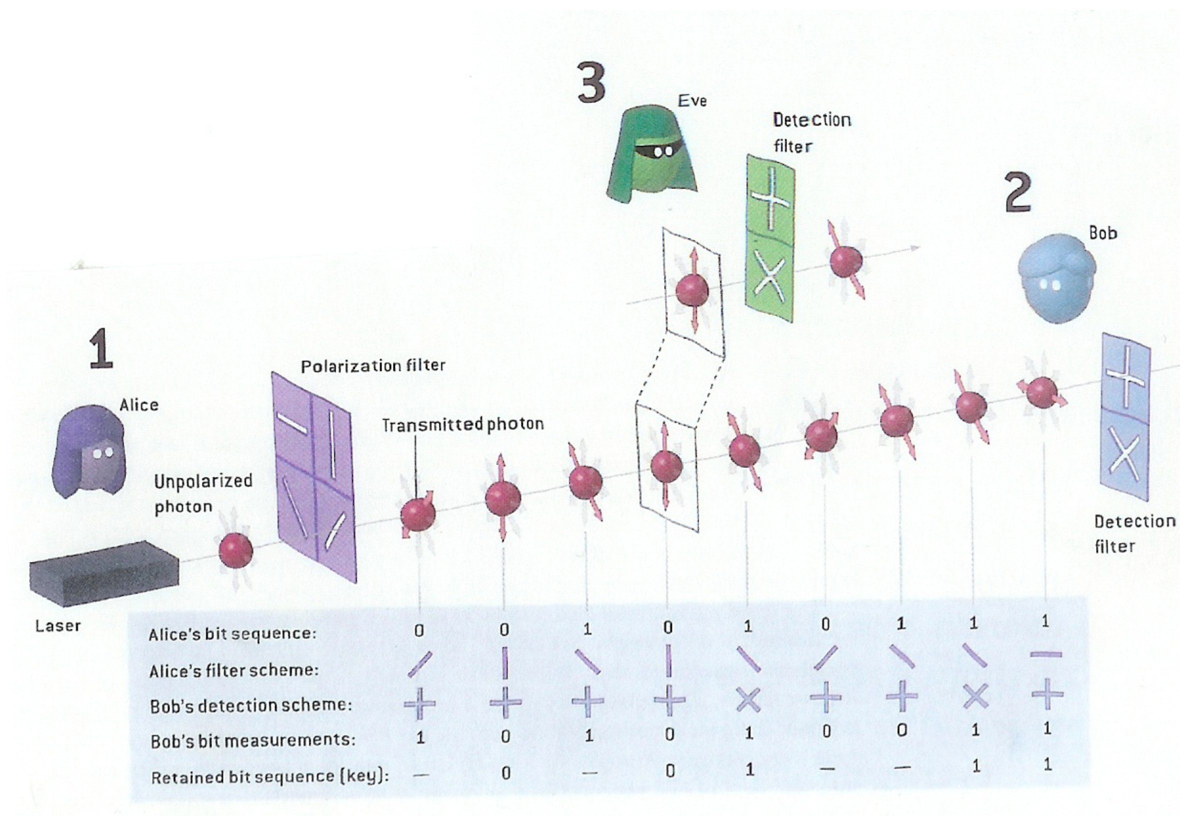
Fiber Optic Communications
Lecture 13: Quantum Cryptography

- Classical cryptography
- Quantum key distribution



Cryptography

- Encoding information for a targeted receiver, and obscuring it for all others
- Key distribution is a major challenge
- One approach: Algorithm (such as RSA) exchanges public keys and then uses public + private keys to encrypt information for correspondent
- Bruce force solution is NP-hard:

$$T(n) = \exp[c(\ln n)^{1/3}(\ln \ln n)^{2/3}]$$

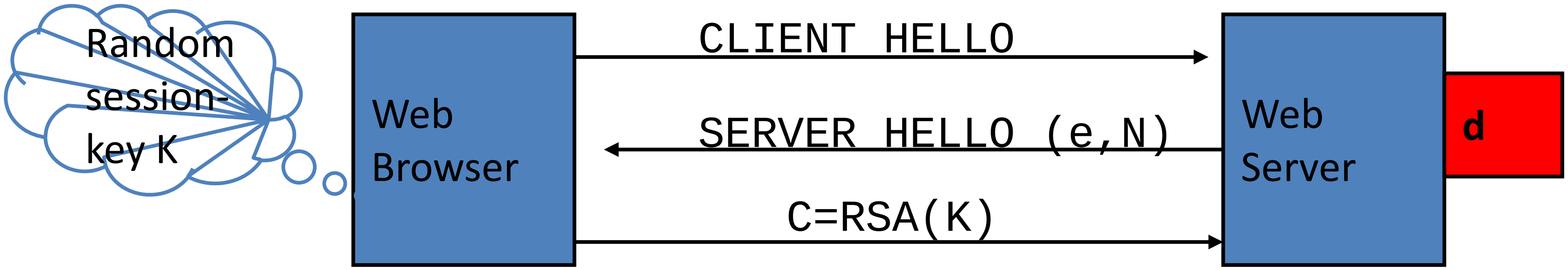
Cryptography

- RSA algorithm now used for most secure internet applications:
  - Public key infrastructure (PKI)
  - SSL/TLS: web certificates
  - Secure e-mail: PGP, GPG, Outlook, etc.

Cryptography Challenges

- Can brute force keys up to 768 bits with sufficient computational power
- With some luck, can cut length of brute force attacks by factors of millions
- As computational power increases, formerly ‘secure’ keys become easier to crack
- RSA algorithm also had a ‘backdoor’ associated with a pseudo-random algorithm (Dual_EC_DRBG) built in as a default
- Therefore, does not satisfy basic security conditions

Simple Attack on Textbook RSA



- Session-key K is 64 bits. View $K \in \{0,\ldots,2^{64}\}$
  Eavesdropper sees: $C = K^e \pmod N$.

- Suppose $\mathbf{K = K_1 \cdot K_2}$ where $K_1, K_2 < 2^{34}$. (prob. ≈20%)
  Then: $\mathbf{C/K_1^{\,e} = K_2^{\,e}} \pmod N$

- Build table: $C/1^e, C/2^e, C/3^e, \ldots, C/2^{34e}$. time: 2^{34}

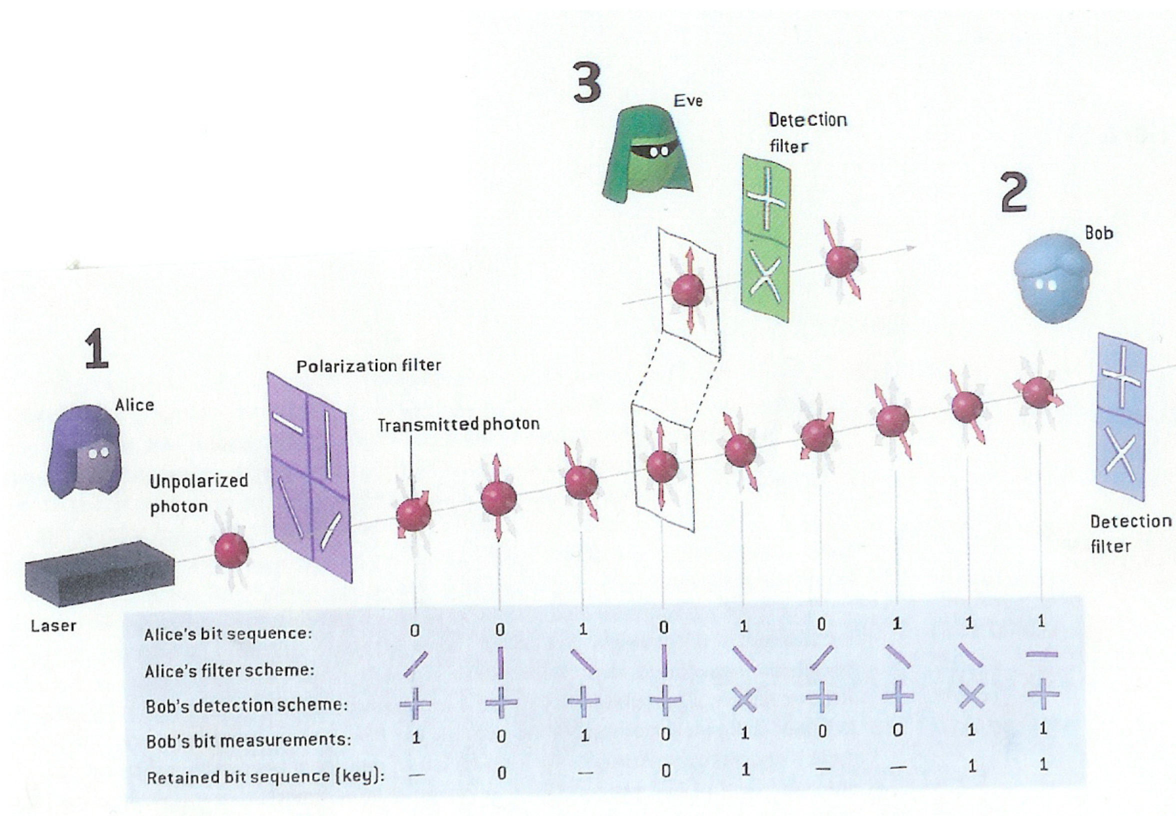
  For $K_2 = 0,\ldots, 2^{34}$ test if $K_2^{\,e}$ is in table. time: $2^{34}\cdot 34$

- Attack time: $\approx 2^{40} << 2^{64}$

Courtesy: Dan Boneh (Stanford University)

Fiber Optic Communications
Lecture 13: Quantum Cryptography

- Classical cryptography
- Quantum key distribution



Quantum Cryptography

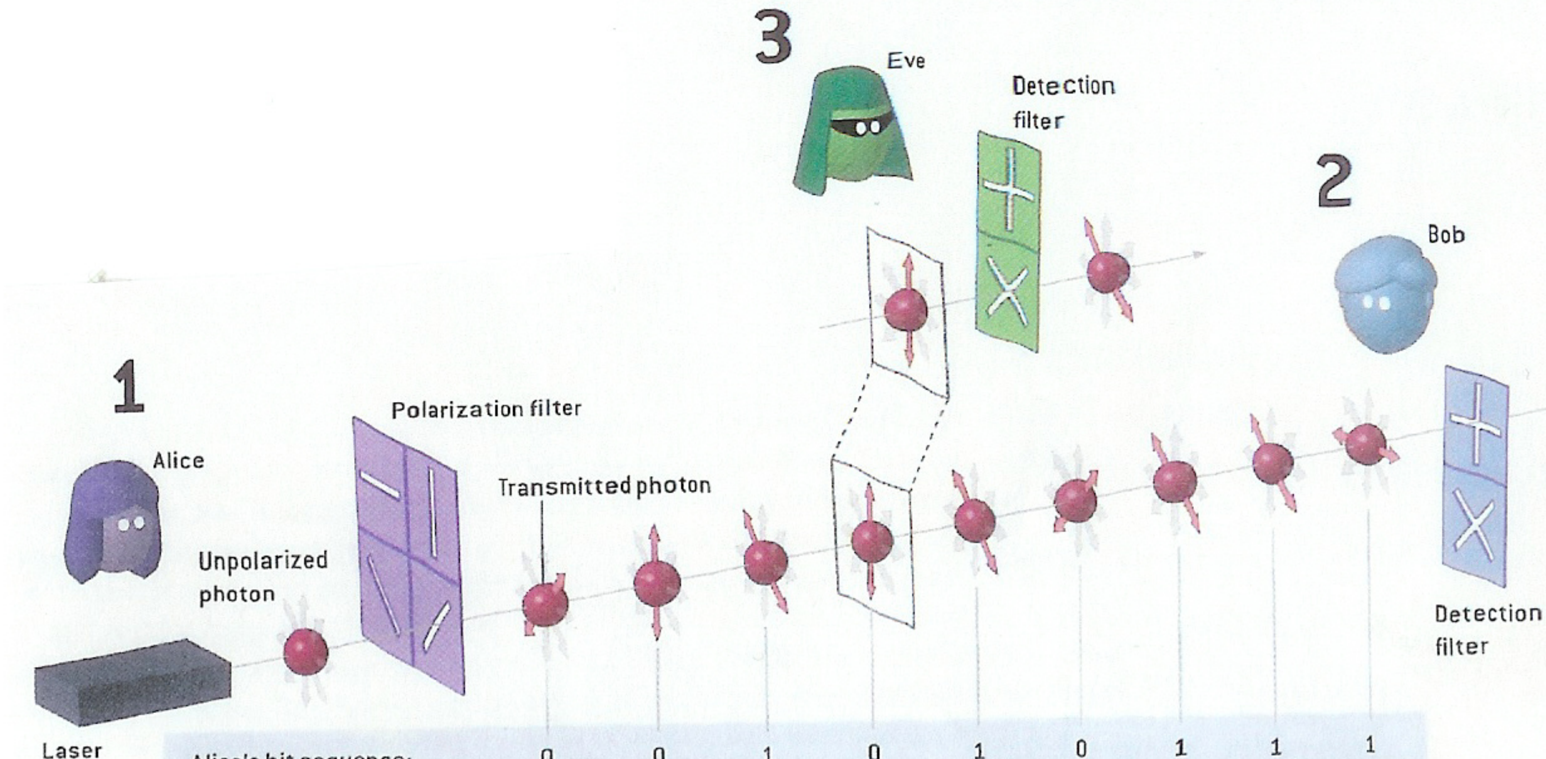
- Here, concept is to use quantum mechanics to solve the key distribution problem
- Unconditionally secure algorithm BB84: proposed by Charles Bennett and Gilles Brassard in 1984.
- Once key is securely received, it can be used to encrypt messages transmitted by conventional channels.

BB84 Setup

- Both Alice and Bob have two polarizers each.
- One has a 0-90 degree basis (+), and one has a 45-135 degree basis ($\times$)

1) Alice uses her polarizers to send randomly photons to Bob in one of the four possible polarizations: 0, 45, 90, or 135 degrees
2) Bob uses his polarizers to measure each polarization of photons he receives. Can use the (+) basis or the ($\times$) basis, but not both simultaneously.

BB84 Implementation



Alice's bit sequence:	0	0	1	0	1	0	1	1	1
Alice's filter scheme:	/	|	\	|	\	/	\	\	—
Bob's detection scheme:	+	+	+	+	×	+	+	×	+
Bob's bit measurements:	1	0	1	0	1	0	0	1	1
Retained bit sequence (key):	—	0	—	0	1	—	—	1	1

Eavesdropper Eve

- If Eve uses the filter aligned with Alice's, she can recover the original polarization of the photon.
- If she uses the misaligned filter, she will receive no information about the photon .
- Also, she will influence the original photon and be unable to retransmit it with the original polarization (no-cloning theorem)
- Thus, Bob will be able to deduce Eve's presence.

Security of quantum key distribution

- Quantum cryptography obtains its fundamental security from the fact that each qubit is carried by a single photon, and each photon will be altered as soon as it is read.

- This makes impossible to intercept message without being detected.

Disadvantages

- Susceptibility to noise: noise looks like an eavesdropping attack, since it degrades quantum coherence

Quantum one-way functions

- Consider a map f: k $\rightarrow |f_k\rangle$.
  - k is the private key
  - $|f_k\rangle$ is the public key
- One-way function: For some maps f, it's impossible (theoretically) to determine k, even given many copies of $|f_k\rangle$
- We can give it to many people without revealing the private key k

Commercial QC providers

- **id Quantique** (Geneva, Switzerland)
  - Optical fiber-based system
  - Tens of km
- **MagiQ Technologies** (New York City)
  - Optical fibers
  - Up to 100 km
- **NEC** (Tokyo): 150 kilometers
- **QinetiQ** (Farnborough, England)
  - Through the air: 10 kilometers.
  - Supplied system to BBN in Cambridge, MA

References

- Gardner, Martin, “Mathematical Games,” Scientific American (Aug. 1977).
- Bennett C. H. & Brassard G., “Quantum cryptography: Public key distribution and coin tossing” (1984).
- Daniel Gottesman, Isaac Chuang, “Quantum Digital Signatures” (2001).