Section 28
MOS Electrostatics & MOScap

28.5 MOScap Exact solution of the electrostatic problem

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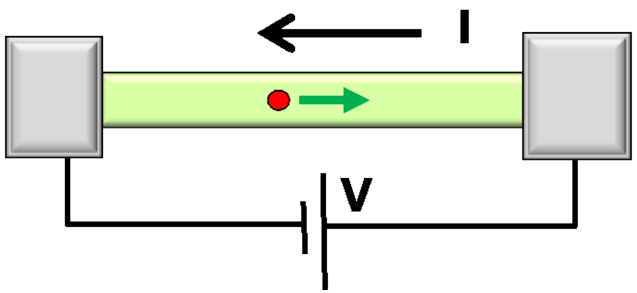
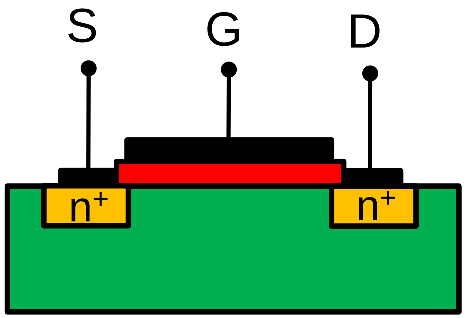


$I = G \times V$

$= q \times n \times v \times A$

charge density velocity area



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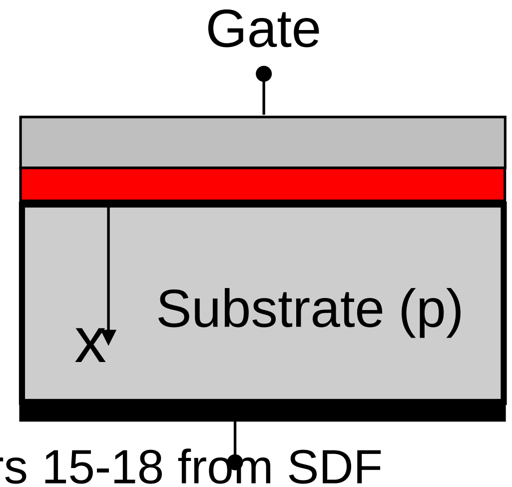
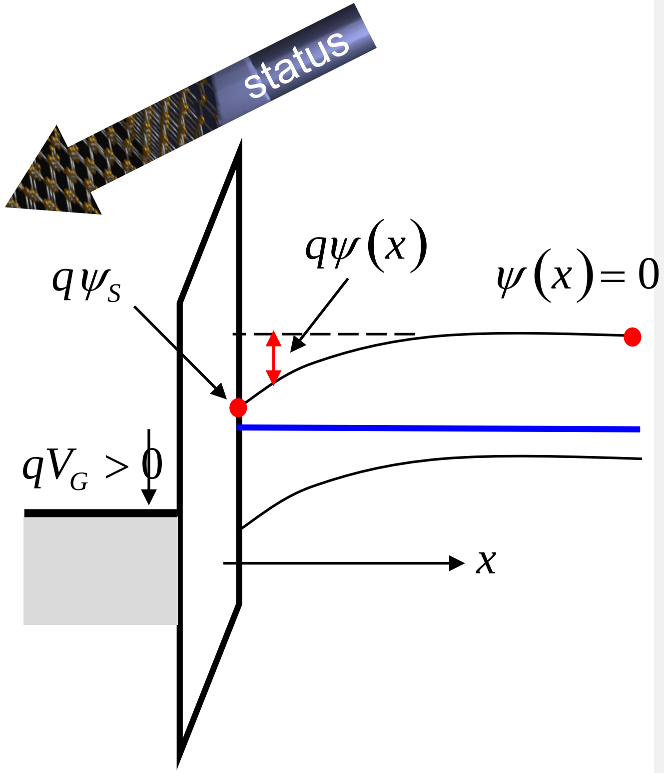
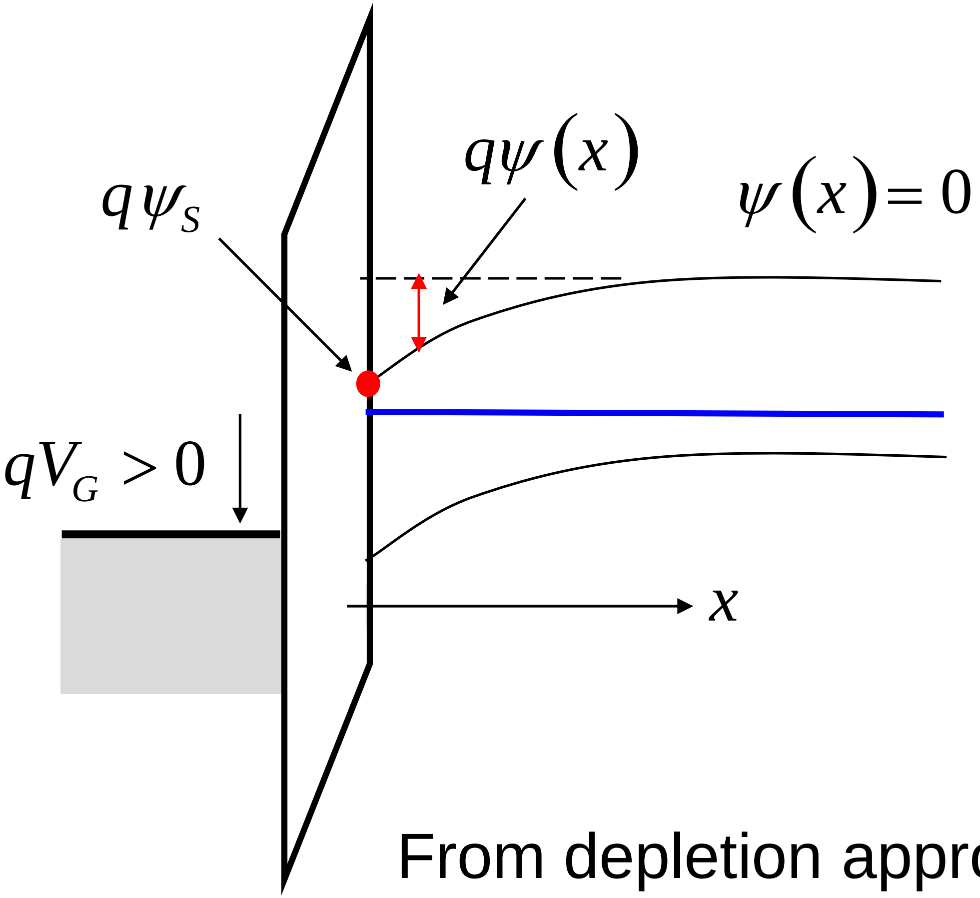




REF: Chapters 15-18 from SDF

A step back: 'Exact' Solution of $Q_S(\psi_S)$



$$\nabla \bullet \vec{D} = \rho$$

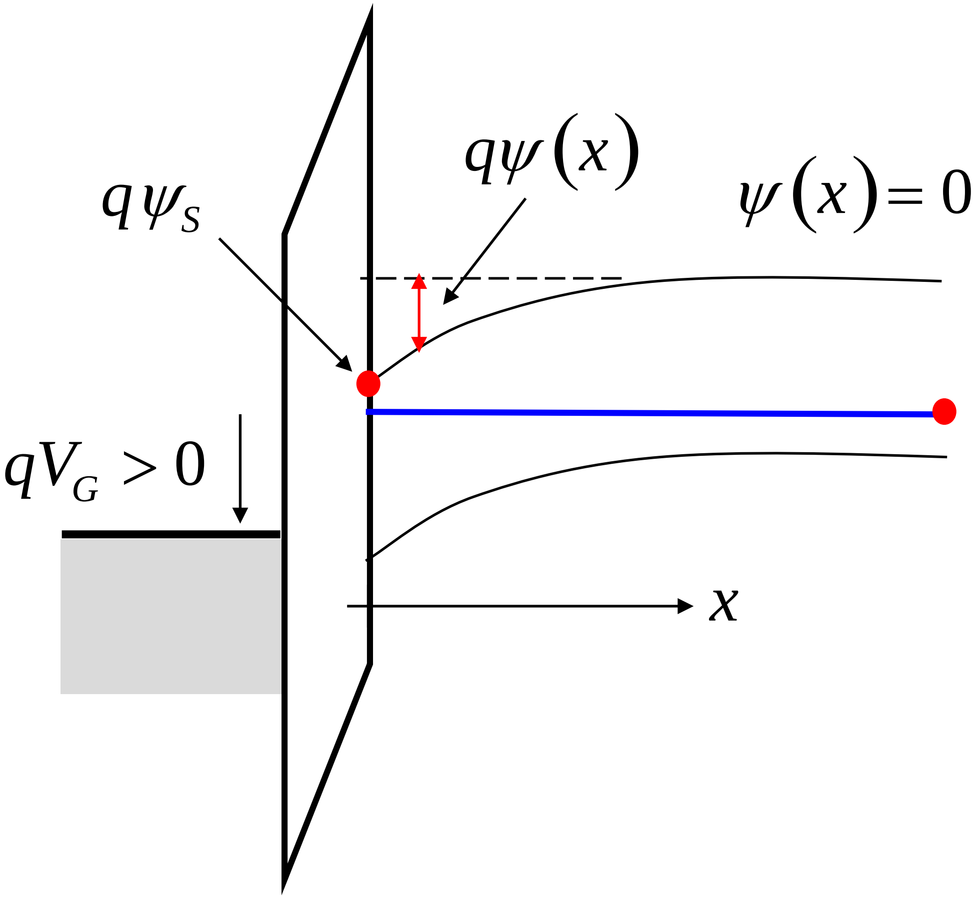
$$\nabla \bullet \left(\vec{J}_n / -q \right) = (G - R)$$

$$\nabla \bullet \left(\vec{J}_p / q \right) = (G - R)$$

$$\left. \frac{d^2\psi}{dx^2} \right| = \frac{-q}{\kappa_s \varepsilon_0} \left[p(x) - n(x) + N_D^+ - N_A^- \right]$$

From depletion approximation … $V_G = \frac{qN_A x_0}{\kappa_{ox}\varepsilon_0} \sqrt{\frac{2\kappa_{ox}\varepsilon_0}{qN_A}} \sqrt{\psi_s} + \psi_s$

Normalized Variable (to save some writing)…



$$\left.\frac{d^2\psi}{dx^2}\right| = \frac{-q}{\kappa_s \varepsilon_0}\left[p(x) - n(x) + N_D^+ - N_A^-\right]$$

$$E_C(x) = \text{constant} - q\psi(x)$$

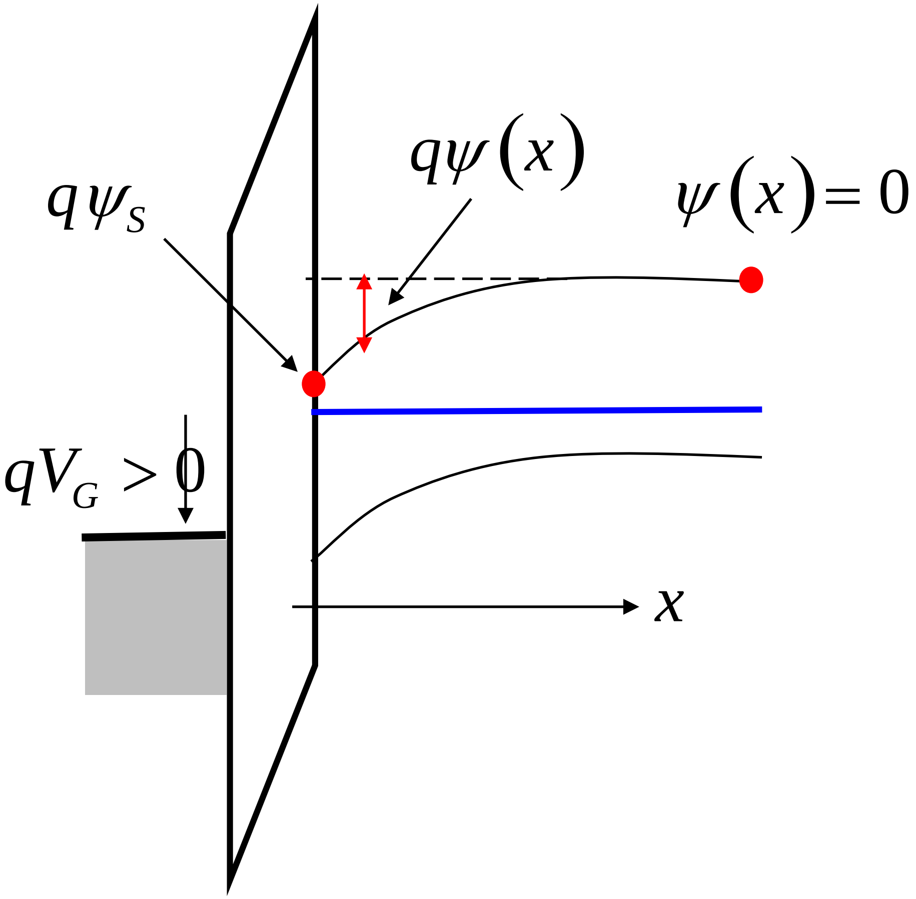
$$\psi(x) = \frac{E_{C,bulk} - E_C(x)}{q}$$

$$u = \frac{\psi(x)}{k_B T / q} = \frac{E_{i(bulk)} - E_{i(x)}}{k_B T}$$

$$u_S = \frac{\psi_S}{k_B T / q} = \frac{E_{i(bulk)} - E_{i(surface)}}{k_B T}$$

$$u_F = \frac{\phi_F}{k_B T / q} = \frac{E_{i(bulk)} - E_F}{k_B T}$$

Normalized Variable (to save some writing!)



$$\left.\frac{d^2\psi}{dx^2}\right| = \frac{-q}{\kappa_s \varepsilon_0}\left[p(x) - n(x) + N_D^+ - N_A^-\right]$$

$$u = \frac{\psi(x)}{k_BT/q} = \frac{E_{i(bulk)} - E_{i(x)}}{k_BT} \qquad u_F = \frac{\phi_F}{k_BT/q} = \frac{E_{i(bulk)} - E_F}{k_BT}$$

$$p(x) = n_i e^{[E_i(x) - E_F]\beta} = n_i e^{+(U_F - U)}$$

$$n(x) = n_i e^{-[E_i(x) - E_F]\beta} = n_i e^{-(U_F - U)}$$

$$N_D^+ = n_i e^{[E_F - E_{i,bulk}]\beta} = n_i e^{-(U_F)}$$

$$N_A^- = n_i e^{-[E_F - E_{i,bulk}]\beta} = n_i e^{(U_F)}$$

Assuming either donor or acceptor is present

Poisson-Boltzmann Equation

$$\left.\frac{d^2\psi}{dx^2}\right| = \frac{-q}{\kappa_s\varepsilon_0}\left[p(x)-n(x)+N_D^+-N_A^-\right]$$

$$n(x)=n_ie^{-[E_i(x)-E_F]\beta}=n_ie^{-(U_F-U)} \quad N_D^+=n_ie^{[E_F-E_{i,bulk}]\beta}=n_ie^{-(U_F)}$$

$$p(x)=n_ie^{[E_i(x)-E_F]\beta}=n_ie^{+(U_F-U)} \quad N_A^-=n_ie^{-[E_F-E_{i,bulk}]\beta}=n_ie^{(U_F)}$$

$$\frac{q}{k_BT}\frac{d^2U}{dx^2}=\frac{-qn_i}{\kappa_s\varepsilon_0}\left[e^{+(U_F-U)}-e^{-(U_F-U)}+n_ie^{-U_F}-n_ie^{U_F}\right]\equiv g(U,U_F)$$

Can be evaluated at any U

$$\frac{d}{dx}\left(\frac{dU}{dx}\right)^2=2\left(\frac{dU}{dx}\right)^1\times\frac{d}{dx}\frac{dU}{dx}=2\frac{dU}{dx}\frac{d^2U}{dx^2}$$

$$\left(2\frac{dU}{dx}\right)\times\frac{d^2U}{dx^2}=-\left(\frac{n_ik_BT}{\kappa_s\varepsilon_0}\right)g(U,U_F)\times\left(2\frac{dU}{dx}\right)$$

$$\frac{d}{dx}\left(\frac{dU}{dx}\right)^2dx=-\frac{1}{2L_D^2}g(U,U_F)\left(2\frac{dU}{dx}\right)dx$$

Debye Length

$$\int_0^{-q\mathcal{E}(x)/kT}d\left(\frac{dU}{dx}\right)^2=-\frac{1}{L_D^2}\int_0^{U(x)}g(U,U_F)dU$$

Exact Solution (continued)

$$\int_0^{-q\mathcal{E}(x)/kT} d\left(\frac{dU}{dx}\right)^2 = -\frac{1}{L_D^2}\int_0^{U(x)} g(U,U_F)\,dU$$

$$\left[\frac{q\mathcal{E}(x)}{kT}\right]^2 = \frac{1}{L_D^2}\int_0^{U(x)} g(U,U_F)\,dU \equiv \frac{F^2(U,U_F)}{L_D^2}$$

Exact F requires numerical solution

At the surface

$$\mathcal{E}_s = \frac{k_BT}{qL_D}F(U_S,U_F)$$

Compare …

$$V_G = \frac{qN_Ax_0}{\kappa_{ox}\varepsilon_0}\sqrt{\frac{2\kappa_{ox}\varepsilon_0}{qN_A}}\sqrt{\psi_s} + \psi_s$$

V_{ox}

$$V_G = \psi_s + \left[\frac{\kappa_s}{\kappa_{ox}}\mathcal{E}_s\right]x_0 = \psi_s + \frac{\kappa_s}{\kappa_{ox}}\frac{k_BT}{qL_D}F(U_s,U_F)x_0$$

Recipe for numerical solution ...

$$\left[\frac{q\mathcal{E}(x)}{kT}\right]^2 = \frac{1}{L_D^2}\int_0^{U(x)} g(U,U_F)\,dU \equiv \frac{F^2(U,U_F)}{L_D^2}$$

$$V_G = \psi_s + \frac{\kappa_s}{\kappa_{ox}}\mathcal{E}_s x_0 = \psi_s + \frac{\kappa_s}{\kappa_{ox}}\frac{k_B T}{q L_D} F(U_s,U_F)\, x_0$$

Begin with a surface potential

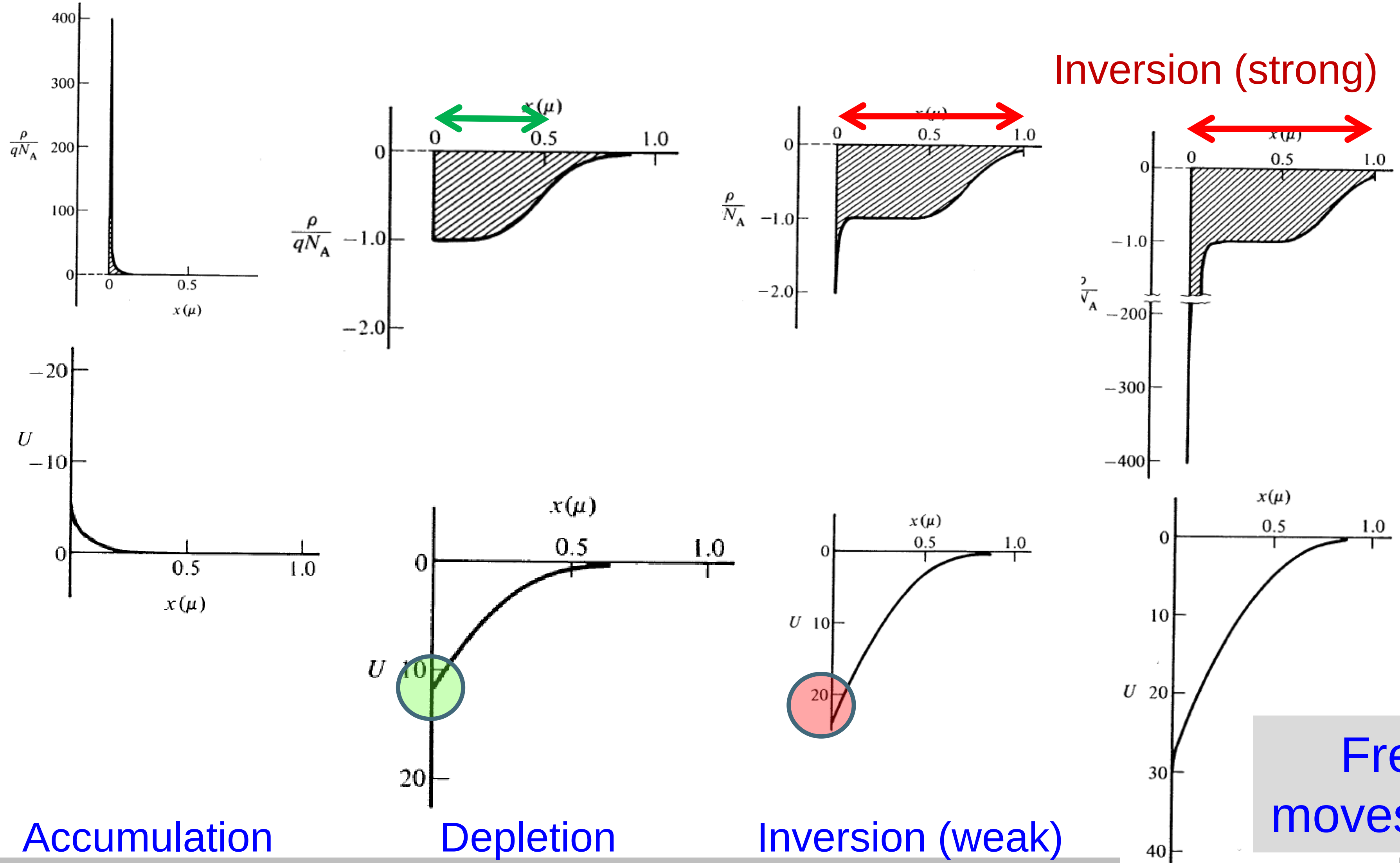
Calculate U_s and then divide U_s by N points.

Calculate $g(U, U_F)$ at those points
and integrate to find $F(U_s,U_F)$

Find V_G.

U: 25→30
Charge density: -2.0 → -400

Inversion (strong)

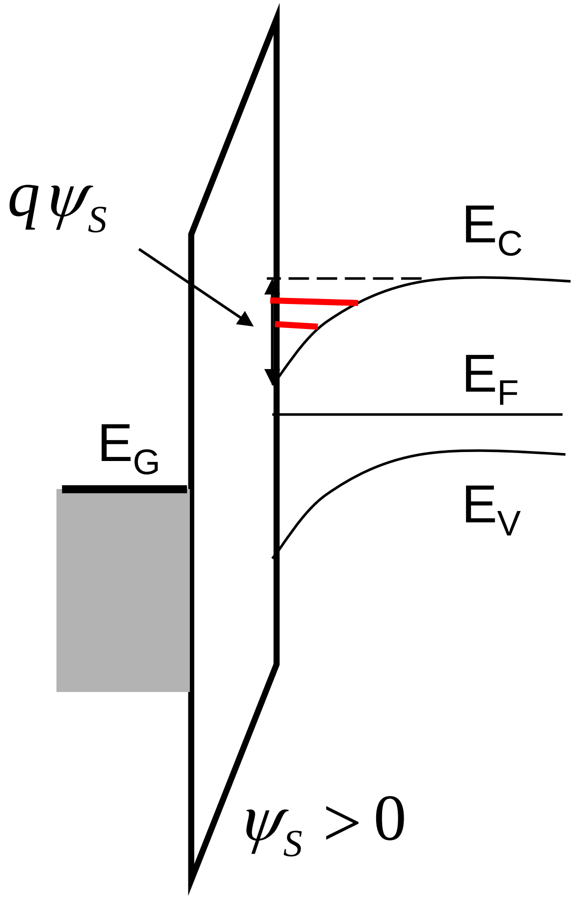


Accumulation Depletion Inversion (weak)

Free charge moves to interface

Huge spike! Hard to resolve numerically! Realistic?

"Exact" solution is not really exact ...



$$\left.\frac{d^2\psi}{dx^2}\right| = \frac{-q}{\varepsilon}\left[p(x) - n(x)\left|\psi(x)\right|^2 + N_D^+ - N_A^-\right]$$

wavefunction, not potential !

Wave function should be accounted for

Bandgap widening near the interface must also should be accounted for.

Assumption of nondegeneracy may not always be valid

"Exact" solution is not really exact ... really

R. Bowen, Chenjing Fernando, Gerhard Klimeck, Amitava Chatterjee, Daniel Blanks, Roger Lake, J. Hu, Joseph Davis, M. Kularni, Sunil Hattangady, I.C. Chen,
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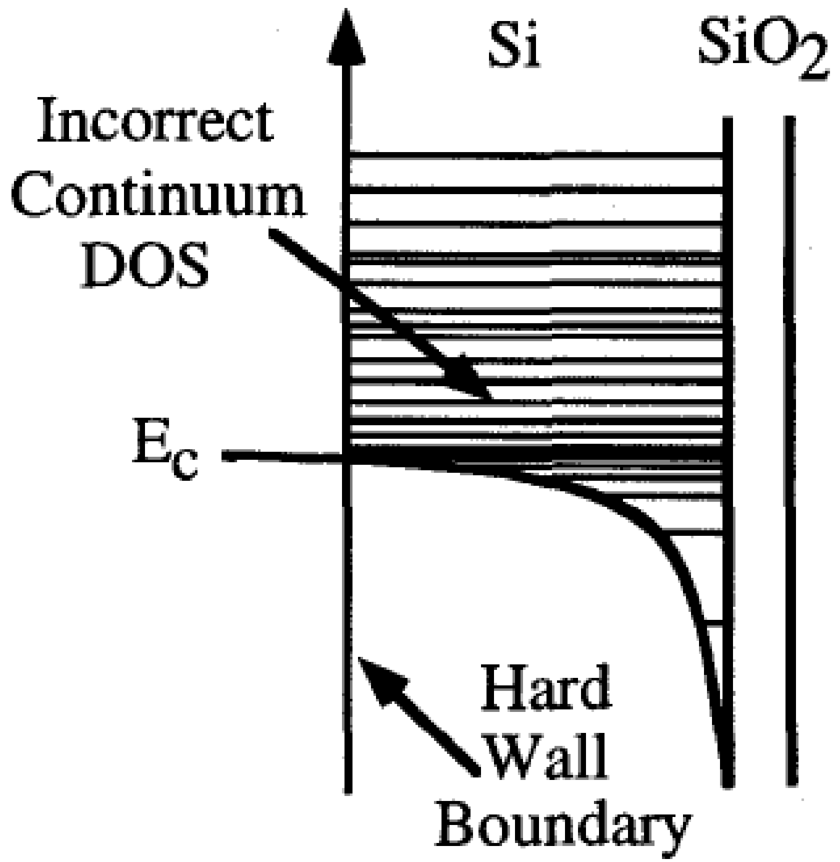

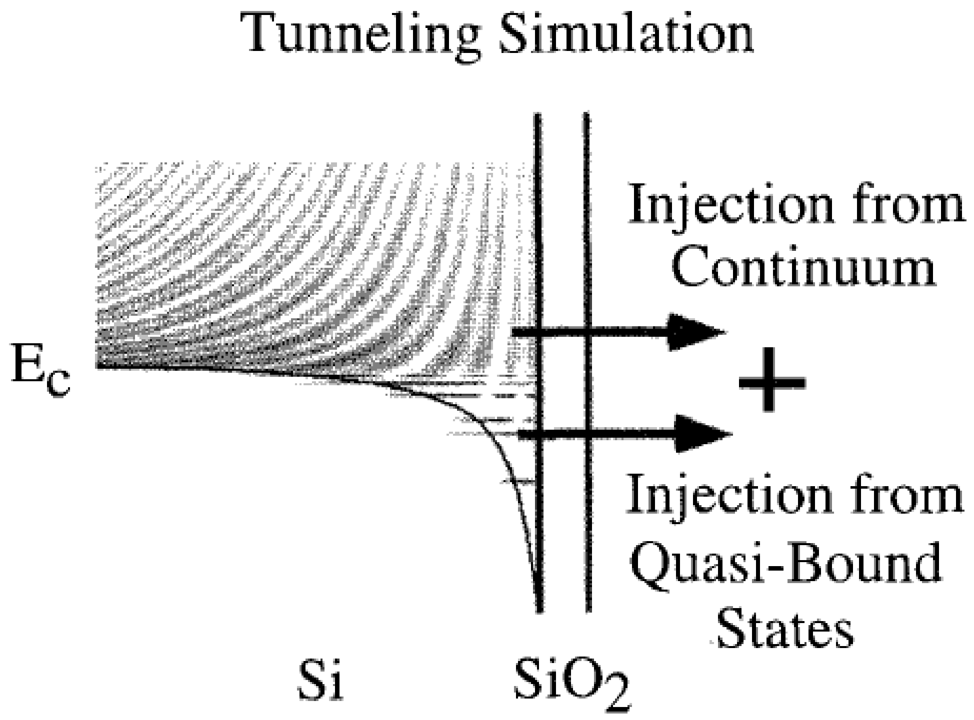

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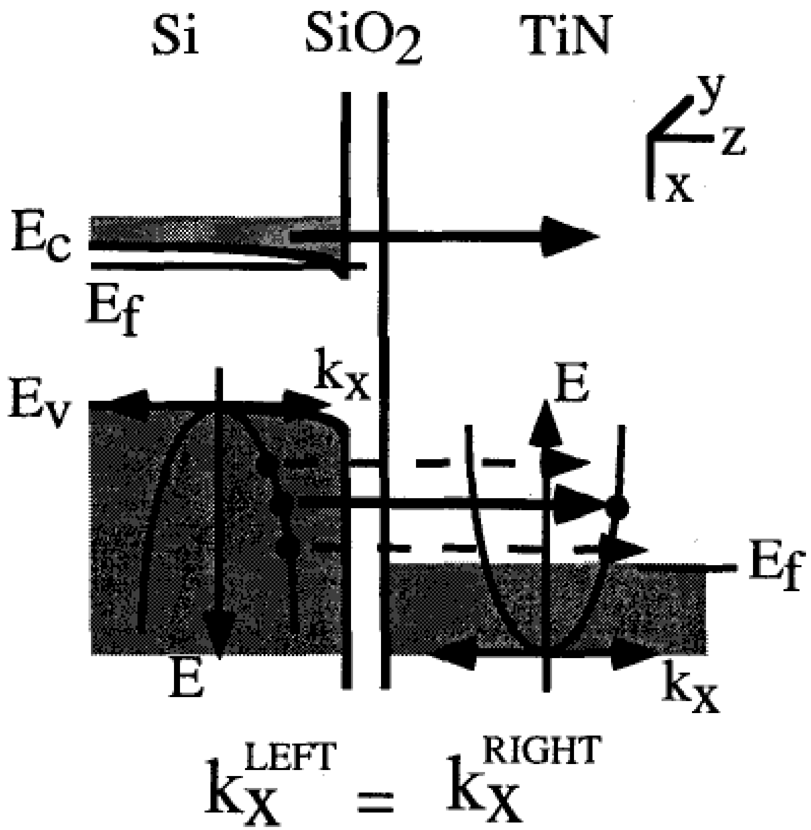

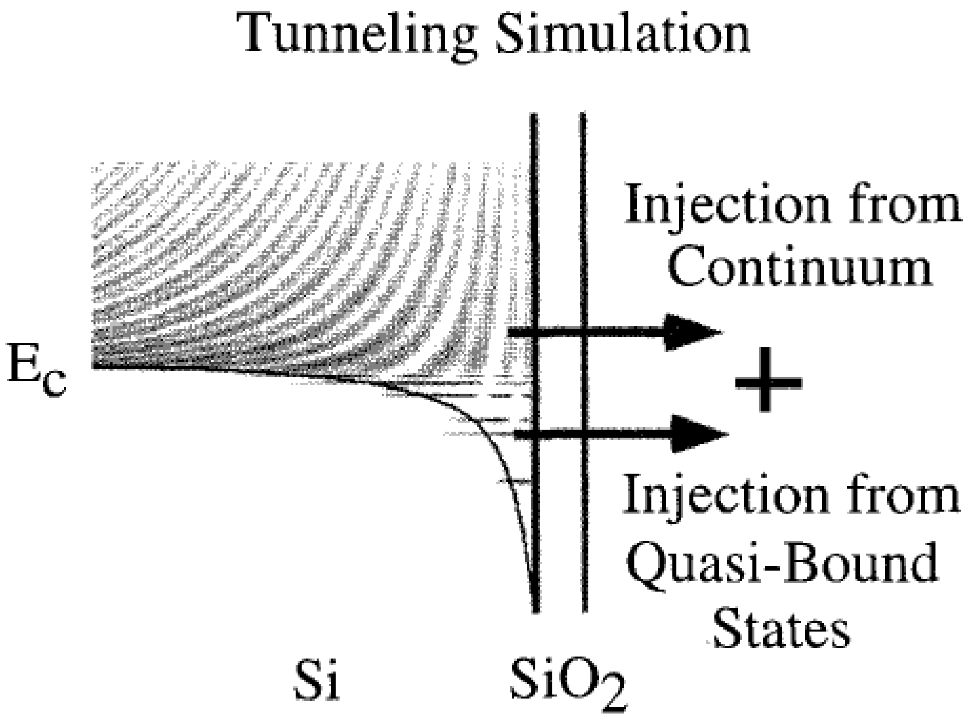

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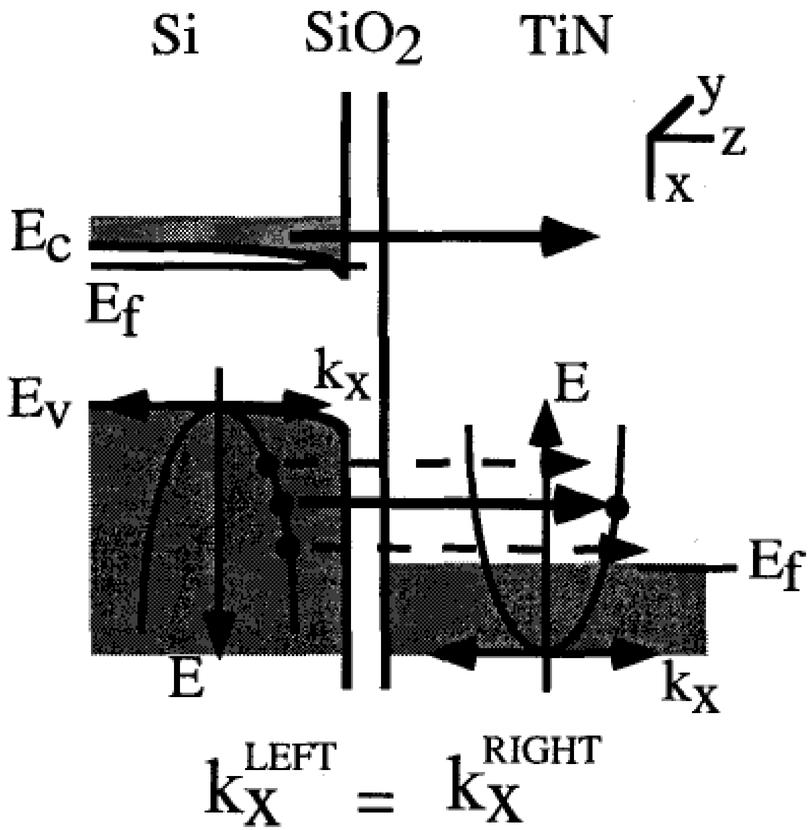

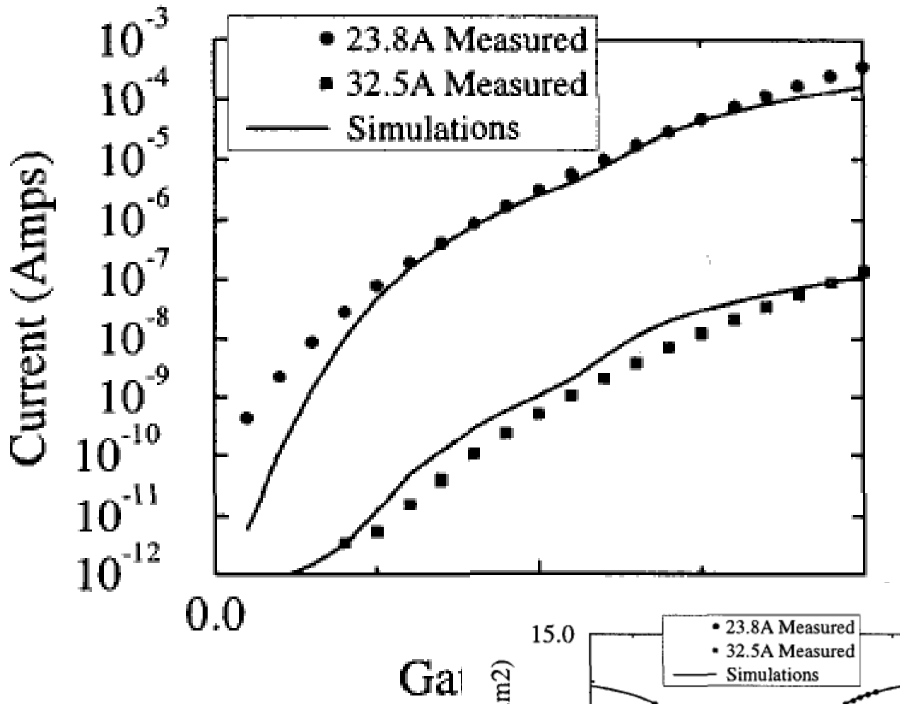

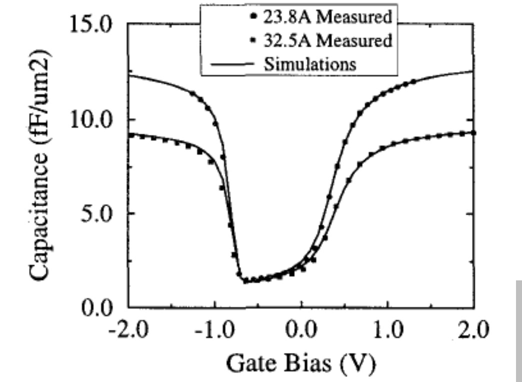

Conclusion

Our discussion today was focused on calculating the induced charge in the depletion and inversion region as a function of gate bias.

We found that we could calculate the tunneling current from the inversion changes by using the thermionic emission theory.

We also discussed the "exact" solution of the MOS-capacitor electrostatics. The "exact" solution is mathematically exact, but not necessarily physically exact solution of the electrostatic problem.

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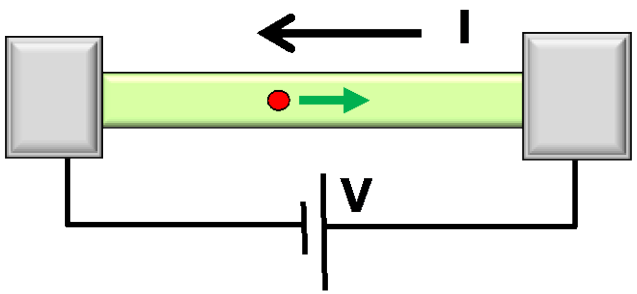
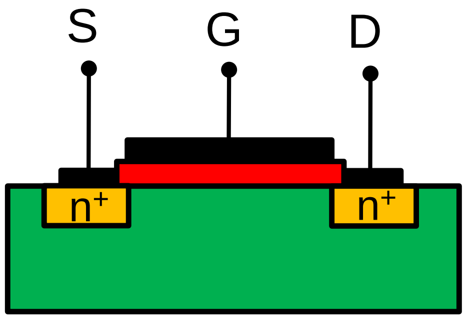


$$I = G \times V$$
$$= q \times n \times v \times A$$

charge density velocity area



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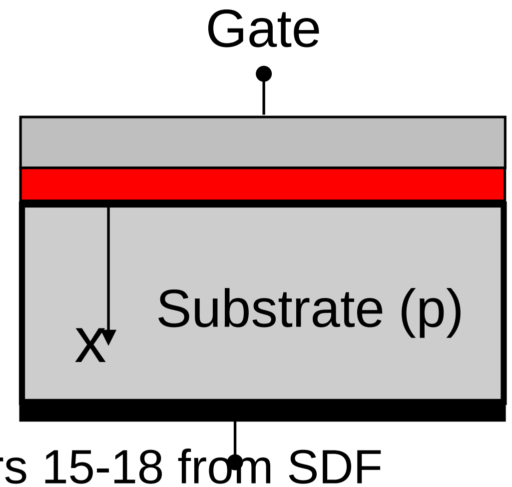

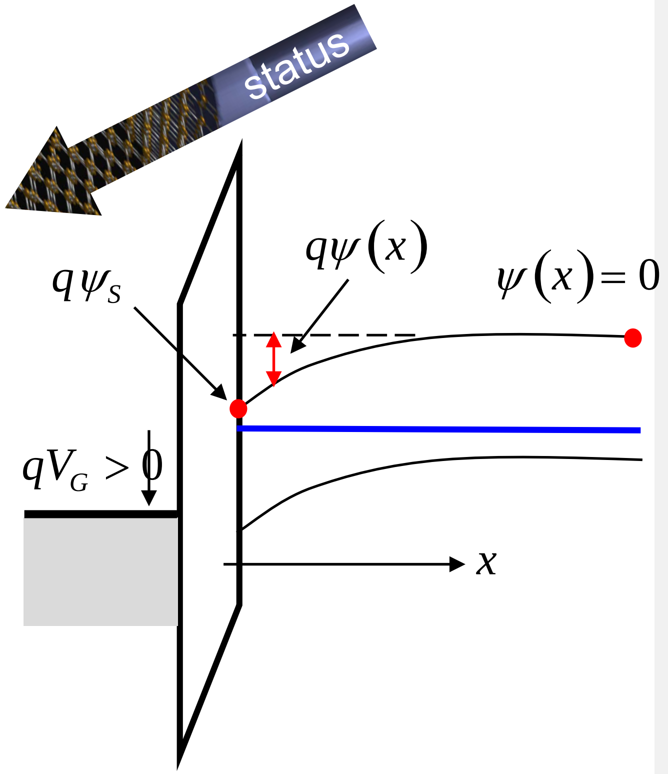

REF: Chapters 15-18 from SDF