

### Solution to Exam 2

1.  $[D] = [v] \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} [v^T]$  where  $v$  is a column vector

$$A = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \det(A - \lambda I) = 0$$

$$\Rightarrow \lambda^2 - 1 = 0 \quad \lambda = \pm 1$$

$$\lambda = 1 \quad (A - \lambda I)x = 0 \quad \text{gives } x = \begin{bmatrix} -i \\ 1 \end{bmatrix} \xrightarrow{\text{normalized}} \frac{1}{\sqrt{2}} \begin{bmatrix} -i \\ 1 \end{bmatrix}$$

$$\lambda = -1 \quad (A - \lambda I)x = 0 \quad \text{gives } x = \begin{bmatrix} i \\ 1 \end{bmatrix} \xrightarrow{\text{normalized}} \frac{1}{\sqrt{2}} \begin{bmatrix} i \\ 1 \end{bmatrix}$$

$$\Rightarrow [v^T] = \frac{1}{\sqrt{2}} \begin{bmatrix} -i & i \\ 1 & 1 \end{bmatrix} \quad \Rightarrow D = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$[v] = \frac{1}{\sqrt{2}} \begin{bmatrix} i & -i \\ 1 & 1 \end{bmatrix}$$

2. Hamiltonian of the system

$$H = \begin{bmatrix} \epsilon & t & 0 & 0 & 0 & 0 \\ t & \epsilon & t & 0 & 0 & 0 \\ 0 & t & \epsilon & t & 0 & 0 \\ 0 & 0 & t & \epsilon & t & 0 \\ 0 & 0 & 0 & t & \epsilon & t \\ 0 & 0 & 0 & 0 & t & \epsilon \end{bmatrix} \quad E\Psi = H\Psi$$

$$E\Psi_n = t\Psi_{n-1} + \Psi_n + t\Psi_{n+1}$$

$$\Psi_n = \Psi_0 e^{ikna} \quad \Rightarrow \quad E = \epsilon + 2t \cos ka$$

from periodic boundary condition

$$ka = \frac{2\pi j}{6} = \frac{\pi j}{3} \quad j = 0, 1, 2, \dots, 5$$

$$E = \begin{aligned} &\epsilon + 2t \\ &\epsilon + t \quad (\text{two fold degenerate}) \\ &\epsilon - t \quad (\text{two fold degenerate}) \\ &\epsilon - 2t \end{aligned}$$

$$E^2 = B K^2$$

circumferential vector  $\vec{c_x}$

$$\vec{K} \cdot \vec{c_x} = 2\pi r$$

$$K_x = \left(\frac{2\pi r}{c}\right)$$

$$\Rightarrow E^2 = B \left[ K_y^2 + \left(\frac{2\pi r}{c}\right)^2 \right]$$

$$= B K^2 + E_{y0}^2$$

$$K = \sqrt{(E^2 - E_{y0}^2)/B}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} [v] = [0] \Leftrightarrow$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = K$$

$$E_{y0} = \left(\frac{2\pi r}{c}\right)^2 B$$

For 1-D solid.

$$N(K) = \frac{1}{\pi} K$$

$$\Rightarrow N(E) = \frac{1}{\pi} \sqrt{\frac{E^2 - E_{y0}^2}{B}}$$

$$D(E) = \frac{d N(E)}{d E}$$

$$= \sum_y \frac{L}{2\pi} \frac{2E}{\sqrt{E^2 - E_{y0}^2}} \frac{1}{\sqrt{B}}$$

$$= \frac{L}{\pi\sqrt{B}} \sum_y \frac{E}{\left[E^2 - \left(\frac{2\pi r}{c}\right)^2 B\right]^{1/2}}$$

and this is called scattering length

$$\frac{L}{B} = \frac{E_{max}}{B} = 0.9$$

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