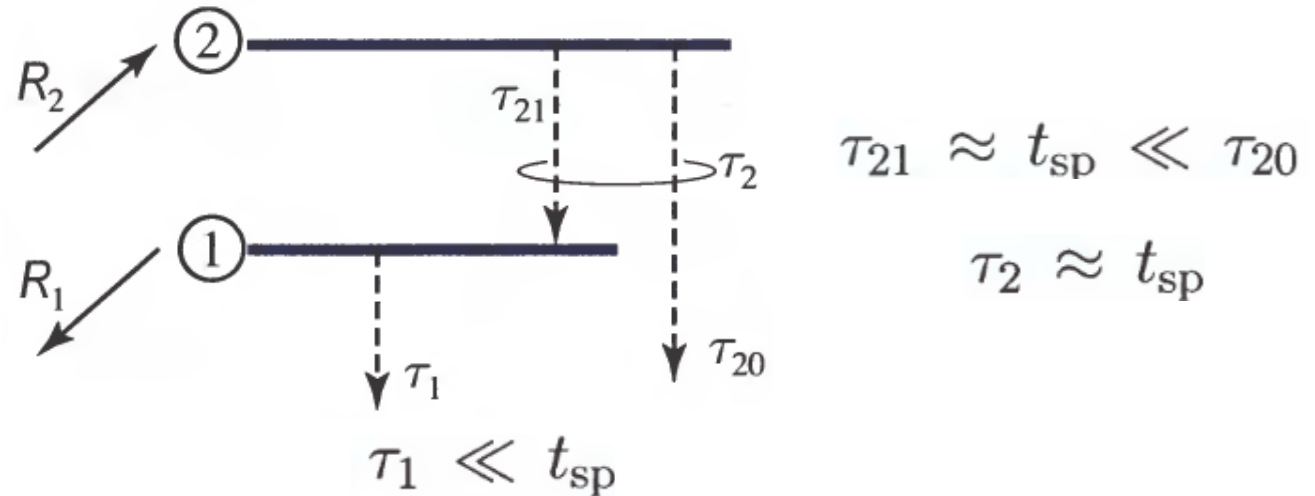
# Rate Equations Continued

$$N_0 = R_2\tau_2 \left(1 - \frac{\tau_1}{\tau_{21}}\right) + R_1\tau_1,$$

A large gain coefficient requires a large population difference (large value of  $N_0$ ). This can be achieved by:

- Large  $R_1$  and  $R_2$
- Long  $\tau_2$  (but  $t_{sp}$ , which contributes to  $\tau_2$  through  $\tau_{21}$ , must be sufficiently short so as to make the radiative transition rate large, as will be seen subsequently)
- Short  $\tau_1$  if  $R_1 < (\tau_2 > \tau_{21}) R_2$

# Rate Equations Continued



- Large  $R_1$  and  $R_2$
- Long  $\tau_2$  (but  $t_{sp}$ , which contributes to  $\tau_2$  through  $\tau_{21}$ , must be sufficiently short so as to make the radiative transition rate large, as will be seen subsequently)
- Short  $\tau_1$  if  $R_1 < (\tau_2 > \tau_{21}) R_2$

# Rate Equations in the Presence of Amplifier Radiation

- The presence of radiation near the resonance frequency enables transitions between levels 2 and 1 to take place via stimulated emission as well as absorption.
- The rate equations must then be extended to include this source of population loss and gain in both levels

$$\frac{dN_2}{dt} = R_2 - \frac{N_2}{\tau_2} - N_2 W_i + N_1 W_i$$

$$\frac{dN_1}{dt} = -R_1 - \frac{N_1}{\tau_1} + \frac{N_2}{\tau_{21}} + N_2 W_i - N_1 W_i$$

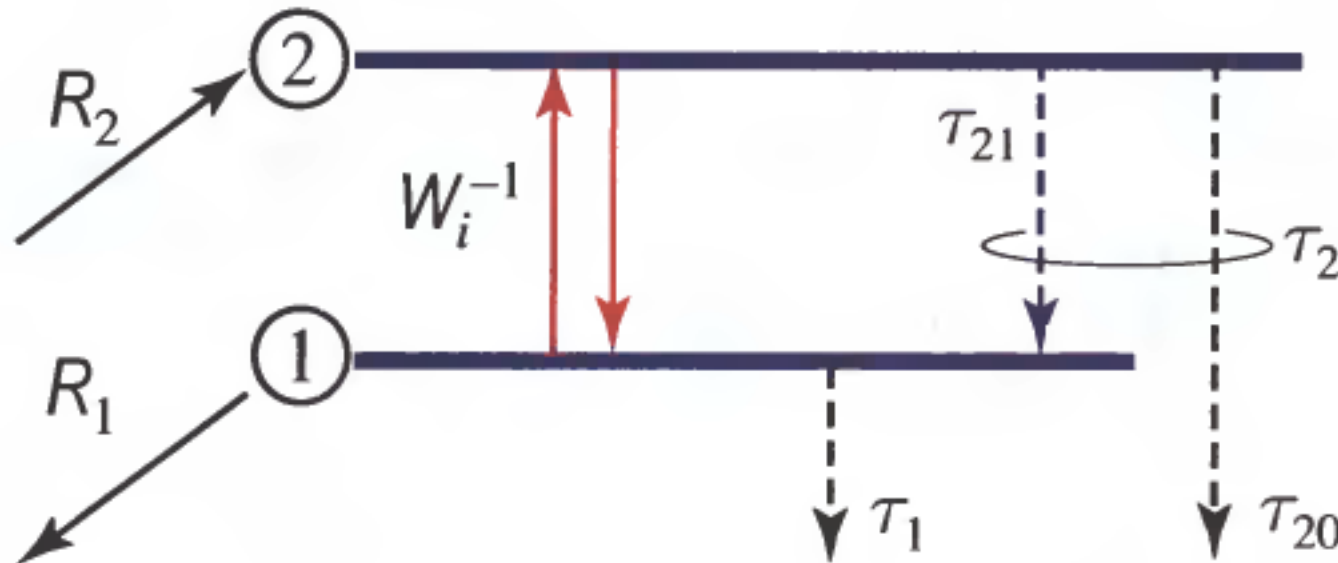
- The population density of level 2 is decreased by stimulated emission from level 2 to level 1 and increased by absorption from level 1 to level 2



# Rate Equations Continued

$$\frac{dN_2}{dt} = R_2 - \frac{N_2}{\tau_2} - N_2 W_i + N_1 W_i$$

$$\frac{dN_1}{dt} = -R_1 - \frac{N_1}{\tau_1} + \frac{N_2}{\tau_{21}} + N_2 W_i - N_1 W_i$$



# Rate Equations Continued

- Under steady-state conditions, these equations can be solved for  $N$  :  $(N_1 - N_2)$

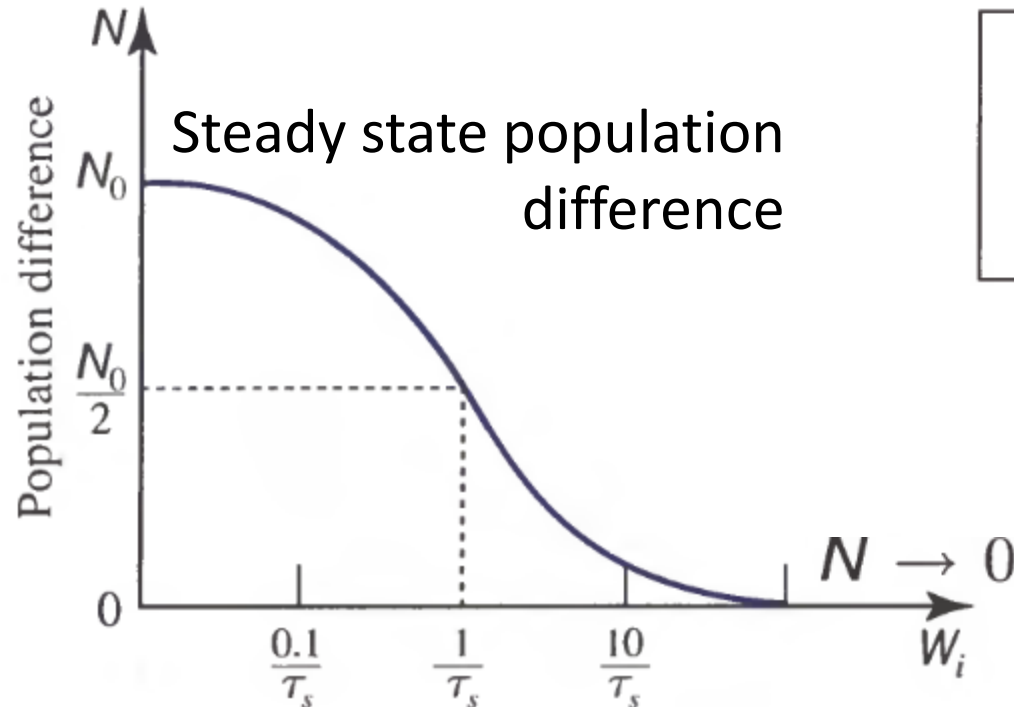
$$N = \frac{N_0}{1 + \tau_s W_i},$$

$$N_0 = R_2 \tau_2 \left(1 - \frac{\tau_1}{\tau_{21}}\right) + R_1 \tau_1,$$

- Where

$$\tau_s = \tau_2 + \tau_1 \left(1 - \frac{\tau_2}{\tau_{21}}\right).$$

# Rate Equations Continued



$$N = \frac{N_0}{1 + \tau_s W_i},$$

$W_i$  - rate of absorption and stimulated emission