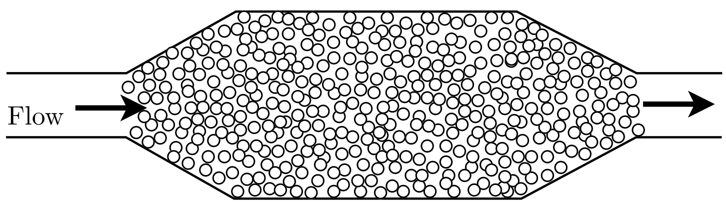
1. Dimensional analysis: catalytic combustion of carbon monoxide

   CuO is a good catalyst for the reaction

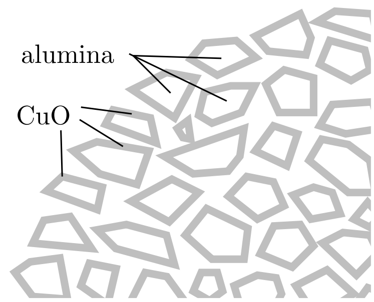
$$2CO(g) + O_2(g) \Rightarrow 2CO_2(g)$$

   CuO is coated on the internal surfaces of porous Al_2O_3 spheres of radius R, so the gases containing CO and O_2 can diffuse in through the pores with diffusivity D and react on the internal CuO surfaces. (The Al_2O_3 is used as a substrate because of its high strength and stability at the high temperatures reached due to combustion; porous CuO would sinter into dense solid spheres at such temperatures.)

Catalytic combustor



Sphere cross-section



   Because the pores are a lot smaller than the sphere, you can treat this as a homogeneous chemical reaction throughout the porous solid. Assume for this problem that O_2 is the limiting reagent, and the rate of reaction is proportional to its concentration with "homogeneous" rate constant k, so $G = -kC_{O_2}$. The concentration of oxygen in the pores just inside the surface of the sphere is approximately equal to that in the gas outside, so that is our outer surface boundary condition.

   If the concentration of oxygen throughout each sphere is very uniform, then the reaction will be occurring on all of the inner pore surfaces within each sphere, so the CuO will be well-utilized. On the other hand, if we are flowing CO gas through this bunch of spheres, the larger the spheres, the less resistance to flow there will be. So we want the spheres to be as large as possible without being so large that there is no reaction at their center.

   (a) Write an expression for a parameter which describes the uniformity of oxygen concentration, in terms of the other problem parameters.
   (b) Identify the number of parameters, and the number of independent units, and use the Buckingham pi theorem to determine the number of independent dimensionless parameters in the problem.
   (c) Write a new expression for the dimensionless uniformity parameter in part 1a in terms of the other dimensionless parameter(s).
   (d) If you double the sphere radius, how much does k have to change in order to yield the same uniformity? (The reaction rate coefficient k tends to be a much stronger function of temperature than diffusivity, so changing the temperature will change D a small amount and k a large amount.)
   (e) Draw a sketch (or sketches) showing how you think that uniformity parameter will vary as a function of the other parameter(s), making sure your sketch captures the transition from strongly non-uniform to roughly uniform concentration in a sphere.
   (f) Sketch the concentration of oxygen in a sphere as a function of radius for large and small values of the other dimensionless parameter(s) of which uniformity is a function. If there is more than one such parameter, make sketches for all combinations of large and small values of each parameter.