1. CD Injection Molding II: Shear Stress

   In Part I, you simplified the Navier-Stokes equations describing liquid polymer flow into a CD mold. The resulting equations (should have) looked like:

$$\frac{1}{r}\frac{\partial}{\partial r}(ru_r) = 0.$$

$$\rho u_r \frac{\partial u_r}{\partial r} = -\frac{\partial p}{\partial r} + \mu \frac{\partial^2 u_r}{\partial z^2}.$$

$$0 = -\frac{1}{r}\frac{\partial p}{\partial \theta},$$

$$0 = -\frac{\partial p}{\partial z} + F_z,$$

   where the r-derivative in the viscous term of the r-momentum equation is removed based on the mass conservation equation. Here you will simplify further and solve the equations.

   Data:

   - Polymer flow rate: $Q = 5 \times 10^{-6} \frac{\mathrm{m}^3}{\mathrm{s}}$.
   - CD mold thickness: $\delta = 1\mathrm{mm}$.
   - Polycarbonate viscosity at injection temperature: $\mu = 200 \frac{\mathrm{kg}}{\mathrm{m \cdot s}}$
   - Polycarbonate density at injection temperature: $\rho = 1200 \frac{\mathrm{kg}}{\mathrm{m}^3}$.

   (a) Solve the mass conservation equation (the top one) to give a general equation for u_r as a function of r.

   (b) Calculate the average velocity in the r-direction at $r = 1\mathrm{cm}$, and calculate the Reynolds number there. What additional assumption can you make, and which term in the r-momentum equation (the second one) does it let you cancel?

   (c) Based on the momentum equations with $F_z = -\rho g$, the pressure field is described by:

$$p = f(r) - \rho g z.$$

   Using this, find a general solution to the r-momentum equation to give u_r as a function of z.

   (d) Reconcile your solutions to parts 1a and 1c to give an expression for u_r as a function of r and z.

   (e) Calculate the shear stress τ_{zr} against the top or bottom face of the mold at $r = 2\mathrm{cm}$.